The anticommutative Latin squares of order 8

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Abstract

Up to isomorphism, there are precisely two Latin squares of order 8 without 3×3 subpatterns of shape $\begin{bmatrix} .\,a\,b \\ a\,.\,c \\ b\,c\,. \end{bmatrix}$. There are no other such squares of order n for $3 \leq n \leq 11$.

1 Anticommutative Latin squares

Let us call a Latin square *anticommutative* when it does not contain 3×3 subpatterns of shape

$$\begin{bmatrix} .\,a\,b \\ a\,.\,c \\ b\,c\,. \end{bmatrix}.$$

Being anticommutative is a property of the main class: it is invariant for paratopy.

There are no anticommutative Latin squares of order n for $3 \leq n \leq 11$, $n \neq 8$. Up to isomorphism (paratopy), there are precisely two anticommutative Latin squares of order 8, let me call them 8A and 8B. Both have an automorphism group of order 384 (viewed as orthogonal array or transversal design) acting transitively on the 64 triples of the design. For square 8A the group permutes {rows, columns, symbols} as Sym(3). For square 8B the group can interchange rows and columns, but fixes the set of symbols.

Square 8A:

0	6	3	5	1	7	4	2
6	0	2	4	5	3	7	1
4	2	0	6	7	1	5	3
3	5	6	0	2	4	1	7
1	7	4	2	0	6	3	5
5	3	7	1	6	0	2	4
7	1	5	3	4	2	0	6
2	4	1	7	3	5	6	0

Square 8B:

0	1	2	5	6	3	4	7
1	0	3	4	7	2	5	6
7	2	0	6	4	5	3	1
4	3	1	0	2	6	7	5
5	6	7	1	0	4	2	3
2	7	5	3	1	0	6	4
3	4	6	7	5	1	0	2
6	5	4	2	3	7	1	0