Algorithms for Model Checking (2IW50)

Lecture 8

Equivalences and Pre-orders: State Space Reduction and Preservation of Properties

Chapter 11, 11.1

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Outline

Equivalences

Pre-orders

Bisimulation Reduction

Summarising

Equivalences

Complexity of model checking arises from:

- State space explosion: the state space is usually much larger than the specification
- Expressive logics have complex model checking algorithms

Ways to deal with the state space explosion:

- equivalence reduction: remove states with identical potentials from a state space
- on-the-fly: integrate the generation and verification phases, to prune the state space
- symbolic model checking: represent sets of states by clever data structures
- partial-order reduction: ignore some executions, because they are covered by others
- abstraction: remove details by working on conservative over-approximation

Equivalences

- A state space reduction reduces model checking complexity.
- Of course, the reduced state space must preserve (an interesting class of) temporal properties.
- This is often characterised by an equivalence relation on Kripke Structures:
  - reduction must yield an 'equivalent" model.
  - "equivalent" models must satisfy the same properties.
- Different instances of this scheme:
  - trace equivalence preserves LTL formulae.
  - strong bisimulation preserves CTL* (and μ-calculus) formulae.
  - simulation preserves ACTL* (and existential μ-calculus) formulae.
  - branching bisimulation preserves CTL*-X formulae.

Equivalences

Let two Kripke Structures over AP be given:

- $M = \langle S, R, S_0, L \rangle$ and
- $M' = \langle S', R', S'_0, L' \rangle$

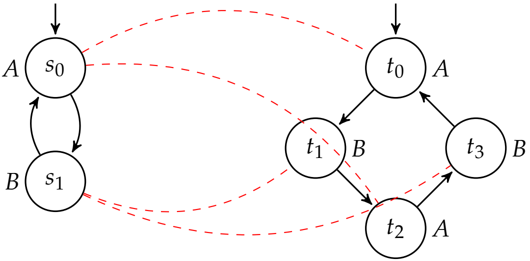
Definition (Strong Bisimulation)

A relation $B \subseteq S \times S'$ is a strong bisimulation relation (also *zig-zag* relation) iff for every $s \in S$ and $s' \in S'$ with sBs':

- $L(s) = L'(s')$
- for all $s_1 \in S$, if sRs_1, then there exists $s'_1 \in S'$ such that $s'R's'_1$ and $s_1Bs'_1$
- for all $s'_1 \in S'$, if $s'R's'_1$, then there exists $s_1 \in S$ such that sRs_1 and $s_1Bs'_1$

Equivalences

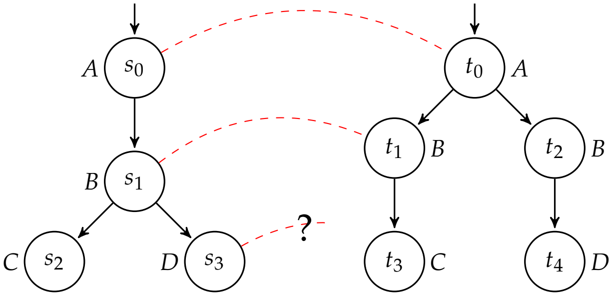
Example



- unwinding and duplication preserves bisimulation
- Sensitive to the moment of choice

Equivalences

Example



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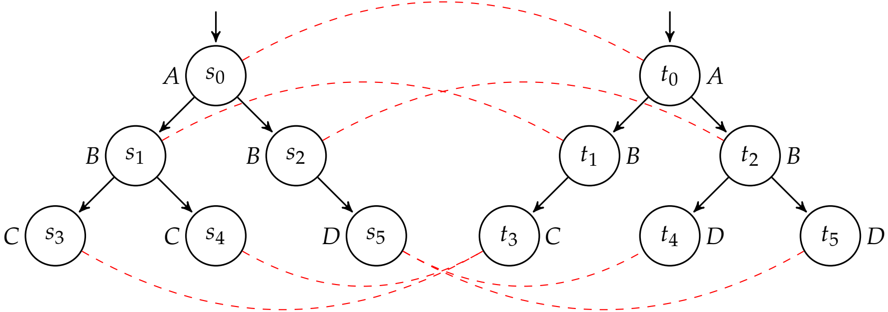
Equivalences

Example



- unwinding and duplication preserves bisimulation
- Sensitive to the moment of choice

Equivalences

Let two Kripke Structures over AP be given:

- $M = \langle S, R, S_0, L\rangle$ and
- $M' = \langle S', R', S_0', L'\rangle$

Definition (bisimilarity)

Two states $s \in S$ and $s' \in S'$ are bisimilar, if for some bisimulation relation B, sBs'.
The Kripke Structures M and M' are bisimilar (notation: $M \equiv M'$) iff there exists a bisimulation relation B, "containing initial states", i.e.:

- $\forall s_0 \in S_0 \; \exists s_0' \in S_0' \; : s_0 B s_0'$
- $\forall s_0' \in S_0' \; \exists s_0 \in S_0 \; : s_0 B s_0'$

Note:

- bisimilarity is an equivalence relation
- the union of bisimulation relations is again a bisimulation relation
- "bisimilarity" itself is the greatest bisimulation relation

Equivalences

Strong bisimulation preserves CTL*:

- Recall the CTL* semantics:
  - $M, s \models f$: state formula f holds in state s,
  - $M, \pi \models f$: path formula f holds along path π.
- Recall that $M \models f$ iff for all $s_0 \in S_0$, $M, s_0 \models f$.

Theorem (14)

If $M \equiv M'$ (i.e. M and M' are bisimilar), then for every CTL* state formula f:

$$M \models f \quad \textit{iff} \quad M' \models f$$

Practical consequence: In order to check $M \models f$, it is safe and sufficient to:

1. Reduce M to M' modulo bisimilarity,
2. Check whether $M' \models f$.

Equivalences

Proof sketch:

Given a relation B, we define that path π corresponds to path π' iff: $\forall i.\ \pi(i)\ B\ \pi'(i)$

Lemma (31)

If B is a bisimulation relation and $s\ B\ s'$ (correction to Lemma 31), then for every $\pi \in \mathbf{path}(s)$ there exists a corresponding path $\pi' \in \mathbf{path}(s')$ (and vice versa).

Next, with structural induction on CTL* formula f one can show: if s and s' are bisimilar and π and π' correspond, then:

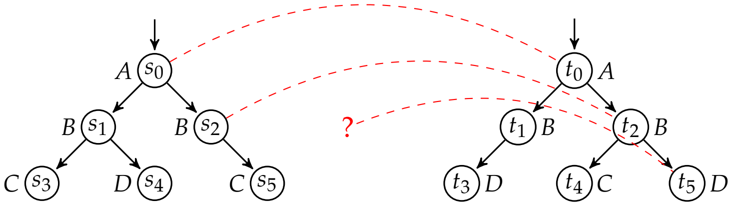
1. $s \models f$ if and only if $s' \models f$
2. $\pi \models f$ if and only if $\pi' \models f$

From this, the theorem follows:
for all M, M' and CTL* formulae f: if $M \equiv M'$ then $M \models f$ iff $M' \models f$.

Equivalences

Theorem (reverse)

If $M \not\equiv M'$ then there exists a formula f in **CTL** *, such that $M \models f$ and $M' \not\models f$.*



- Note that both systems have the same paths.
- There is no bisimulation relation between these two systems containing the initial states.
- Indeed, the following CTL formula holds in (the initial state of) the right system, but not on the left: $\mathsf{A}\,\mathsf{X}\,(b \wedge \mathsf{E}\,\mathsf{X}\,d)$
- We will see later that using E is essential.

Outline

Equivalences

Pre-orders

Bisimulation Reduction

Summarising

Pre-orders

- bisimilar models have the same behaviour, so they make true exactly the same properties.
- Idea: If we allow to really forget information, we may:
  - reduce the state space further, but:
  - preserve only a smaller class of formulae.
- We say that system M' simulates system M if M' has at least the behaviour of M.

Let two Kripke Structures be given:

- $M = \langle AP, S, R, S_0, L \rangle$ and
- $M' = \langle AP', S', R', S'_0, L' \rangle$, with $AP \subseteq AP'$.

Definition (Simulation Relation)

A relation $H \subseteq S \times S'$ is a simulation relation iff for every $s \in S$ and $s' \in S'$ with $s\ H\ s'$:

- $L(s) \cap AP' = L'(s')$
- for all s_1, if $s\ R\ s_1$, then there exists s'_1 such that $s' R' s'_1$ and $s_1\ H\ s'_1$.

Pre-orders

Definition (Simulation)

M' simulates M (written: $M \sqsubseteq M'$) iff there exists a simulation relation H, such that

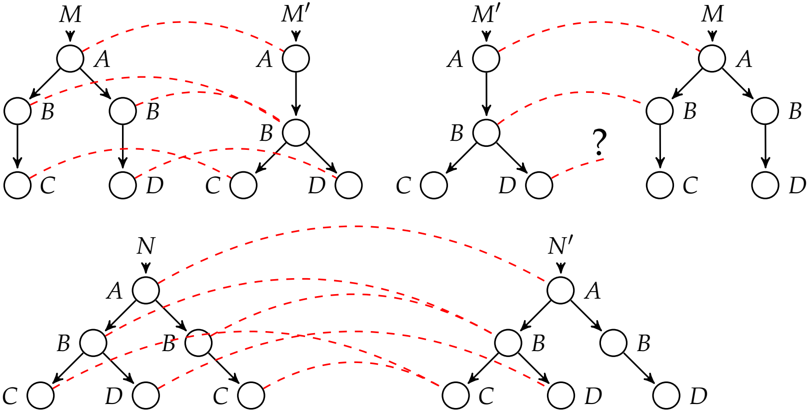
$$\forall s_0 \in S_0.\ \exists s_0' \in S_0'.\ s_0\ H\ s_0'$$

This defines an equivalence relation as follows: $M \sim M'$ iff $M \sqsubseteq M'$ and $M' \sqsubseteq M$.

Note:

- $\sqsubseteq$ is a pre-order on Kripke Structures (i.e. it is reflexive and transitive, but not necessarily symmetric).
- Warning:
  - it is possible that $M \sim M'$ but still $M \not\equiv M'$
  - In words: if two systems simulate each other, they need not be bisimilar.
  - Intuitively: the two simulations may use a different H, while a bisimulation requires one B.

Pre-orders



- $M \sqsubseteq M'$ but not $M' \sqsubseteq M$;
- $N \sim N'$ but $N \not\equiv N'$.

Pre-orders

Definition (ACTL*)

ACTL* (see p.31) is the fragment of CTL* with only universal path quantifiers, no existential path quantifiers.

Note:

- This only makes sense for formulae in positive normal form, i.e. negations only occur directly in front of atomic propositions.
- Examples: $\mathsf{A}\,\mathsf{F}\,Gp, \mathsf{A}\,\mathsf{G}\,(p \to \mathsf{A}\,\mathsf{X}\,q)$ are in ACTL*, but $\mathsf{A}\,\mathsf{G}\,(p \to \mathsf{E}\,\mathsf{X}\,q)$ is not.
  Careful: $(\mathsf{A}\,\mathsf{G}\,p) \to (\mathsf{A}\,\mathsf{G}\,q)$ is not in ACTL*, because actually:

$$\begin{aligned}(\mathsf{A}\,\mathsf{G}\,p) \to (\mathsf{A}\,\mathsf{G}\,q) &\equiv \neg(\mathsf{A}\,\mathsf{G}\,p) \vee (\mathsf{A}\,\mathsf{G}\,q)\\ &\equiv (\mathsf{E}\,\mathsf{F}\,\neg p) \vee (\mathsf{A}\,\mathsf{G}\,q)\end{aligned}$$

Pre-orders

Simulation preserves ACTL*:

Theorem
If $M \sqsubseteq M'$ (i.e. M' simulates M), then for every ACTL state formula f over AP':*

$$\text{if } M' \models f \quad \text{then} \quad M \models f$$

Practical consequence: In order to check $M \models f$, it is safe to find an approximation M' with $M \sqsubseteq M'$ and check that $M' \models f$.

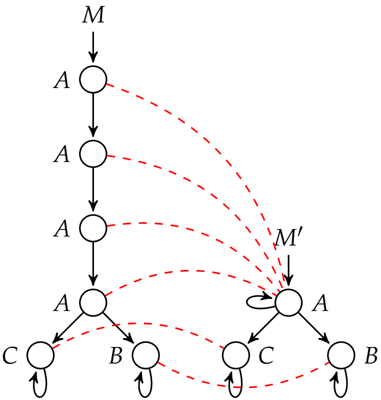
However: if $M' \not\models f$, we obtain no information about $M \models f$ — it may or may not hold.

In the previous example, we had: $N \sim N'$ but $N \not\equiv N'$. Hence:

- N and N' satisfy the same ACTL* formulae
- N and N' do not satisfy the same CTL formulae
- They can only be distinguished using operator E .

Pre-orders

Example



- Observe that $M \sqsubseteq M'$ with H indicated left.
- Note that $M' \models \mathsf{A}\,\mathsf{G}\,\neg d$ and hence $M \models \mathsf{A}\,\mathsf{G}\,\neg d$.
- Note that $M' \not\models \mathsf{A}\,\mathsf{F}\,(b \vee c)$, but actually $M \models \mathsf{A}\,\mathsf{F}\,(b \vee c)$. This shows that some information is really lost.
- Note: $M \models \mathsf{A}\,\mathsf{X}\,a$ but $M' \not\models \mathsf{A}\,\mathsf{X}\,a$ (wrong direction) conclusion: $M' \not\sqsubseteq M$.
- Note: $M' \models \mathsf{E}\,\mathsf{X}\,b$, but $M \not\models \mathsf{E}\,\mathsf{X}\,b$ (not in ACTL*).

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Bisimulation Reduction

Computing Bisimulation Equivalence:

Let two Kripke Structures be given:

- $M = \langle AP, S, R, S_0, L \rangle$ and
- $M' = \langle AP, S', R, S'_0, L' \rangle$.

Define a sequence of relations $s\ B_i^*\ s'$ iff s and s' cannot be distinguished within i steps:

- $s\ B_0^*\ s'$ if and only if $L(s) = L'(s)$.
- $s\ B_{n+1}^* s'$ if and only if:
  1. $s\ B_n^*\ s'$, and
  2. $\forall s_1$ with $R(s, s_1)$, $\exists s'_1$ with $s'\ R'\ s'_1$ and $s_1\ B_n^*\ s'_1$.
  3. $\forall s'_1$ with $R'(s', s'_1)$, $\exists s_1$ with $s\ R\ s_1$ and $s_1\ B_n^*\ s'_1$.
- Let $B^* := \bigcap_i B_i^*$

Clearly, $B_i^* \supseteq B_{i+1}^*$, so B^* can be computed by fixed point iteration.

Actually, this can be implemented symbolically by OBDDs

Bisimulation Reduction

- Actually: B^* is the largest bisimulation between M and M'.
- So: if s and s' are bisimilar, then $s\ B^*\ s'$.
- To test if $M \equiv M'$: check if for each $s_0 \in S_0$ there exists an $s'_0 \in S'_0$ such that $s_0\ B^*\ s'_0$.
- By carefully splitting equivalence classes, the procedure can run in $\mathcal{O}(|R| \times \log(|S|))$ time (Paige-Tarjan).
- Similar ideas apply to checking $M \sqsubseteq M'$.

The algorithm can be modified for state space reduction as follows:

- The equivalence classes of B^* form the states of the reduced state space (minimal modulo bisimulation).
- The transitions between two classes are derived from the transitions between elements of these classes.

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Summarising

- Bisimulation is an equivalence relation.
- Bisimulation preserves CTL^* formulae.
- Simulation is a pre-order.
- Simulation preserves ACTL^* formulae only, and only in one direction.
- Simulation allows for more reduction but sometimes crucial information is lost.
- Bisimulation and Simulation reduction can be computed in polynomial time.

Possible improvement: Instead of:

1. generating state space
2. reducing state space
3. model checking reduced state space,

it would be better to generate a smaller state space immediately.