Proposed content course on "Local convergence, giants and small-world properties in configuration models"

(based on Random Graphs and Complex Networks Volume 2)

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In this document, we propose an outline for a course on connectivity properties in configuration models, based on Random Graphs and Complex Networks Volume 2. We focus on the local limit, the giant component and small- and ultra-small-world properties in configuration models.

(1) Start with a recap of the configuration model, by discussing some of their properties in [II, Chapter 1]. Particularly [II, Section 1.3.3] is then convenient. Should you wish to brush up more details on configuration models, you can discuss the basics in [I, Chapter 7]. Should you be interested in the related model of uniform random graphs with prescribed degrees, then also [II, Section 1.3.4] is useful.

(2) Discuss the theory of local convergence in [II, Chapter 2]. Cover the basic notation of rooted graphs and their metric properties in [II, Section 2.2], the notion of local convergence of deterministic graphs in [II, Section 2.3], and that of random graphs in [II, Section 2.4].

(3) Continue with consequences of local convergence in [II, Section 2.5]. Here, you can pick those consequences that you like best.

(4) Discuss the relation between the size of the giant and the local limit in [II, Section 2.6]. This is useful as intuition, and the proof is relatively straightforward.

(5) Discuss local convergence of configuration models in [II, Section 4.2]. Time permitting, you can choose to include the analysis for uniform random graphs with prescribed degrees in [II, Section 4.2.3] and [II, Section 4.2.4].

(6) Treat the size of the giant in configuration models in [II, Section 4.3]. There are two proofs, a 'giant is almost local proof in [II, Section 4.3.1], and a continuous-time exploration proof in [II, Section 4.3.2]. Take your pick, or do both.

(7) Treat the connectivity of configuration models in [II, Section 4.4], since it is interesting and at the same time, the proof is quite simple. It suffices to treat [II, Theorem 4.24], whose proof is quite simple. Time permitting, you can treat some of the later extensions.

(8) Close the course with the small-world properties of the configuration model in [II, Chapter 7]. [II, Section 7.2] gives an outline of the results, which are then proved in [II, Section 7.3]. The lower bounds on distances are the simplest, and can be found in [II, Section 7.3.2]. The upper bounds on distances in [II, Section 7.3.3] either rely on the 'giant is almost local' proof, or in second moment methods for the existence of paths. Should you choose to treat the latter (which is not necessary), it may be convenient to briefly discuss the analysis for rank-1 inhomogeneous random graphs in [II, Section 6.5.1]. The ultra-small properties of the configuration model are proved in [II, Section 7.3.4], which is quite short, and [II, Section 7.3.5], which proves an upper bound on the diameter of the core of high-degree vertices.

(9) Time permitting, you could discuss extensions to generation growth of infinite-mean branching processes in [II, Section 7.4], the diameter of configuration models in [II, Section 7.5], related distance results in [II, Section 7.6], and/or directed configuration models in [II, Section 9.2.2].