Exercises Algorithms for Model Checking (Part I)

1 CTL*, CTL and LTL

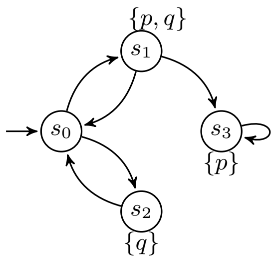

Figure 1:

1. For each of the CTL* formulae below, indicate whether it is (syntactically) a formula in LTL and/or CTL or neither. Determine for each formula in which states of the Kripke Structure of Fig. 1 it holds.

   (a) p,

   (b) $\mathsf{E}\ [q\ \mathsf{R}\ p]$,

   (c) $\mathsf{E}\ \mathsf{F}\ \mathsf{G}\ p$,

   (d) $\mathsf{A}\ \mathsf{G}\ \mathsf{F}\ p$,

   (e) $\mathsf{A}\ \mathsf{G}\ \mathsf{E}\ \mathsf{F}\ p$,

   (f) $\mathsf{A}\ \mathsf{G}\ \mathsf{F}\ (p \wedge \mathsf{X}\ q)$,

   (g) $\mathsf{A}\ \mathsf{G}\ (\neg q \vee \mathsf{F}\ p)$,

   (h) $\mathsf{A}\ ((\mathsf{G}\ p) \vee (\mathsf{F}\ q))$

2. For each pair of CTL* formulae below, if possible, give a Kripke Structure in which both are valid, a Kripke Structure in which both are not valid, and a Kripke Structure in which only one of them is valid.

   (a) p and $\mathsf{A}\ \mathsf{F}\ p$

   (b) $\mathsf{A}\ \mathsf{F}\ \mathsf{A}\ \mathsf{G}\ p$ and $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ p$

   (c) $\mathsf{A}\ \mathsf{F}\ \mathsf{A}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{F}\ \mathsf{X}\ p$

   (d) $\mathsf{A}\ \mathsf{X}\ \mathsf{E}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{X}\ \mathsf{X}\ p$

   (e) $\mathsf{A}\ \mathsf{X}\ \mathsf{A}\ \mathsf{X}\ p$ and $\mathsf{A}\ \mathsf{X}\ \mathsf{X}\ p$

   (f) $\mathsf{A}\ [p\ \mathsf{U}\ q]$ and $\mathsf{A}\ [\neg q\ \mathsf{R}\ \neg p]$

3. Consider LTL, CTL and CTL*. State for each of the claims below whether they hold or not.

(a) Every CTL^* formula is equivalent to either an LTL formula or a CTL formula.

(b) The language LTL is more expressive than CTL.

(c) The language CTL is more expressive than LTL.

4. Express that along all paths, proposition p holds infinitely often and $\neg p$ holds infinitely often.

5. Express that along all paths, proposition p holds infinitely often and $\neg p$ only holds finitely often.

6. Prove using the semantics of CTL^*, or disprove using a Kripke Structure, the following equivalences:

   (a) $\mathsf{A}\ [\phi\ \mathsf{U}\ \psi] \equiv \neg(\mathsf{E}\ [\neg\psi\ \mathsf{U}\ \neg(\phi \vee \psi)] \vee \mathsf{E}\ \mathsf{G}\ \neg\psi)$

   (b) $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ p \equiv \mathsf{A}\ \mathsf{G}\ \mathsf{F}\ p$

2 Model Checking CTL and Fair CTL

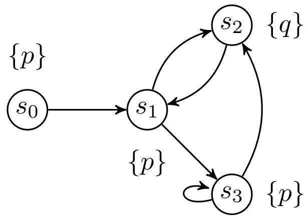

Figure 2:

1. For each of the CTL formulae below, if possible, draw a Kripke Structure in which the formula holds, a Kripke Structure in which it does not hold, but in which it does hold fairly with an appropriate fairness constraint. Also provide this fairness constraint.

   (a) $\mathsf{A}\ \mathsf{G}\ \mathsf{A}\ \mathsf{F}\ (\neg p \vee q)$

   (b) $q \wedge \mathsf{A}\ \mathsf{F}\ q \wedge \neg(\mathsf{E}\ [\neg q\ \mathsf{R}\ \neg p])$

   (c) $\neg A\mathsf{F}\ p \vee \mathsf{E}\ \mathsf{G}\ (\neg p \vee q)$

   (d) $(p \vee \mathsf{A}\ \mathsf{F}\ p) \wedge \neg\mathsf{E}\ \mathsf{G}\ p$

2. Determine for each of the following CTL formulae in which states of the Kripke Structure of Fig. 1 it holds using the symbolic model checking algorithm for CTL, using explicit set notation to represent sets of states, rather than BDDs.

   (a) p,

   (b) $\mathsf{E}\ [q\ \mathsf{R}\ p]$,

   (c) $\mathsf{A}\ \mathsf{G}\ \mathsf{E}\ \mathsf{F}\ p$,

   (d) $\mathsf{A}\ \mathsf{G}\ p \vee \mathsf{A}\ \mathsf{F}\ q$

   (e) $\mathsf{A}\ \mathsf{F}\ q$

   (f) $\mathsf{A}\ [q\ \mathsf{R}\ p]$

3. Extend the Kripke Structure of Fig. 1 with the Fairness constraints $F = \{\ \{s_1\}, \{s_2\}\ \}$. In which states do the formulae of exercise 2 *fairly* hold? Repeat the exercise using fairness constraint $F = \{\ \{s_3\}\ \}$.

4. Answer Exercises 2 and 3 for the Kripke Structure in Fig. 2 instead of the Kripke Structure of Fig. 1.

5. Prove that $\mathsf{A}\ \mathsf{F}\ f = \mu Z.f \cup \mathsf{A}\ \mathsf{X}\ Z$.

3 Counterexamples and Witnesses

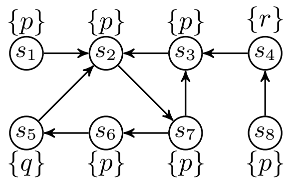

Figure 3:

1. Consider the Kripke Structure in Fig. 3.

   (a) Fairness constraint: $\neg r$ and q. Check that $s_1 \models_F \mathsf{E}\ \mathsf{G}\ (p \vee q)$.

   (b) Construct a witness for $s_1 \models_F \mathsf{E}\ \mathsf{G}\ (p \vee q)$, using the techniques for symbolic model checking.