Examination Algorithms for Model Checking

02 July, 2008, 09:00 – 12:00

Important notes:

- The exam consists of four questions.
- Weighting: 1: **30**, 2: **15**, 3: **30**, 4: **25**.
- Carefully read the questions. The book, the course notes and other written material may be used during this examination.

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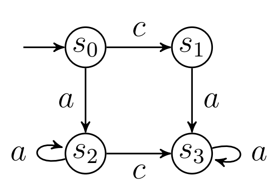
1. Consider the following Labelled Transition System, with action alphabet $\{a, c\}$.



Consider the following propositional μ-Calculus formula: $\mu X.\ (\nu Y.\ ([c]X \wedge [a]Y))$

(a) Encode the problem of $s_0 \models \mu X.\ (\nu Y.\ ([c]X \wedge [a]Y))$ into a Boolean Equation System. Note: solving the BES is not required.

(b) Use the Emerson-Lei algorithm to determine in which states of the LTS the above formula holds. Make sure to clearly mark every approximation step.

2. Consider the four temporal logics LTL, CTL, ACTL and the *alternation-free* μ-calculus (i.e. formulae with dependent alternation depth 1). State for each of the five below claims whether they hold or not. Motivate your answer by providing counterexamples or a formal justification.

(a) Every LTL formula is a valid ACTL formula.

(b) Every property described by an ACTL formula can be described by an alternation-free μ-calculus formula.

(c) The language LTL is more expressive than CTL.

(d) Every property described by an alternation-free μ-calculus formula can be described by a CTL formula.

(e) The lower bound time complexity of the model checking problem for the alternation-free μ-calculus is exponential in the number of states.

3. Consider the process described by LPE C below, where div is the division operator on natural numbers, $*$ is the multiplication operator and odd and even indicate whether a natural number is odd and even, respectively.

$$\begin{array}{rcll} C(n{:}\mathbb{N}) & = & \mathsf{odd}(n) \wedge n > 1 & \longrightarrow \mathsf{up} \cdot C(1 + 3 * n) \\ & + & \mathsf{even}(n) & \longrightarrow \mathsf{down} \cdot C(n \;\mathsf{div}\; 2) \\ & + & n = 1 & \longrightarrow \mathsf{level} \cdot C(n) \end{array}$$

Let ϕ be the following first-order modal μ-calculus formula:

$$\mu X.\ (\langle \mathsf{true} \rangle \mathsf{true} \wedge [\neg \mathsf{level}] X)$$

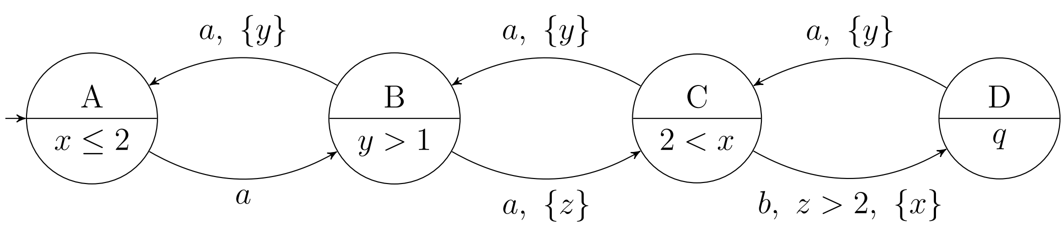
(a) What does property ϕ express in natural language?

(b) Verify whether the PBES given below can be the result (up to simplifications that preserve logical equivalence) of the transformation of ϕ and C to a PBES, i.e. the transformation $\mathbf{E}(\phi)$. If yes, clearly indicate how the (sub)expressions in the PBES relate to the (sub)expressions in ϕ. If no, indicate a (sub)expression of ϕ that demonstrates the mismatch.

$$\begin{array}{rcl} \mu \tilde{X}(n{:}\mathbb{N}) & = & ((\mathsf{odd}(n) \wedge n > 1) \implies \tilde{X}(1 + 3 * n)) \\ & \wedge & (\mathsf{even}(n) \implies \tilde{X}(n \;\mathsf{div}\; 2)) \\ & \wedge & ((\mathsf{odd}(n) \wedge n > 1) \vee \mathsf{even}(n) \vee n = 1) \end{array}$$

(c) Can instantiation of the PBES be used to obtain a BES that codes the solution to $\tilde{X}(3)$? If so, perform the instantiation and compute the value to $\tilde{X}(3)$ by solving the BES. Clearly mark every step you take in solving the BES. If instantiation cannot be used, give a formal explanation why it cannot be used.

4. Consider the Timed Automaton with locations A, B, C and D, three clocks x, y, z, two actions a and b and atomic proposition q. Location A is the initial location.



Answer the following questions:

(a) Is the Timed Automaton non-Zeno? If so, give a proof. If not, give a Zeno path.

(b) Can the Timed Automaton timelock? If so, give a path that leads to the timelock. If not, give a brief (at most ten lines) textual motivation for your answer.

(c) Does the formula $\mathsf{E}\ \mathsf{F}\ q$ hold? Motivate your answer.

Good luck. □