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AFDELING ZUIVERE WISKUNDE
(DEPARTMENT OF PURE MATHEMATICS)

ZN 76/77 AUGUSTUS

A.E. BROUWER

THE t-DESIGNS WITH v < 18

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The t-designs with v < 18

by

A.E. Brouwer

Below we list the known and unknown t-designs without repeated blocks for $2 \le t < k \le \frac{1}{2}v$, $\lambda \le \frac{1}{2}\lambda_+ = \frac{1}{2}\binom{v-t}{k-t}$ and $v < 18$.

This table is a preliminary one - we hope to be able to extend it much further, at least up to v = 24 and perhaps even to v = 28. Comments, additions and corrections and also references for t-designs with $18 \le v \le 28$ are very much appreciated.

KEY WORDS & PHRASES: *t-design*

Table of t-designs without repeated blocks,
2 <= t < k <= v/2, λ <= λ+/2,
A.E. Brouwer, 770629

v	t	k	λ-	λ+	group	order	remarks
6	2	3	2	4	PSL5	60	ext of 1-(5,2,2) - unique
7	2	3	1	5	PGL3.2	168	λ=1: PG(2,2) (Fano) - unique
							λ=2: 2 disjoint copies of Fano - unique
8	2	3	6	6			
		4	3	15			see 3-(8,4,λ/3).
							λ=3: 4 nonisomorphic systems [NANDI]
8	3	4	1	5			ext of 2-(7,3,λ) - unique
							λ=1: Hadamard 3-design
9	2	3	1	7			λ=1: AG(2,3) unique - has (2 different)
							packings of 7 disjoint copies [KIRKMAN]
		4	3	21	C9	9	λ=3: 3 disjoint copies
							λ=3: 11 nonisomorphic systems [STANTON e.a.]
9	3	4	6	6			
10	2	3	2	8	C9p	9	λ=2
					C10	10	λ=4
		4	2	28			λ=2: 3 nonisomorphic systems [NANDI]
							λ>2: der of 3-(11,5,λ)
		5	4	56	C9p	9	λ=4: 6 disjoint copies
							λ=28: ext of 1-(9,4,28)
							λ=4: 21 nonisomorphic systems [van LINT e.a.]
10	3	4	1	7			der of 4-(11,5,λ)
							λ=1: MG(2,3) - unique
		5	3	21			ext of 2-(9,4,λ)
							λ=3: 7 nonisomorphic systems [GIBBONS]
10	4	5	6	6			
11	2	3	3	9			der of 3-(12,4,λ)
		4	6	36			der of 3-(12,5,λ)
		5	2	84	C11	11	λ=2: 2 disjoint copies - unique [HUSAIN]
							λ=4: 6 disjoint copies (5*(λ=4)+2*(λ=2))
					D11	22	all λ except λ=2,6,8
11	3	4	4	8			der of 4-(12,5,λ)
		5	2	28			λ=2: does not exist [DEHON, OBERSCHELP]
					C11	11	λ>2
					D11	22	λ>6
11	4	5	1	7	SA11	55	λ=1: 2 disjoint copies [WITT] - unique
					S4mS4	24	λ=3 [BROUWER]
12	2	3	2	10			der of 3-(13,4,λ)
		4	3	45	C11p	11	λ=3: at least 7 disjoint copies
		5	20	120			der of 3-(13,6,λ)
		6	5	210			see 3-(12,6,λ*2/5)
12	3	4	3	9			der of 4-(13,5,λ)
		5	6	36	SA11	55	λ=6,18
					S3*C4	24	λ=12
		6	2	84			ext of 2-(11,5,λ)
							λ=2: Hadamard 3-design
12	4	5	4	8	PGL5*E2	120	λ=4 [DENNISTON]
		6	2	28			λ=2 does not exist (see 3-(11,5,2))
							λ=4,8,12: see 5-(12,6,λ/4)
							λ=14: ext of 3-(11,5,14)
							λ=6,10: unknown
12	5	6	1	7			ext of 4-(11,5,λ)
							λ=1: unique [WITT]

13	2	3	1	11			der of 3-(14,4,λ)
							λ=1: 2 nonisomorphic systems with autgps of order 39, 6.
		4	1	55	C13	13	disjoint: 4*(λ=1)+8*(λ=5)
							λ=1: PG(2,3) - unique
		5	5	165			der of 3-(14,6,λ)
		6	5	330	GA13	156	disjoint: 1*(λ=5)+1*(λ=10)+3*(λ=15)+9*(λ=30)
13	3	4	2	10	GA13	156	λ=4
					D13	26	λ=2,4
		5	15	45			der of 4-(14,5,λ)
		6	20	120	GA13	156	
13	4	5	3	9			der of 5-(14,6,λ)
		6	6	36	SA13	78	λ=12 [KRAMER & MESNER]
							λ=6,18: unknown
13	5	6	4	8			λ=4: unknown
14	2	3	6	12	SA13p	78	
		4	6	66			der of 3-(15,5,λ)
		5	20	220			der of 3-(15,6,λ)
		6	15	495			der of 3-(15,7,λ)
		7	6	792	SA13p	78	disjoint: 2*(λ=6)+2*(λ=12)+2*(λ=18)+9*(λ=36)
14	3	4	1	11	SA7*E2	21	λ=1: 2 disjoint solutions [BAYS & DE WECK]
							4 nonisomorphic systems with autgps of order 42,14,6,6 [MENDELSOHN & HUNG]
					SA13p	78	λ=2,3,4,5
		5	5	55			λ=5: unknown
					PSL13	1092	λ=10,15,25
							λ=20: residual of 4-(15,5,4)
		6	5	165	GA13p	156	
		7	5	330			ext of 2-(13,6,λ)
14	4	5	10	10			
		6	15	45	GA13p	156	!
		7	20	120			λ=20: unknown
							λ=40: see 5-(14,7,12)
							λ=60: ext of 3-(13,6,60)
14	5	6	3	9	C13m3p	39	!
		7	6	36			λ=12: ext of 4-(13,6,12)
							λ=6,18: unknown
14	6	7	4	8			λ=4: unknown - ext of 5-(13,6,4)
15	2	3	1	13			der of 3-(16,4,λ)
							λ=1: 80 nonisomorphic systems [HALL & SWIFT]
		4	6	78			der of 3-(16,5,λ)
		5	2	286			λ=2 does not exist [HALL & CONNOR]
							λ>2: der of 3-(16,6,λ)
		6	5	715	PGL5b	120	disj: (λ=5)+(λ=10)+(λ=20)+(λ=40)+5*(λ=60)
							for λ>5 also res of 3-(16,6,λ*2/5)
		7	3	1287	PGL5b	120	all λ except λ=6,9
							λ=3: SBIBD - 5 nonisomorphic systems [CHAT & SHRIKHANDE]
							perm: 29 disjoint copies of λ=3
15	3	4	12	12			
		5	6	66	A6b	360	λ=12,30 !
					(SA7*C2)p	42	λ=6 !
							λ=18,24: see 4-(15,5,λ/6) !
		6	20	220			der of 4-(16,7,λ)
		7	15	495			λ=90,105,120,135,165,...,225: see 4-(15,7,λ/3)
					CA13pp	156	λ=15,30,45,60,75,90,105,120,135 !
					S6b	720	λ=60,75,90,135,150,165,180,225,240 [K & M]
15	4	5	1	11			λ=1: does not exist [MENDELSOHN & HUNG]
							λ=2,5: unknown
					PSL5b	60	λ=4 !
					(SA7*C2)p	42	λ=3 !
		6	5	55	PSL13pp	1092	λ=15 !
					SA13pp	78	λ=10 !
							λ=5,20,25: unknown

		7	5	165	GA13pp	156	λ=30,35,40,45,55,60,65,70,75 !
							λ=5,10,15,20,25,50,80: unknown
15	5	6	10	10			
		7	15	45			λ=15: unknown
15	6	7	3	9			λ=3: unknown
16	2	3	2	14			der of 3-(17,4,λ)
		4	1	91			der of 3-(17,5,λ) except perhaps for λ=3
					S4[C2]	384	λ=3
							λ=1: AG(2,4) - unique
		5	4	364			der of 3-(17,6,λ)
		6	1	1001	A4[C2]	192	λ>19 (disj: λ=2,6,8,2*12,15,2*24,32,36,6*48,96)
							λ=1: does not exist [FISCHER]
							λ=2: SBIBD - 3 nonisomorphic systems [HUSAIN]
							λ=3: nonresidual example [BHATTACHARYA]
							or residual of 2-(25,9,3)
							perm: using λ=2,3 one finds all λ<65
		7	14	2002			der of 3-(17,8,λ)
							λ>28: see 3-(16,7,λ*5/14)
		8	7	3003			see 3-(16,8,λ*3/7)
16	3	4	1	13			λ=4,5: der of 4-(17,5,λ)
					GF16am3	80	λ=1: 2 disjoint solutions - planes in AG(4,2)
					C3p[C2]	48	λ=1,3,4,6
		5	6	78			der of 4-(17,6,λ)
		6	2	286			λ=2: does not exist (see 2-(15,5,2))
							λ=6 or λ>10: der of 4-(17,7,λ)
					A6bp	360	λ=4,12,16,22,30,34,36,40,42,46,52,54,60 etc
					PSL7*E2	336	λ=8 and λ=12 and all λ>16 !
					GA16	240	λ=6,10,16,20,26,30,36,40 etc
		7	5	715			λ=15 or λ>25: res of 4-(17,7,λ*2/5)
					GA16	240	λ=15 and λ=20 and all λ>25
					PGL5bp	120	λ=15,25,30 etc
							λ=5,10: unknown
		8	3	1287			ext of 2-(15,7,λ)
							λ=3: Hadamard 3-design
16	4	5	12	12			
		6	6	66			λ=12,24: der of 5-(17,7,λ)
							λ=18: see 5-(16,6,3)
					GA16	240	λ=30 !
							λ=6: unknown
		7	20	220	GA16	240	λ=20,40,60,100 !
							λ=80: der of 5-(17,8,80)
		8	15	495			see 5-(16,8,λ/3) but:
					qma16	960	λ=75,240
							λ=15,30,45,60,150: unknown
16	5	6	1	11	PSL7*E2	336	λ=3 !
							λ=1: does not exist (see 4-(15,5,1))
							λ=2,4,5: unknown
		7	5	55	GA16	240	λ=15 !
							λ=5,10,20,25: unknown
		8	5	165			ext of 4-(15,7,λ), but:
							λ=5,10,15,20,25,50,80: unknown
16	6	7	10	10			
		8	15	45			λ=15: unknown
16	7	8	3	9			λ=3: unknown - ext of 6-(15,7,3)
17	2	3	3	15	GA17	272	disjoint: (λ=3)+2*(λ=6)
		4	3	105	GA17	272	disjoint: (λ=3)+3*(λ=6)+7*(λ=12)
		5	5	455	GA17	272	disjoint: (λ=5)+3*(λ=10)+21*(λ=20)
		6	15	1365	GA17	272	disjoint: 7*(λ=15)+42*(λ=30)
		7	21	3003	GA17	272	disjoint: 7*(λ=21)+68*(λ=42)
		8	7	5005	GA17	272	disjoint: (λ=7)+(λ=14)+[illegible]*(λ=28)+85*(λ=56)
17	3	4	2	14	GA17	272	disjoint: (λ=2)+3*(λ=4)
		5	1	91	GA17	272	all λ except λ=1,3
					SA17	136	λ=1: 2 disjoint solutions
							λ=3: unknown

		6	4	364	GA17	272	
		7	7	1001	GA17	272	all λ except λ=7
							λ=7: unknown
		8	14	2002	GA17	272	
17	4	5	1	13	GA17	272	disjoint: 2*(λ=4)+(λ=5) [KRAMER]
							λ=1,2,3,6: unknown
							λ=1: if existing then autgp is e17 [DENNISTON]
		6	6	78			der of 5-(18,7,λ) [KRAMER]
		7	2	286	GA17	272	λ>10 !
					PGL16	4080	λ=6,40,46,60,66,100,106,110,116 [KRAMER]
							λ=2: does not exist (see 2-(15,5,2))
							λ=4,8,10: unknown
		8	5	715	PGL16	4080	λ=15,80,95,100,115,120,135,160,175,200,215, 220,235,240,255,260,275,280,295,320,335, 340,355 [KRAMER]
					GA17	272	λ=30,...,55,70,...,95,120,...,130 !
							λ=5: does not exist [DELSARTE]
							some other values of λ are known:
							res of 5-(18,8,λ*2/5): λ=110,150,155,165 etc
							der of 5-(18,9,λ): λ=140 etc
							λ=10,15,20,25,55,60,65,85,105,145 etc: unknown
17	5	6	12	12			
		7	6	66	GA17	272	λ=12,24 !
							λ=6,18,30: unknown
		8	20	220	PGL16	4080	λ=80 [KRAMER]
							all other λ are unknown
17	6	7	1	11			λ=1: does not exist (see 4-(15,5,1))
							all other λ are unknown
		8	5	55			all λ unknown
17	7	8	10	10			
18	2	3	2	16	C17o	17	λ=2: 6 disjoint copies

Explanation:

abbreviations:

der	derived design
ext	extension
res	residual design
aut	automorphism
gp	group
disj	disjoint
...	including all intermediate admissable values of lambda
BIBD	balanced incomplete block design
SBIBD	symmetric BIBD

group denotations:

E#	identity (as permutation group on # elements) - order 1
C#	cyclic group - order #
D#	dihedral group - order 2 * #
S#	symmetric group - order #!
A#	alternating group - order #! / 2
GA#	general affine group - order # * (# - 1)
GAd.q	general affine group on AG(d,q)
SA#	special affine group - order # * (# - 1) / 2
PGL#	general projective group - order (# + 1) * # * (# - 1)
PGLd.q	general projective group on PG(d,q)
PSL#	special projective group - order (# + 1) * # * (# - 1) / 2
S4mS4	representation of S4 of degree 11 - order 24 (permute points, bipartitions and triples of a 4-set)
C13m3	subgroup of GA13 generated by x -> x+1 and x -> 3x - order 39
GF16am3	subgroup of GA16 generated by x -> x+1 and x -> ax, where a ** 5 = 1 - order 80

Now let G and H be permutation groups acting on sets X and Y, resp. with degrees m = |X| and n = |Y|. Then we define:

G+H	direct sum acting on X+Y - degree m+n - order |G| * |H|
G*H	direct product acting on X*Y - degree m*n - order |G| * |H|
Gp	representation of the abstract group G with degree m+1 obtained by adding a point fixed by all group elements (i.e. G+E1) - order |G|
Gb	representation of the abstract group G with degree m*(m-1)/2 acting on the unordered pairs of X - order |G|
G[H]	group of degree n ** m and order |G| * (|H| ** m) obtained by letting G permute the coordinates of the direct product of m copies of H
G~H	group of degree n * m and order |G| * (|H| ** m) obtained by letting G permute the summands of the direct sum of m copies of H (wreath product)

geometry denotations:

PG(d,q)	projective geometry of dimension d over GF(q)
AG(d,q)	affine geometry of dimension d over GF(q)
MG(d,q)	moebius geometry of dim d over GF(q) (d=2: inversive plane)

symbols:

v	number of points of the design
b	number of blocks of the design
k	blocksize
t	strength
λ	lambda: each t-set is covered by λ blocks
λ-	smallest lambda satisfying the divisibility conditions
λ+	lambda of the trivial design (each k-set is a block)

remarks:

ext	a t-(2*k+1,k,λ) design is extendable to a (t+1)-(2*k+2,k+1,λ) design when t is even, or when t is odd and λ = λ+/2 [ALLTOP]
der	derive from a t-(v,k,λ) design a (t-1)-(v-1,k-1,λ) design
res	the residual of a t-(v,k,λ) design is a (t-1)-(v-1,k,λ*(v-t+1)/(k-t+1)-λ) design
group	usually the group given is not the full automorphism group of the design (e.g. we give S^A11 for the unique 4-(11,5,1) which has M11 as full automorphism group)
perm	if a t-(v,k,λ) exists then it can be chosen disjoint from a given collection of d k-sets when v! > b * d * k! * (v-k)! i.e. when d < λ+/λ.

Hadamard 3-design: a 3-(4r,2r,r-1) exists whenever a Hadamard matrix of order 4r exists.

! this design (or: the design with smallest λ among these) seems to be new.

List of unknown designs with small v
A.E. Brouwer, 770828

v < 11: all known

v = 11: 3-(11,5,2) does not exist [DEHON e.a.]
all others are known

v = 12: 4-(12,6,2) does not exist
4-(12,6,λ) are unknown for λ = 6,10
all others are known

v = 13: 4-(13,6,λ) are unknown for λ = 6,18
5-(13,6,4) is unknown (and extendable to 6-(14,7,4) [ALLTOP])
all others are known

v = 14: unknown:
3-(14,5,5), 4-(14,7,20), 5-(14,7,λ) λ=6,18, 6-(14,7,4).

v = 15: nonexistent:
2-(15,5,2) [HALL & CONNOR]
4-(15,5,1) [MENDELSOHN & HUNG]
unknown:
4-(15,5,λ) λ = 2,5
4-(15,6,λ) λ = 5,20,25
4-(15,7,λ) λ = 5,10,15,20,25,50,80
5-(15,7,15)
6-(15,7,3)

v = 16: nonexistent:
2-(16,6,1) [FISCHER]
3-(16,6,2)
5-(16,6,1)
unknown:
3-(16,7,λ) λ = 5,10
4-(16,6,6)
4-(16,8,λ) λ = 15,30,45,60,150
5-(16,6,λ) λ = 2,4,5
5-(16,7,λ) λ = 5,10,20,25
5-(16,8,λ) λ = 5,10,15,20,25,50,80
6-(16,8,15)
7-(16,8,3)

v = 17: nonexistent:
4-(17,7,2)
4-(17,8,5) [DELSARTE]
6-(17,7,1)
note:
4-(17,5,1) cannot have automorphisms [DENNISTON]
unknown:
3-(17,5,3)
3-(17,7,7)
4-(17,5,λ) λ = 1,2,3,6
4-(17,7,λ) λ = 4,8,10
4-(17,8,λ) λ = 5,10,20,25,60,65,105,145,170,180,185,210,...,330
t-(17,k,λ) t > 4
but known:
5-(17,7,λ) λ = 12,24 [BROUWER]
5-(17,8,80) [KRAMER]

v = 18: nonexistent:
4-(18,6,1) [WITT]
5-(18,8,2)
5-(18,9,5)

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