

# Module Architecture Control using Relation Algebra

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# Questions

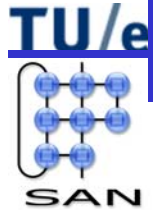
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- Development artefacts of a system?
- How to maintain consistency:
  - between these artefacts (inter)?
  - between elements of an artefact (intra)?
- This lecture:
  - consistency between architecture and implementation
- SAN:
  - consistency of designs [Lange et al 05]
  - generalization [Muskens et al 05].



# Goals

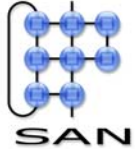


- Student understands:
  - the need for module architecture control;
  - how module architecture control can be performed;
  - the basics of relation algebra.
- Student can apply relation algebra on a simple example.



# Overview

TU/e



- Context and motivation
  - Software architecture – recap
  - Application domain
- Module architecture notions
- Relation algebra
- Verification
- Conclusion
- References



# Overview

TU/e



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# Software architecture – recap



Stakeholders

- **End-User:**
  - behavior, performance, security, reliability
- **Customers:**
  - low cost, timely delivery
- **Product-Management:**
  - features, short time to market, low cost, parity with products
- **Development:**
  - low cost, employability
- **Maintenance:**
  - modifiability

# Software architecture – recap

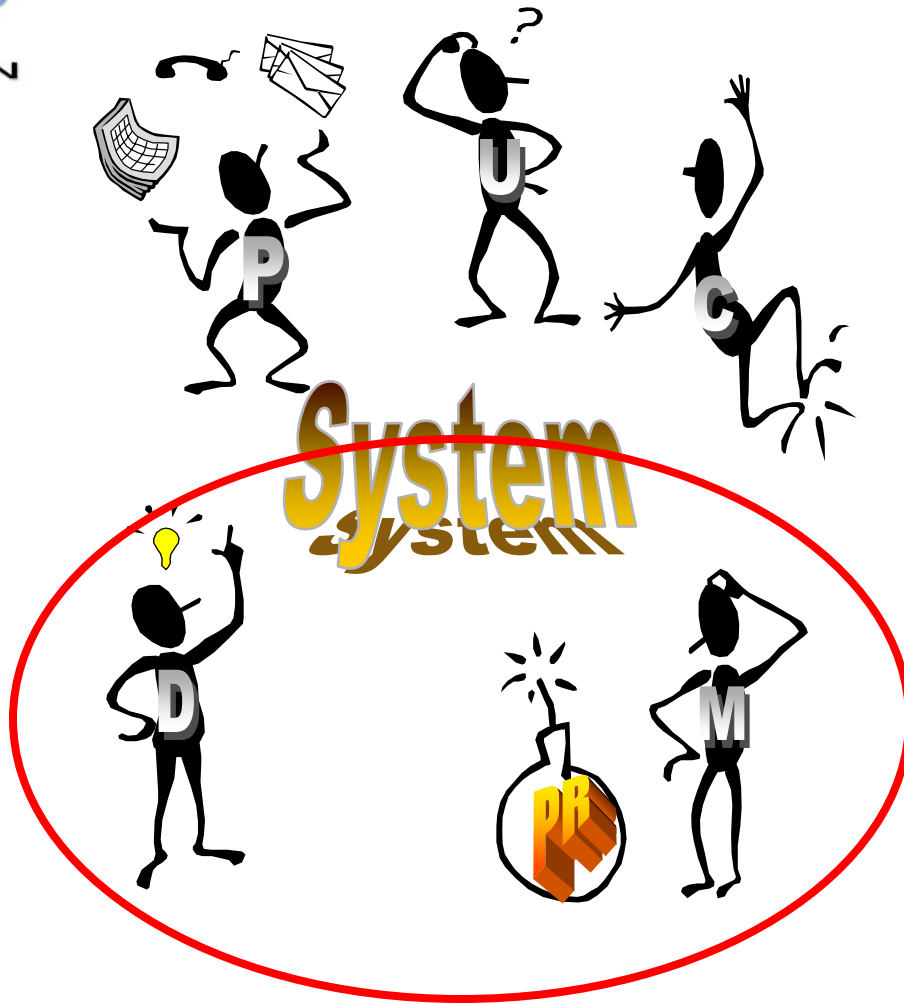


Stakeholders

- **End-User:**
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  - modifiability

Product view

# Software architecture – recap



Stakeholders

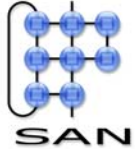
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  - modifiability

Development view





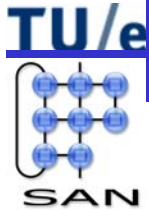
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# Software architecture – recap

- **Software architecture:**
  - earliest artifact
  - means for mutual communication
  - enables analysis of concerns
  - manifests concerns as system qualities
- **Software architecture is vital !**

[Bass et al 1995]



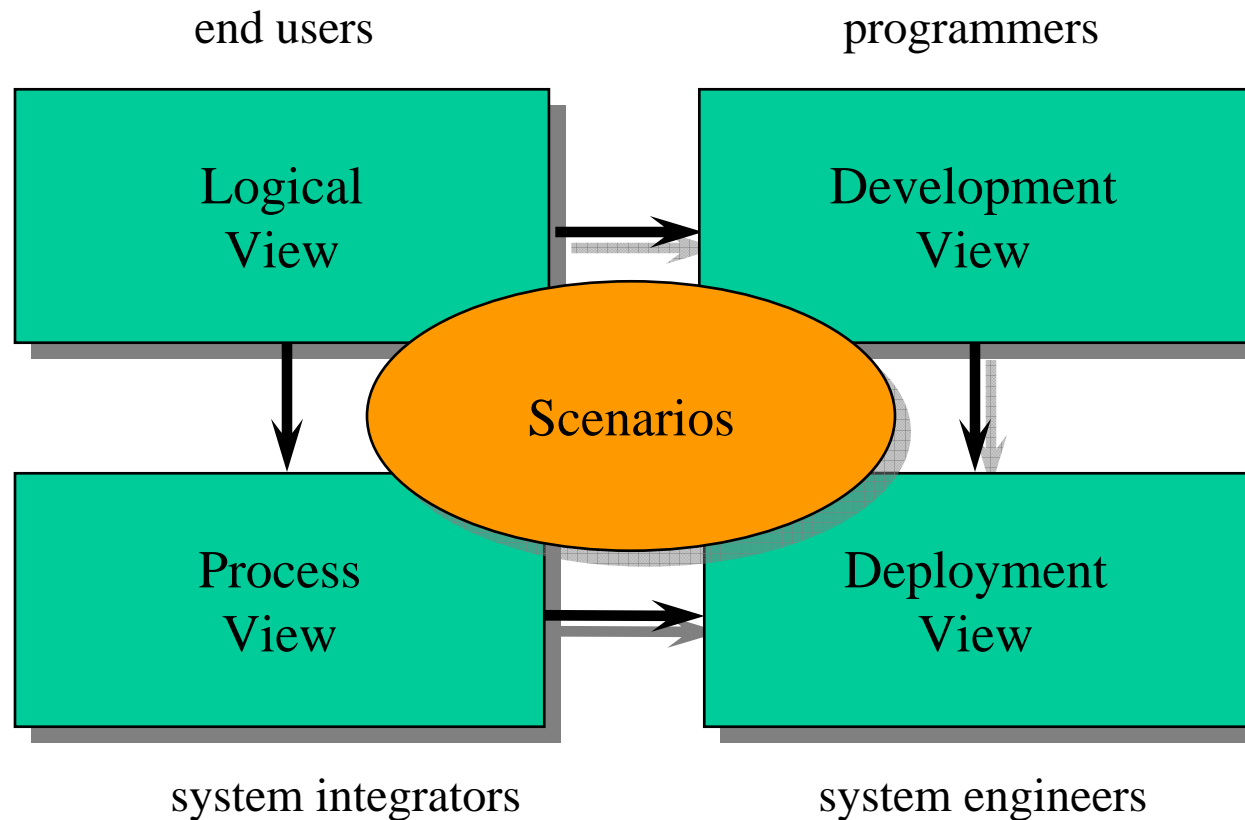
# Software architecture – recap

## Need for (system) architecture

- *If a project has not achieved a system architecture, including its rationale, the project should not proceed to full-scale system development. Specifying the architecture as a deliverable enables its use throughout the **development** and **maintenance** process.*

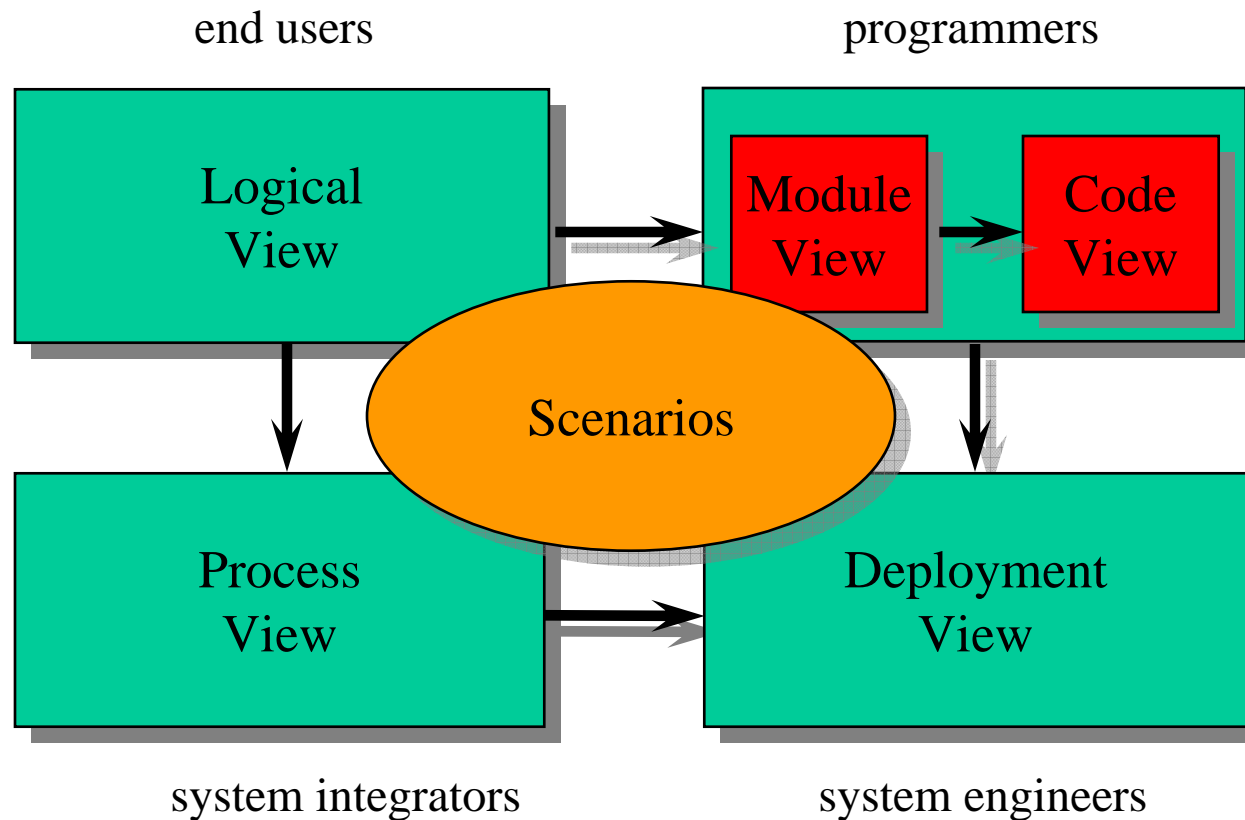
[Boehm 1995]

# Software architecture – recap



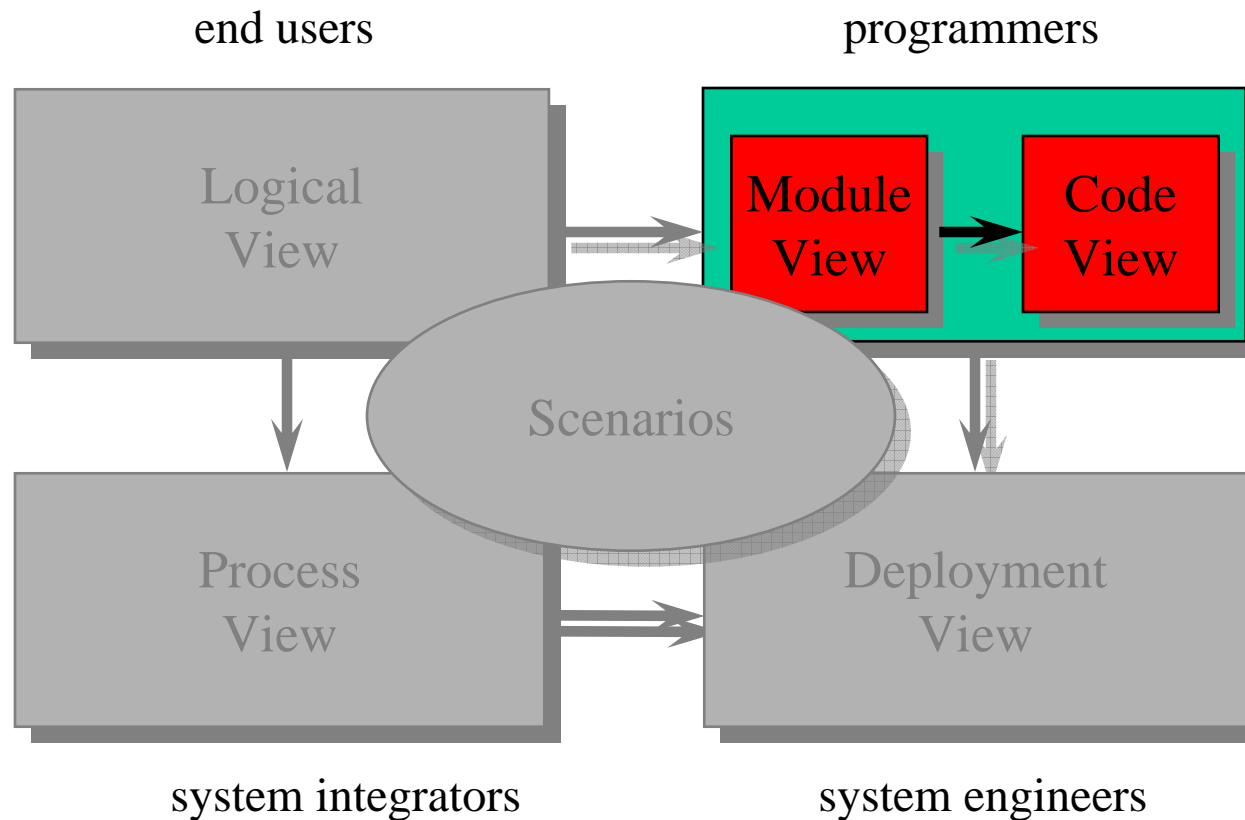
**4+1 View Model [Kruchten 95]**

# Software architecture – recap

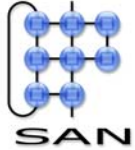


## 4+1 View Model Revisited

# Software architecture – recap

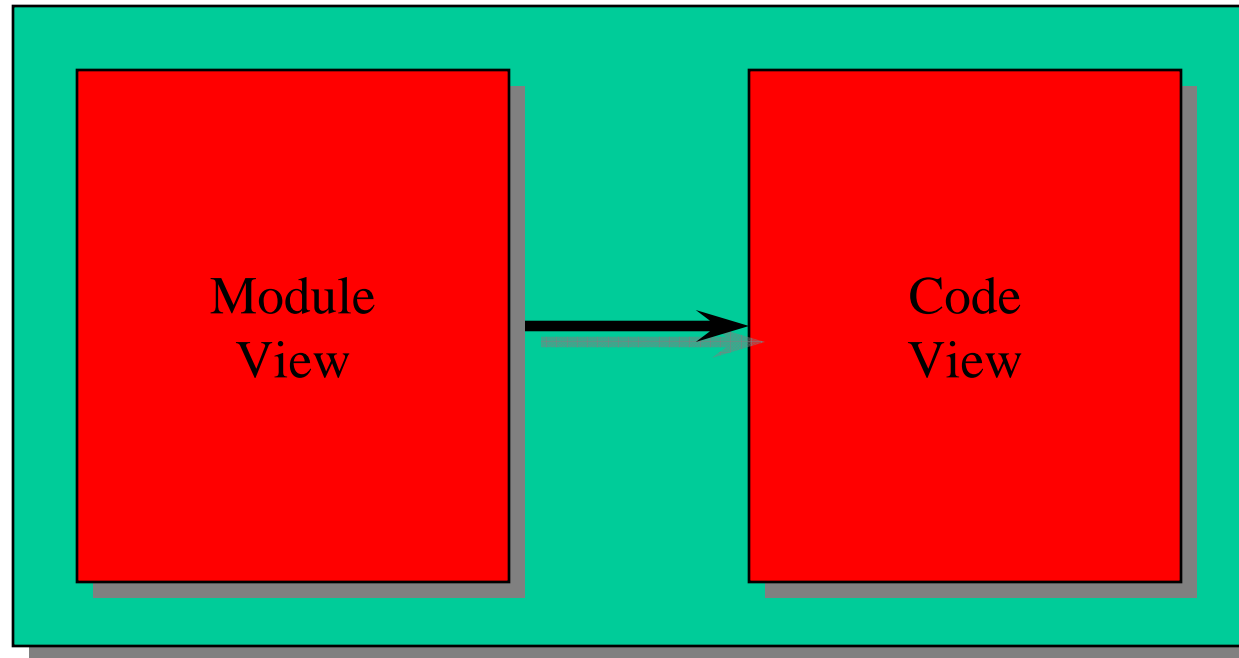


## 4+1 View Model Revisited



# Software architecture – recap

programmers



## Development View



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# Software architecture – recap

- A software architecture characterization:
  - Components (or [architectural] entities);
  - Connections;
  - Constraints.
- Module view:
  - System, Subsystems, Components;
  - Part-of relation and uses relation;
  - Layering, orthogonally.
- Code view:
  - Directories, Files;
  - Directory structure, location of files, and include relation;
  - File name conventions and file length constraints.



# Overview

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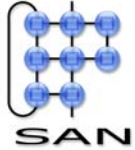


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# Application domain

- Telecommunications domain
- SOPHO: Philips' family of PBXs
  - 100 - 1M telephony lines
  - origin dating back to early 1980's
  - economic lifetime ~ 15 years
  - maintenance obligations  $\geq$  10 years
  - 5 K files, 2.5 MLOC in C++
  - successful  $\Rightarrow$  asset
    - $\Rightarrow$  careful to maintain this legacy



# Application domain

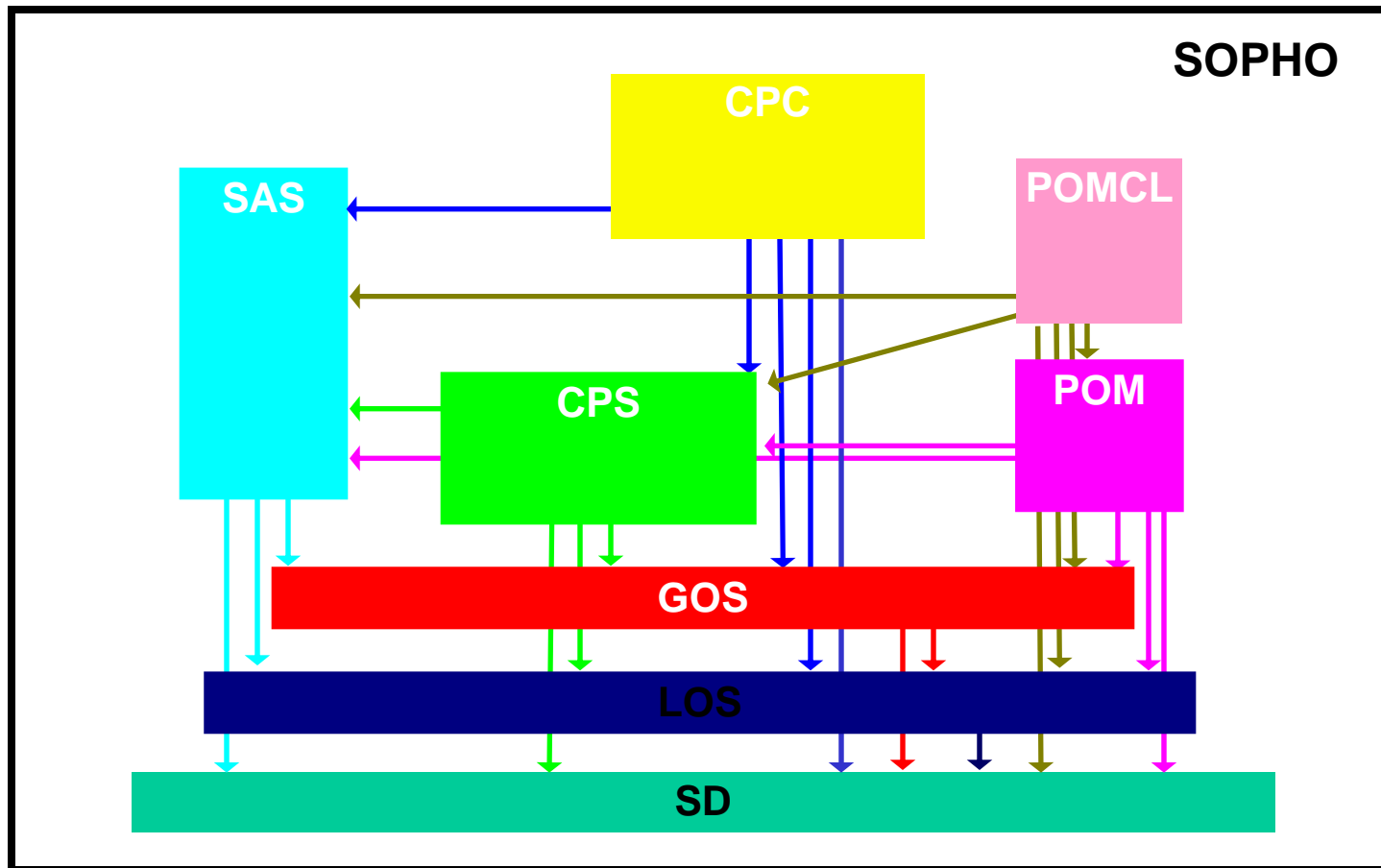
- Complexity of legacy systems
  - hard to understand (e.g. size);
  - documentation out-of-date;
  - gap between intrinsic and experienced complexity.
- Architecture vital, but not maintained ....:
  - recovery;
  - analysis;
  - verification and control;
  - (improvements).
- Need for architectural support !



# Application domain

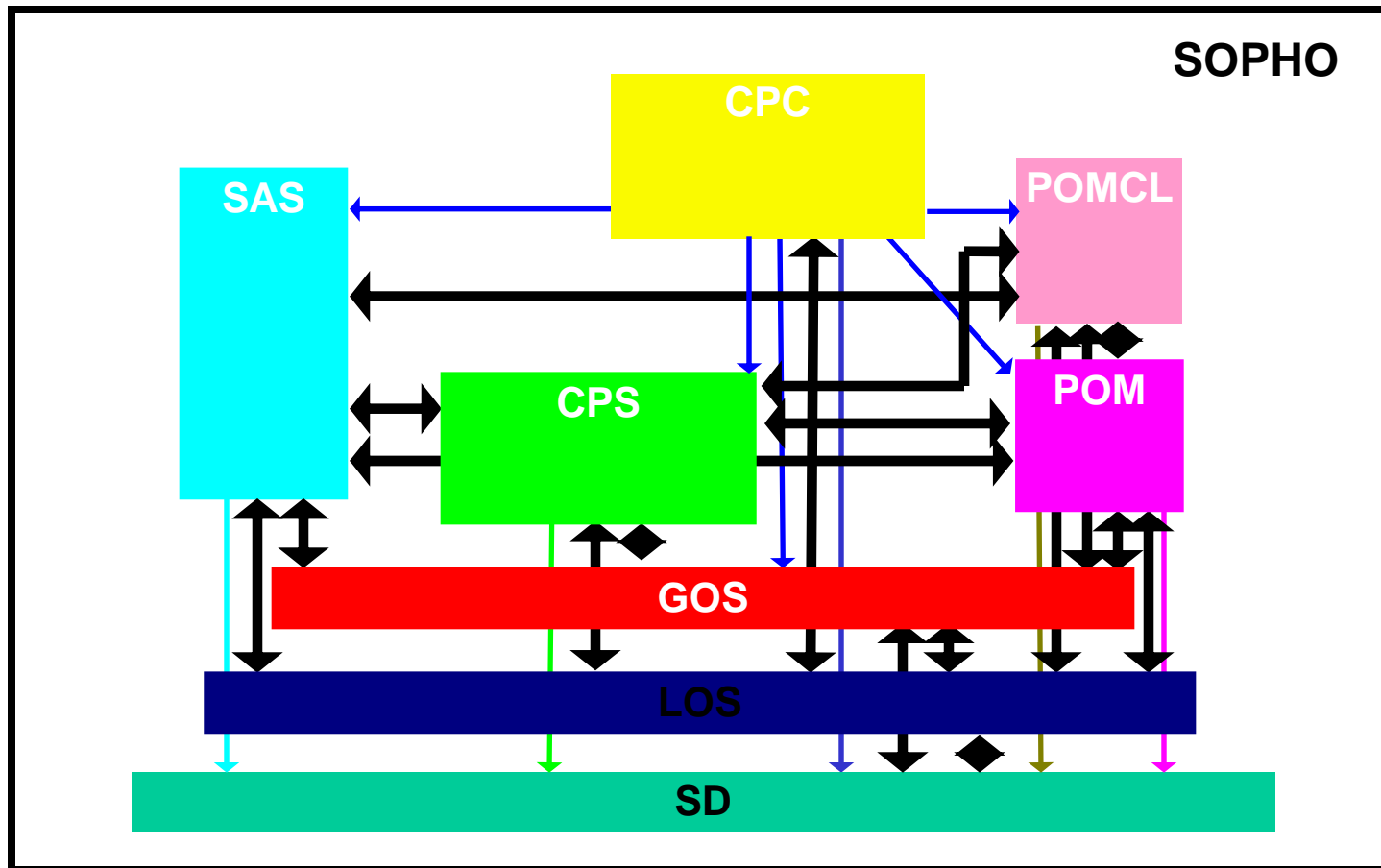
- Characteristics (development view):
  - module view
    - 8 K architectural entities;
    - organised in an **unbalanced** tree, depth 5 – 12;
    - **layered** system, consisting of 8 subsystems.
  - code view
    - 1 directory with 5 K files, and 2.5 MLOC in C++;
    - 35 K include statements;
    - File names based on “12 NCs”,  
file length varies from 100 to 20 K lines.

# Application domain



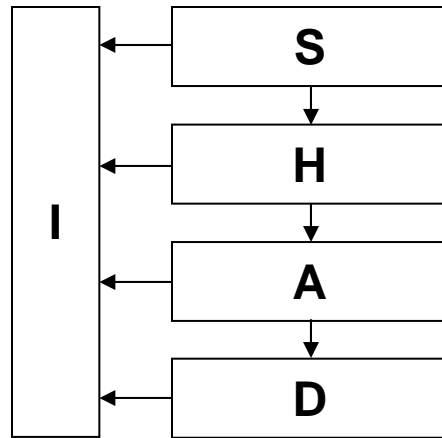
**“Intended” module architecture  
(documentation + software architects)**

# Application domain

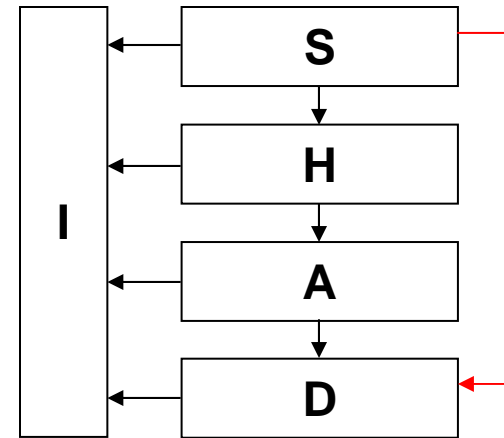


**“Derived” module architecture  
(extracted from the implementation)**

# Conformance



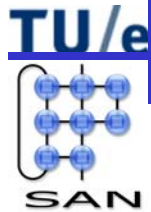
Intended



Extracted

Causes when “intended” and “extracted” differ:

1. “intended” is wrong (e.g. out-of-date): improve;
2. “extracted” is wrong: improve;
3. implementation is optimized for, e.g., speed  $\Rightarrow$  refinement.



# Application domain

- Ensure conformance to an architecture !
  - Keep the architecture up-to-date
- Approach using relation algebra (RPA):
  - Represent the “intended” architecture in RPA.
  - Extract the “derived” architecture from the implementation, and represent in RPA.
  - Express “conformance” in RPA.
  - Ensure conformance by means of verification (using RPA) and improvements (i.e. control).



# Overview

TU/e



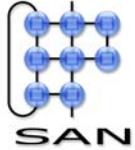
- Context and motivation
- **Module architecture notions**
- Relation algebra
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# Overview

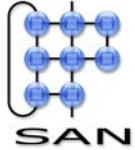
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- Context and motivation
- Module architecture notions
  - Module diagram
  - Decomposition tree
  - Lifting
  - Hiding
  - Lowering
  - Weights
- Relation algebra
- Verification
- Conclusion
- References



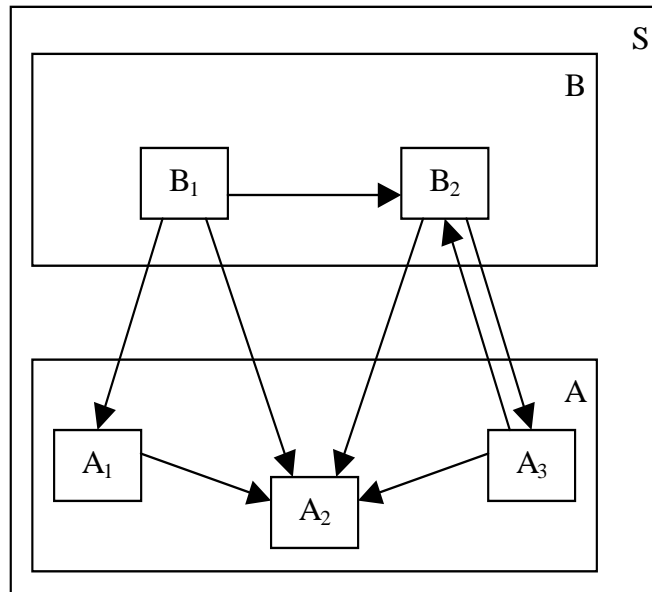
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# Module architecture notions

- Module diagram
- Decomposition tree
- Lifting
- Hiding
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- Weights

# Module diagram

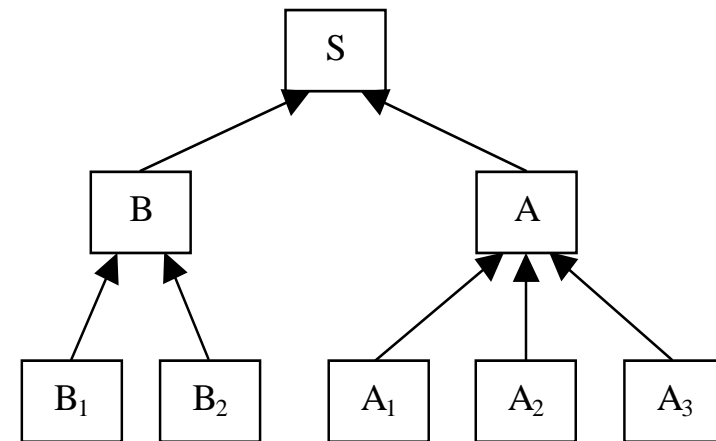
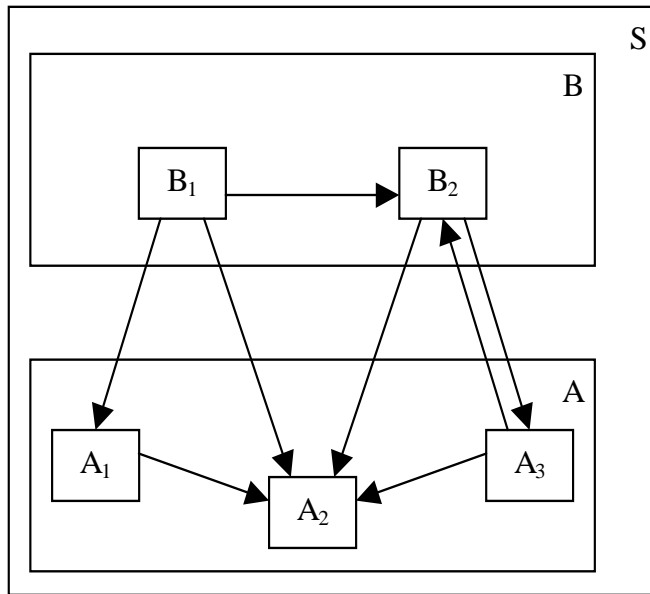


- Visualises a system's architecture
- “Boxes-in-boxes” representation
- Boxes represent entities
- Arrows represent dependencies

System S is *not* layered

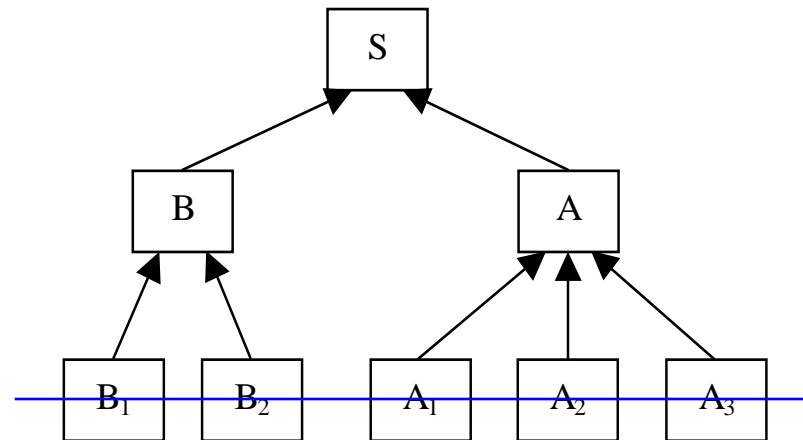
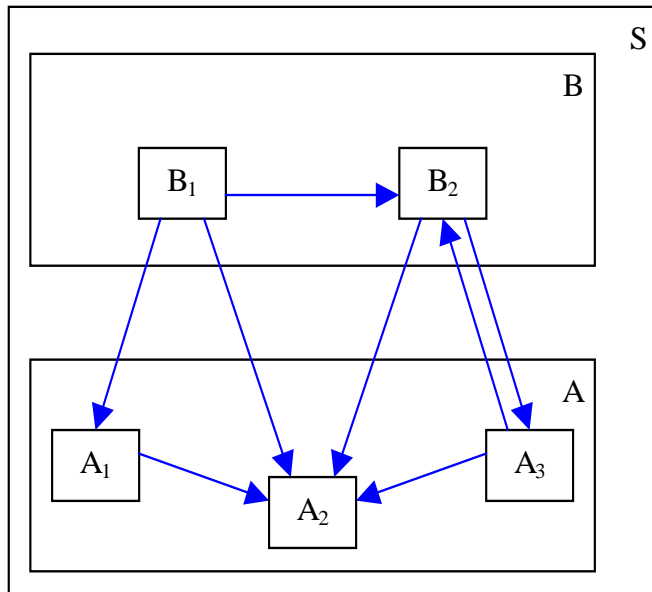


# Decomposition tree



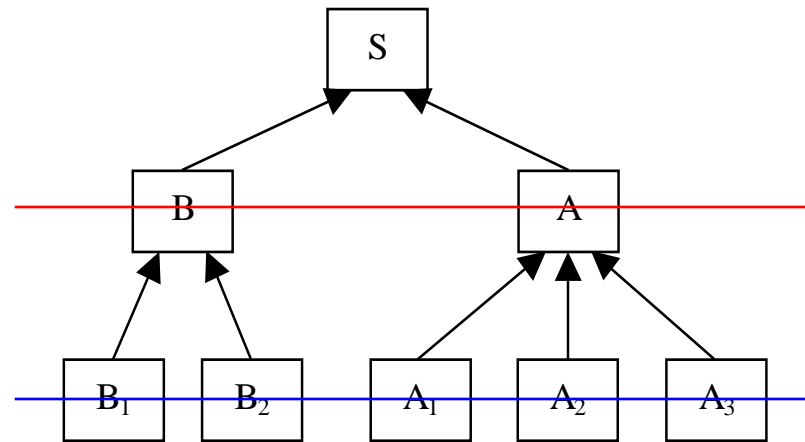
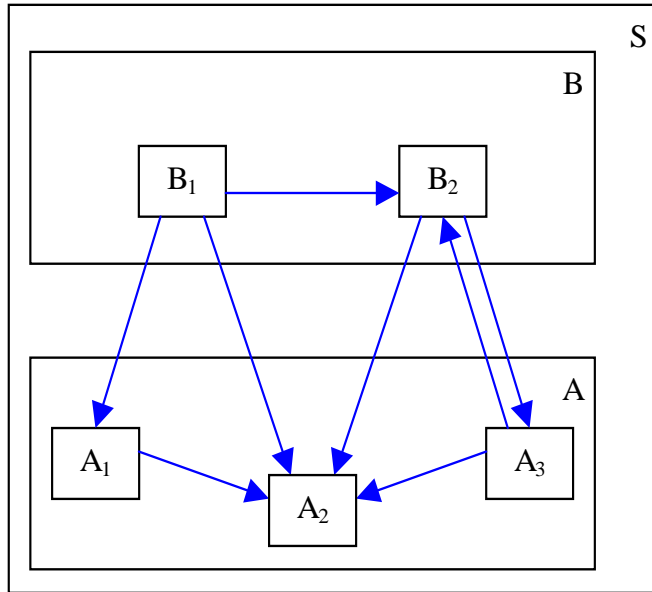
System S is *balanced*, and  
the decomposition tree has *3 levels*

# Lifting (1)



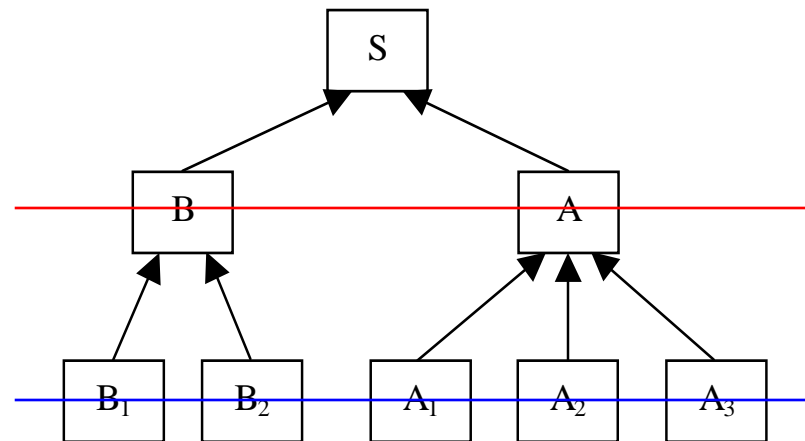
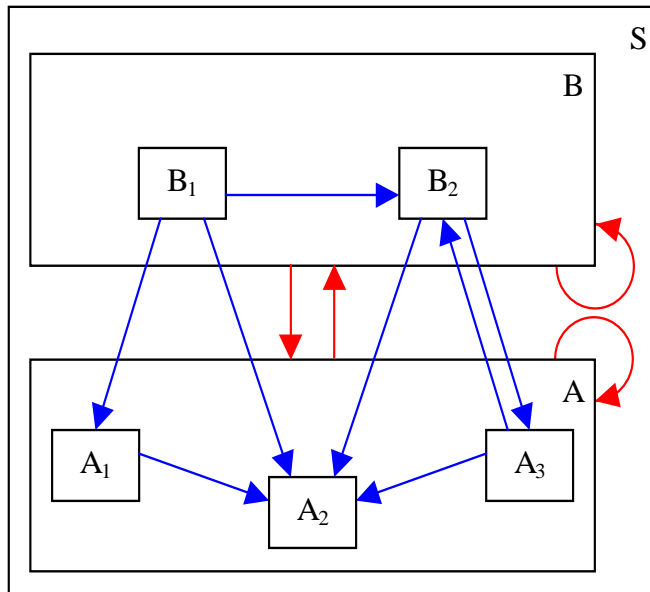
Uses relation corresponds with a level (*tree-cut*)

# Lifting (1)



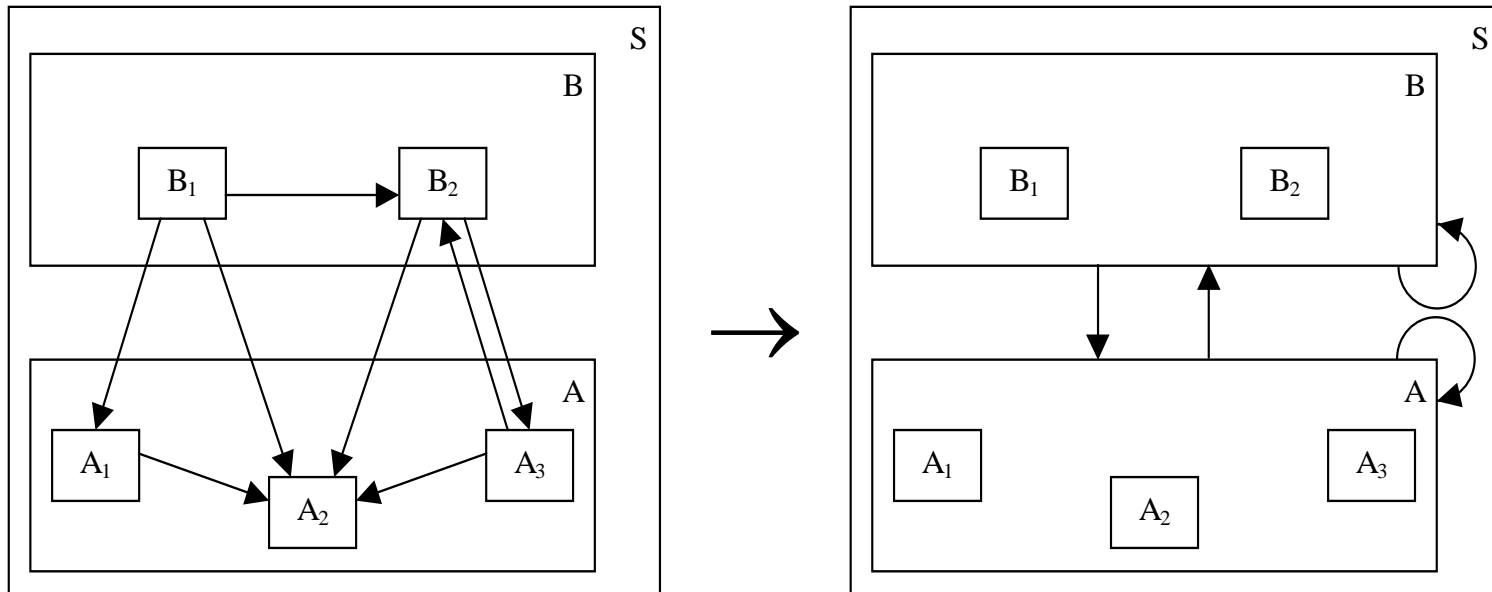
Transform a relation to a higher level.

# Lifting (1)



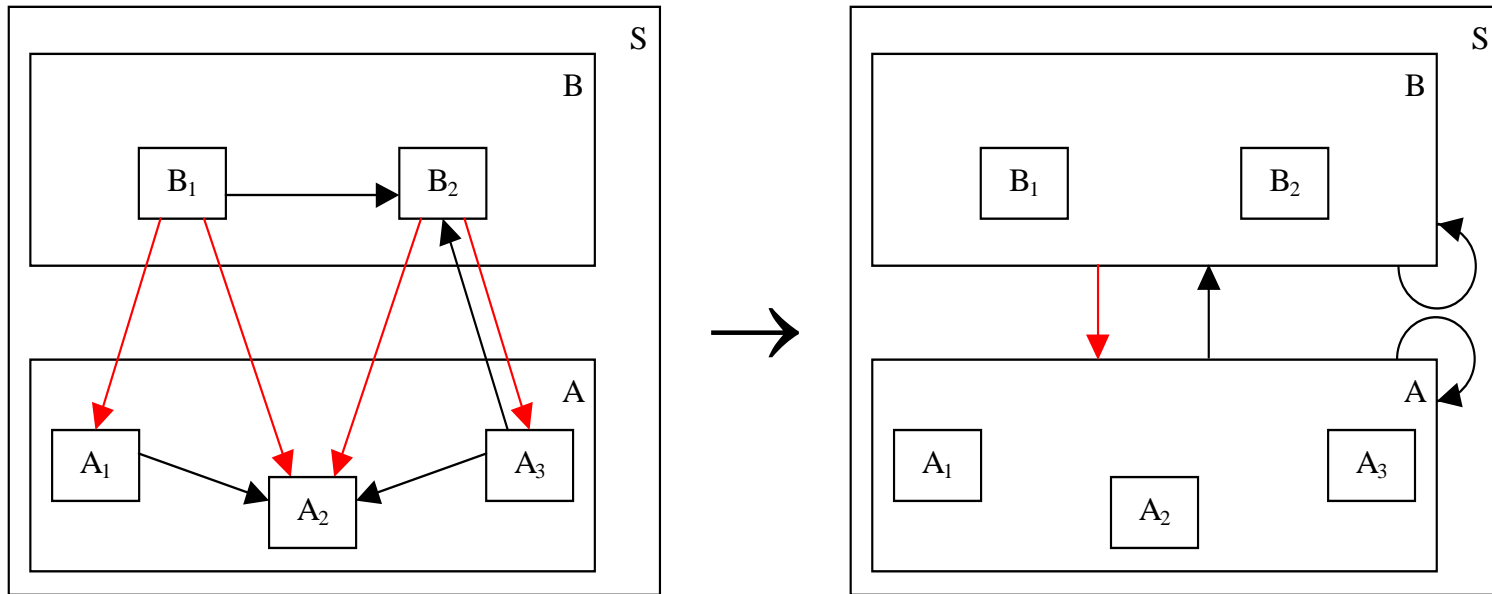
Transform a relation to a higher level, i.e. *replace both the source and the destination of each arrow by its enclosing entity.*

# Lifting (2)

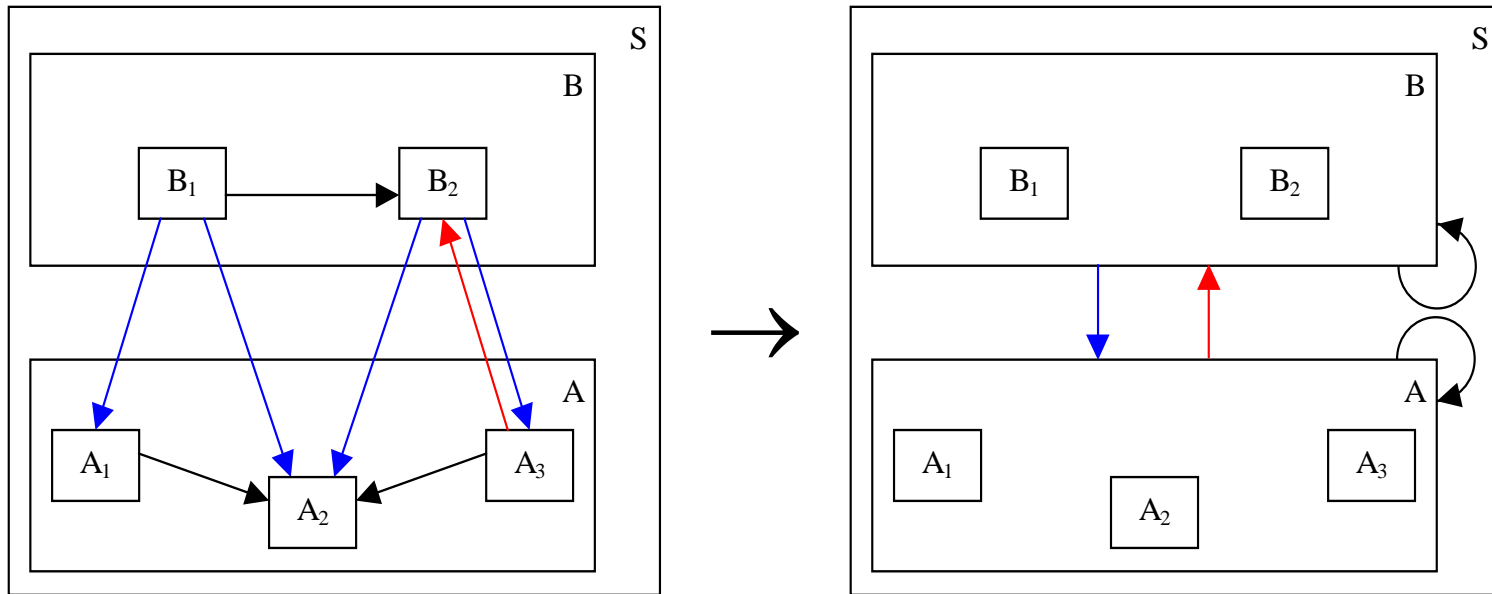




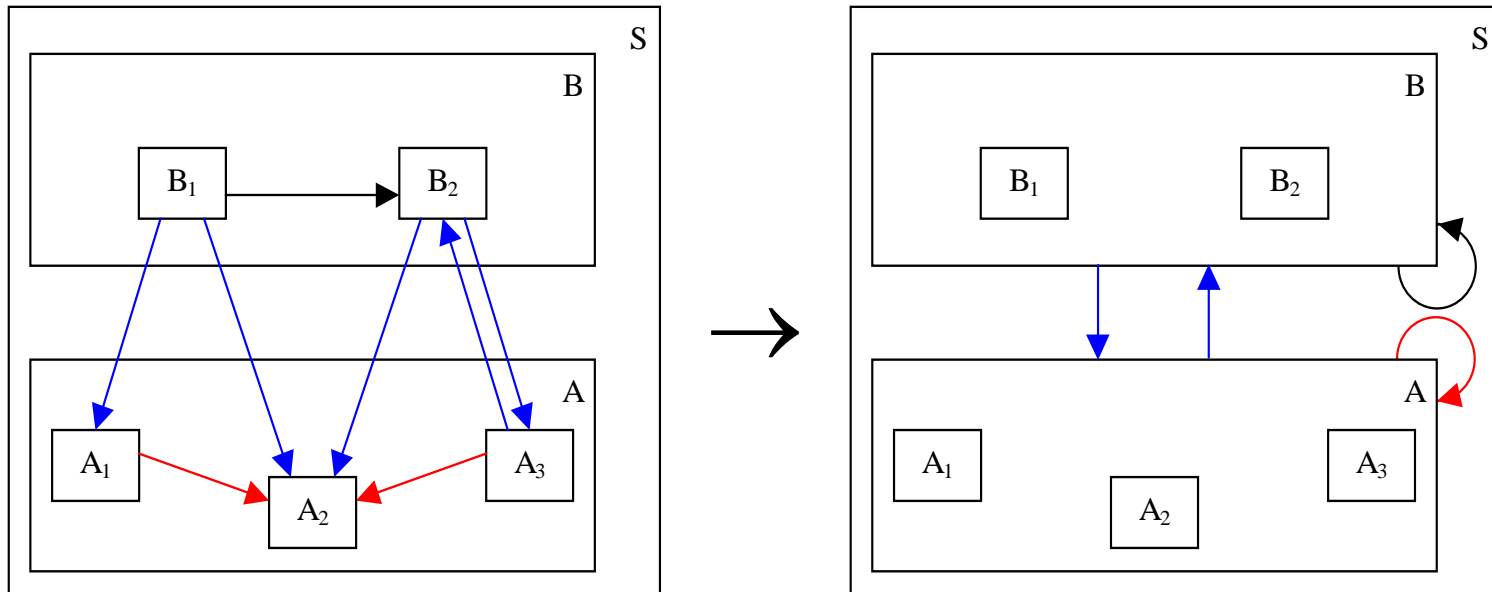
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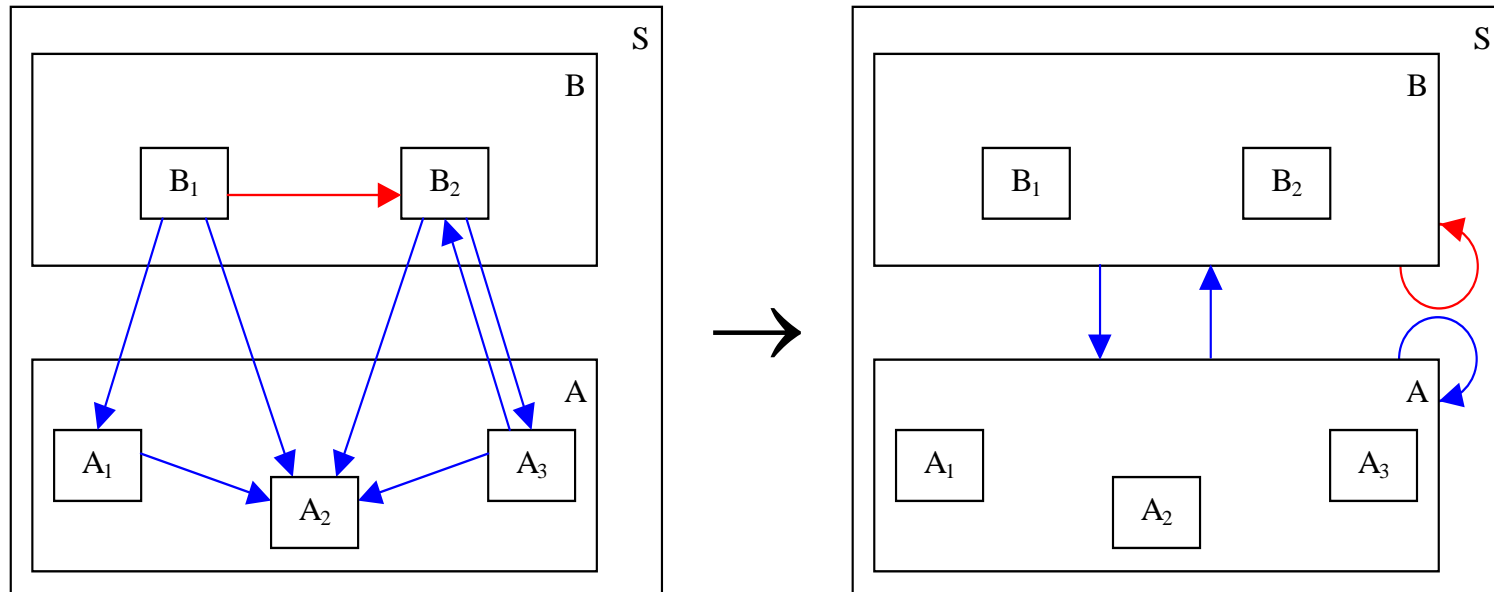
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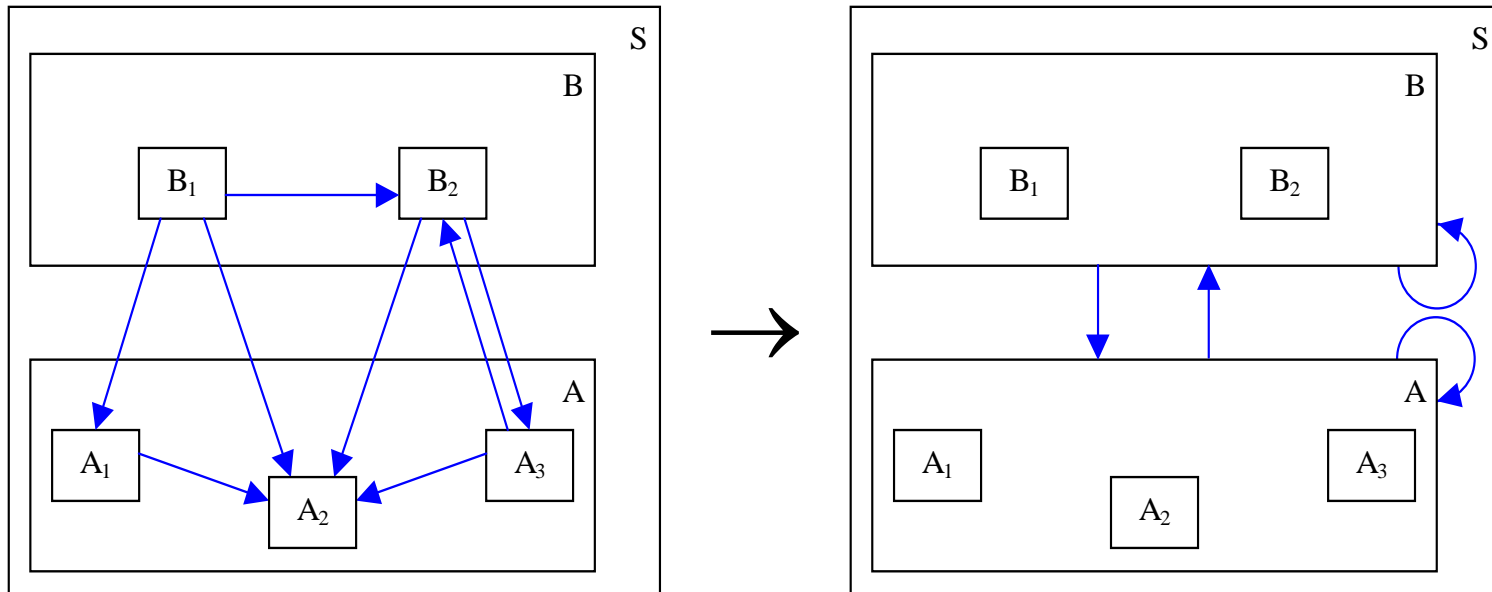
# Lifting (2)



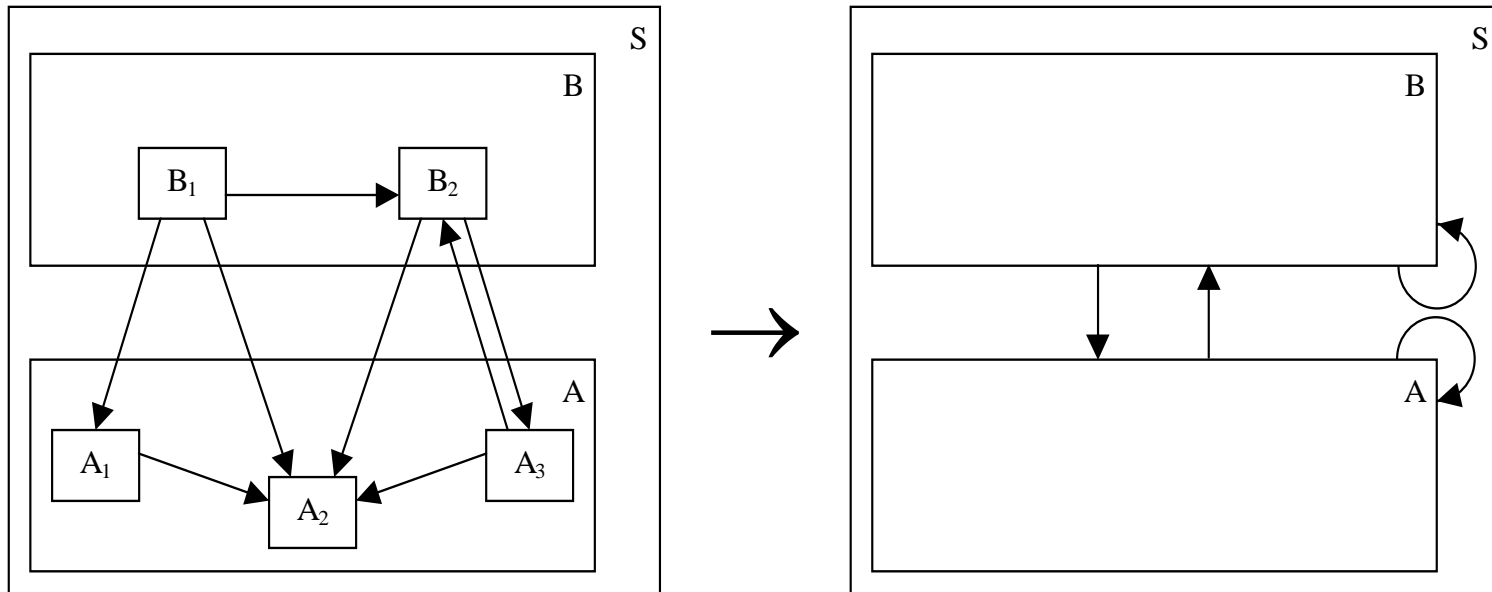
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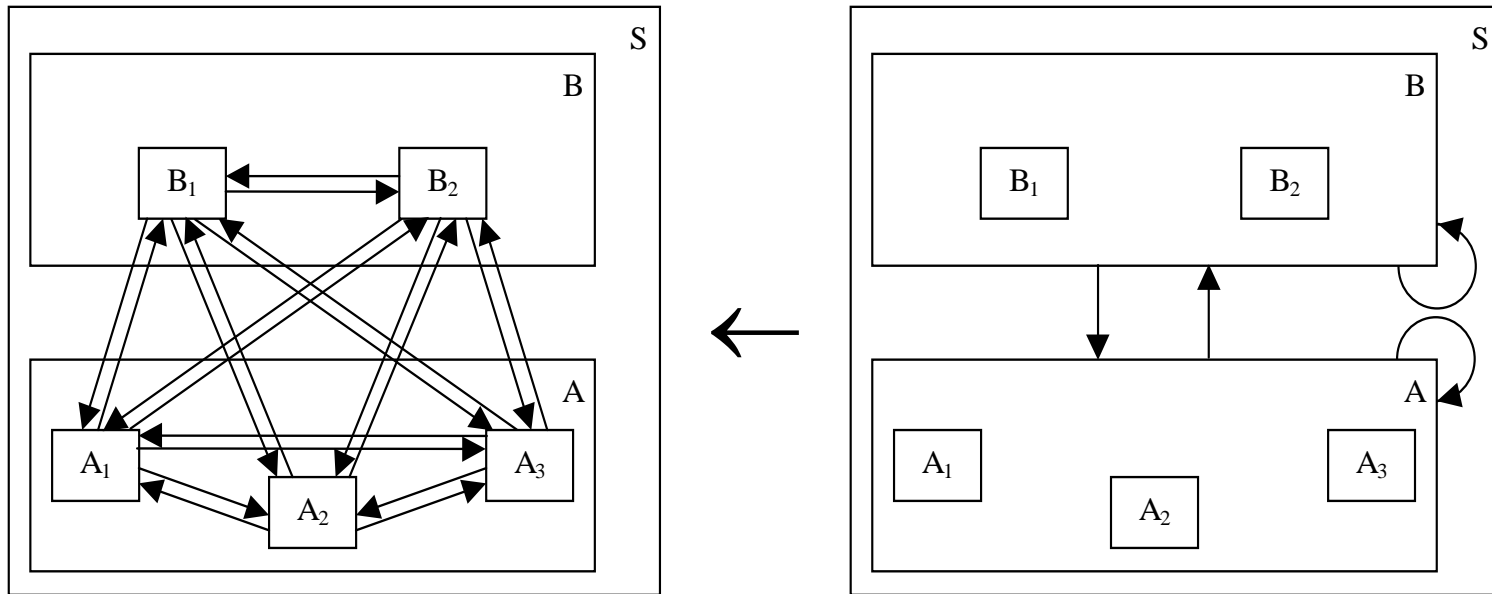


# Hiding



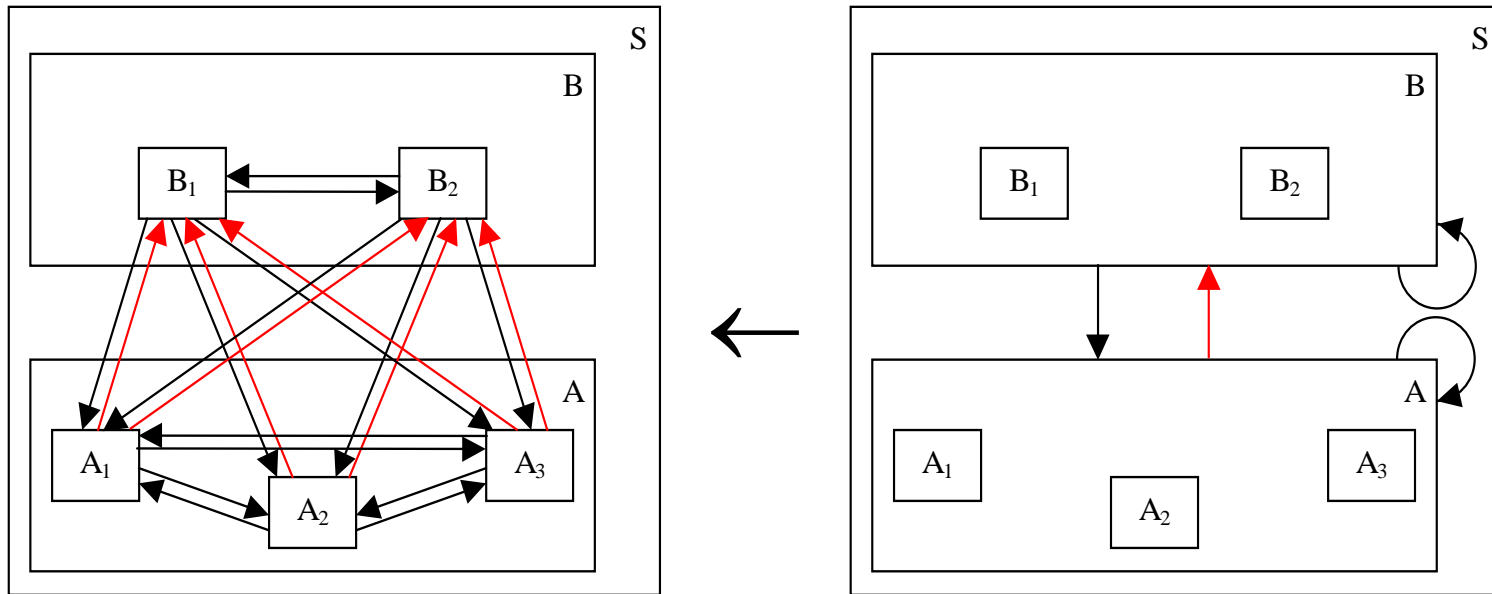
Hiding the decomposition structure  
of both  $A$  and  $B$

# Lowering



both cases a complete graph.

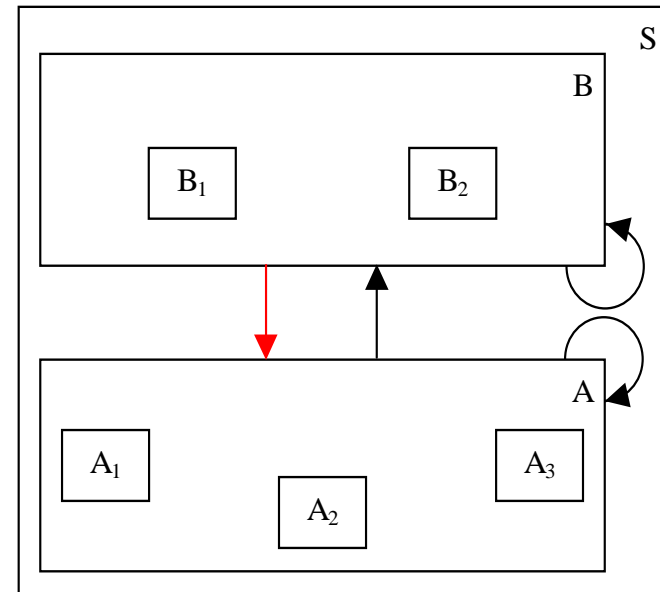
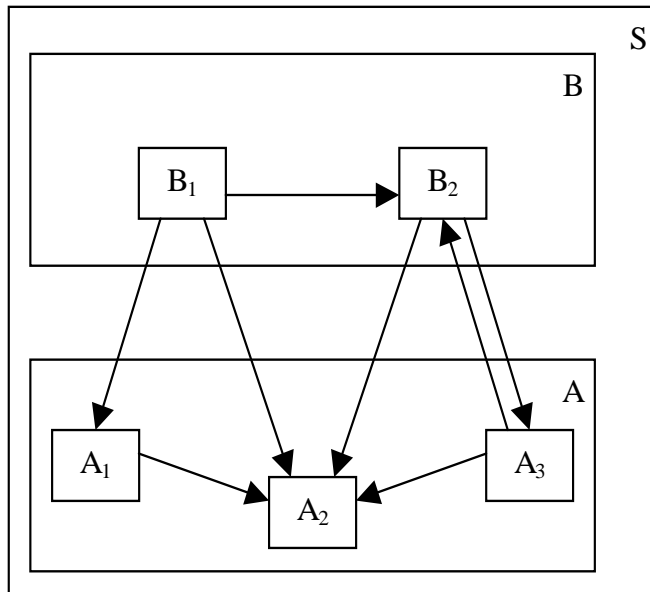
# Lowering



Application: architectural *verification*,  
e.g. layering

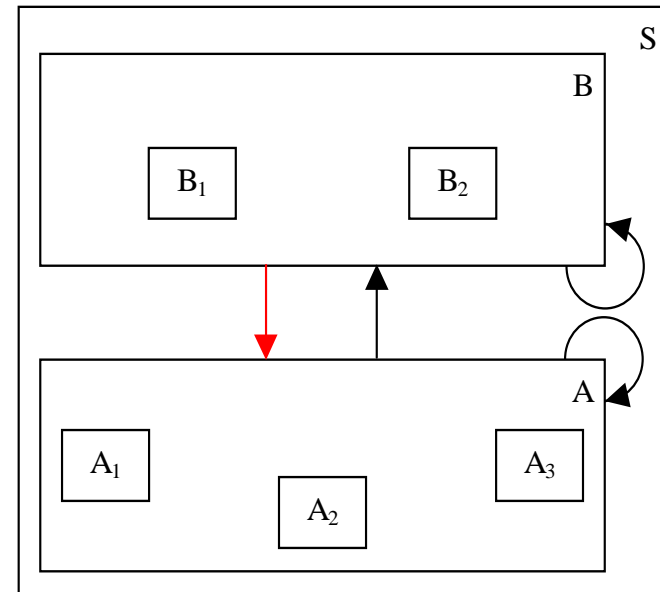
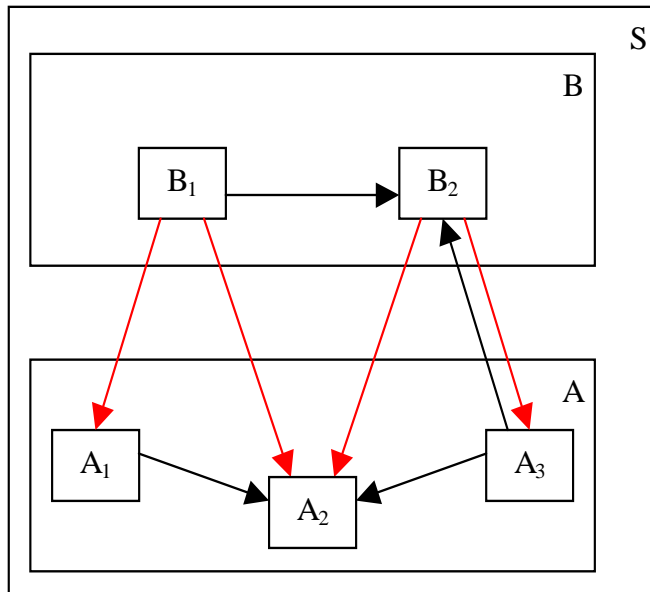


# Weights



Which value to be associated ?

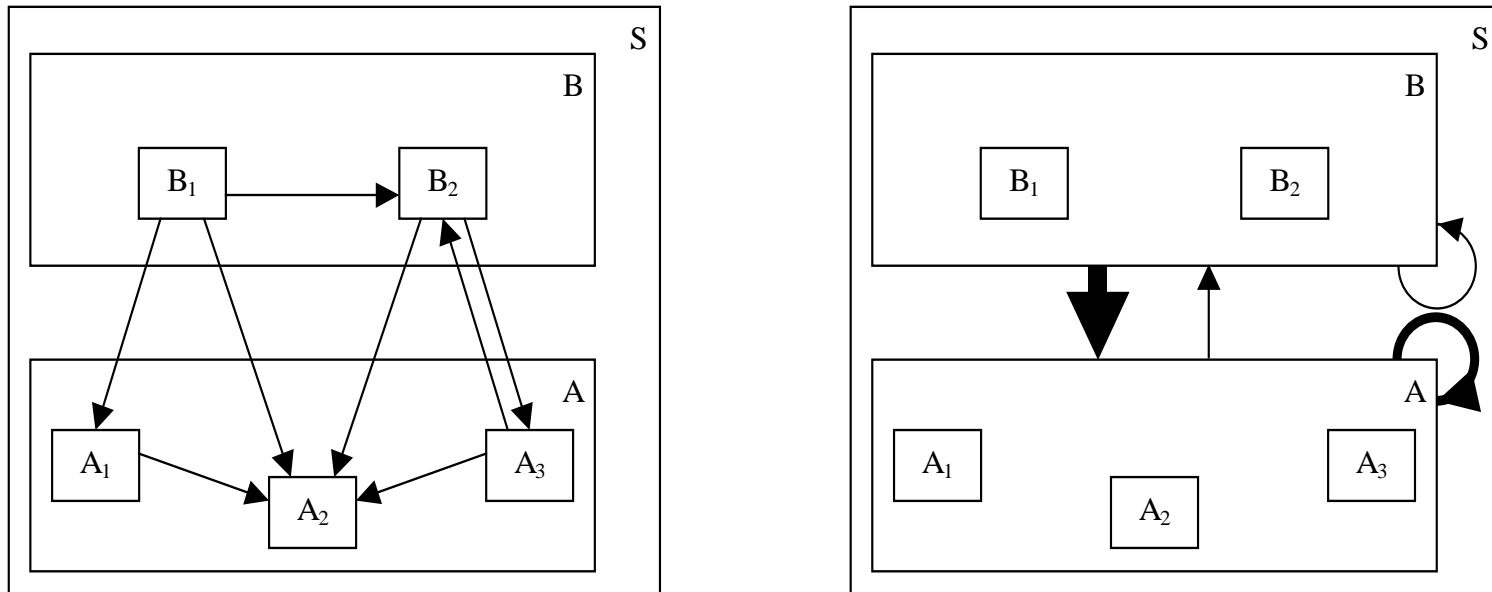
# Weights



4: number of *uses* relations

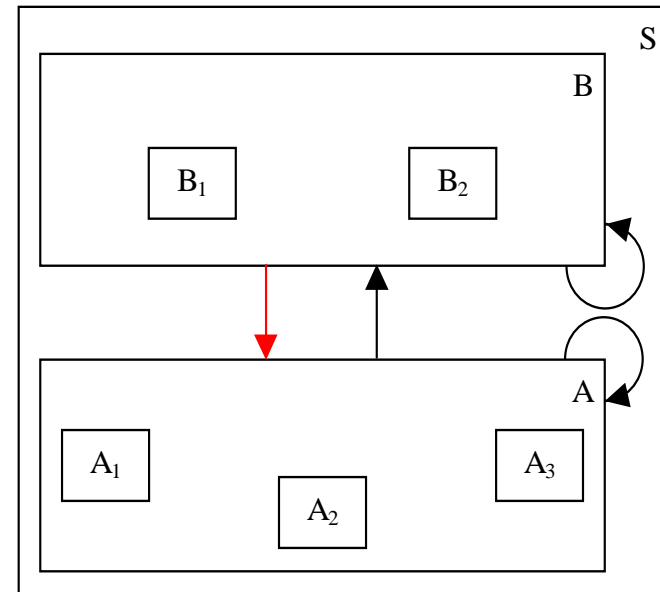
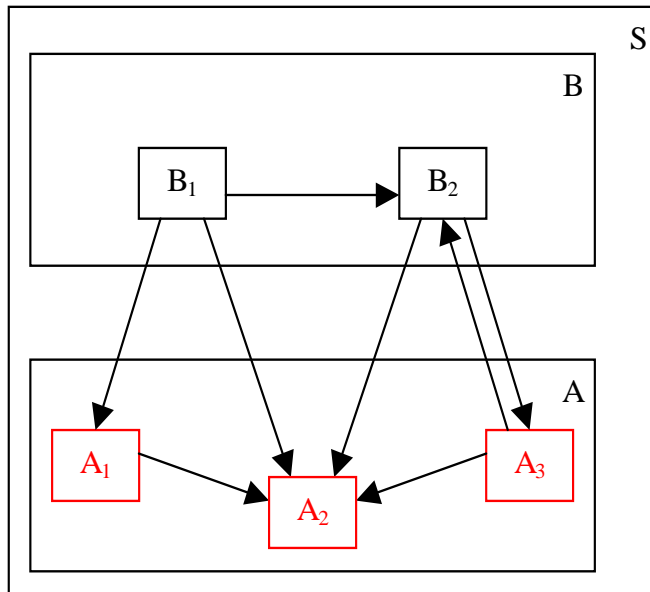
*size-oriented weight*

# Weights



Fisheye view of the  
*size-oriented weight*

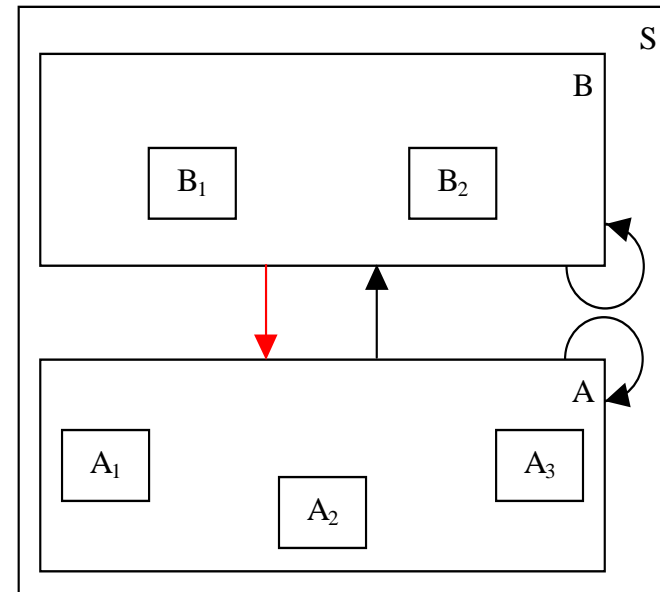
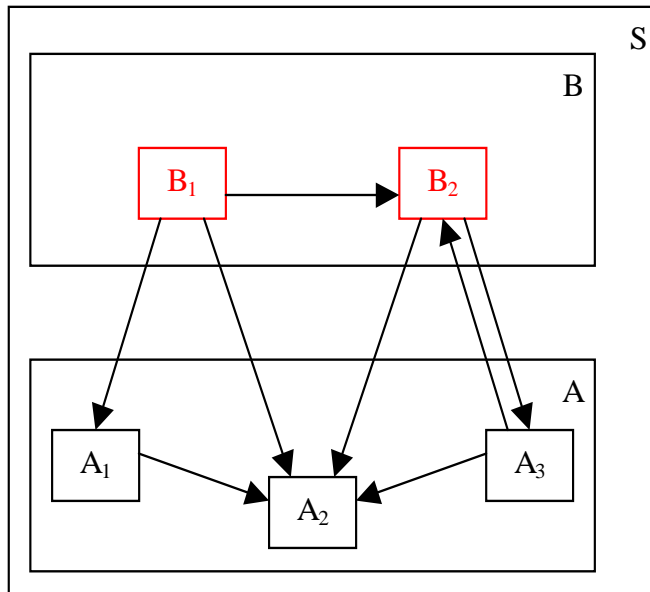
# Weights



**3**: number of *used entities* ( $A_1$ ,  $A_2$  and  $A_3$ )

*fan-in-oriented weight*

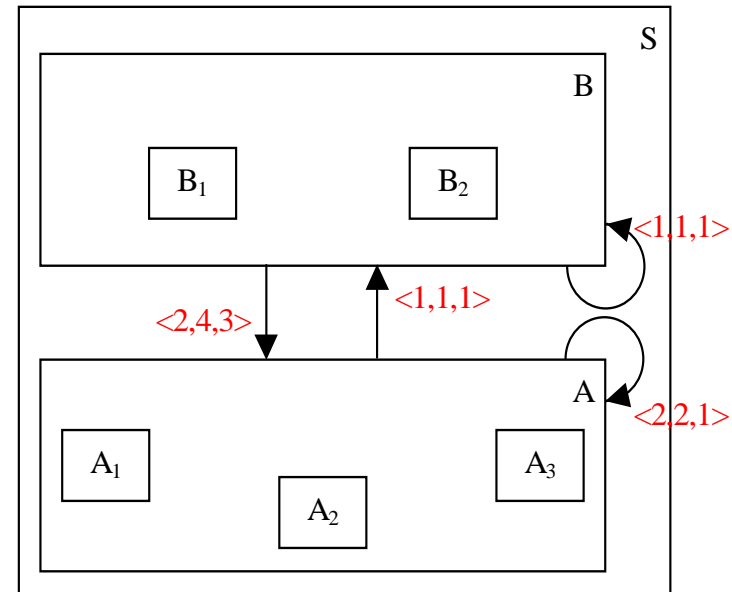
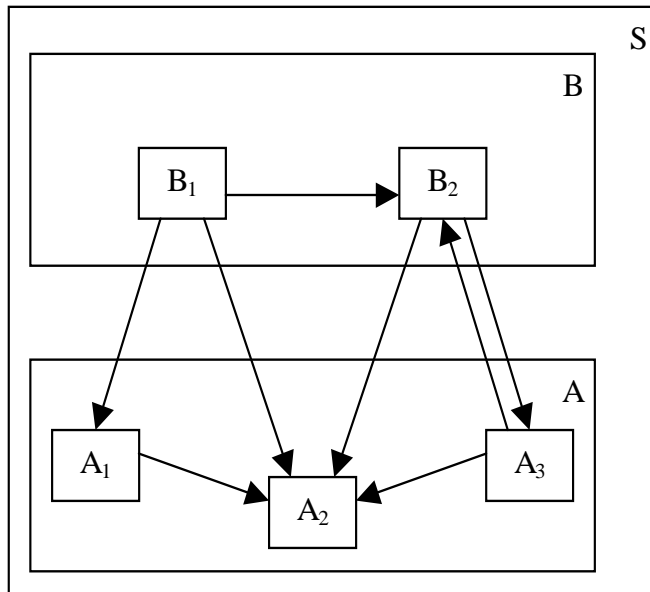
# Weights



**2**: number of *using entities* ( $B_1$  and  $B_2$ )

*fan-out-oriented weight*

# Weights



Each weight has its merits  
during architectural analysis



# Overview

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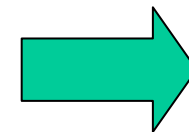


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- **Relation algebra**
  - Usage
  - Overview
  - Examples
  - Application
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# Relation Algebra: Usage

- Visualisation and view calculations
  - reverse architecting purposes
- Relational calculus
  - software architecture analysis
- Architectural rules
  - software architecture verification
- Architectural metrics
  - software architectural quality assurance
- (formal basis of tools)







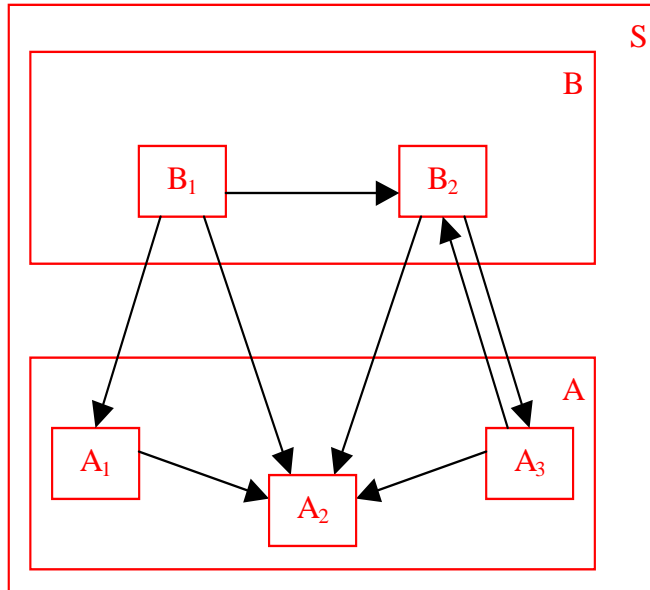
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# Relation Algebra: Overview

- Sets
  - $\emptyset, \cup, \cap, -, \dots$
- Relations: sets of pairs
  - ; (composition),  $^{-1}, +, *, |, \uparrow$  (lifting),  $\downarrow$  (lowering)
- Multi-sets: bags
  - $\sqcap, \sqcup$
- Multi-relations: bags of pairs

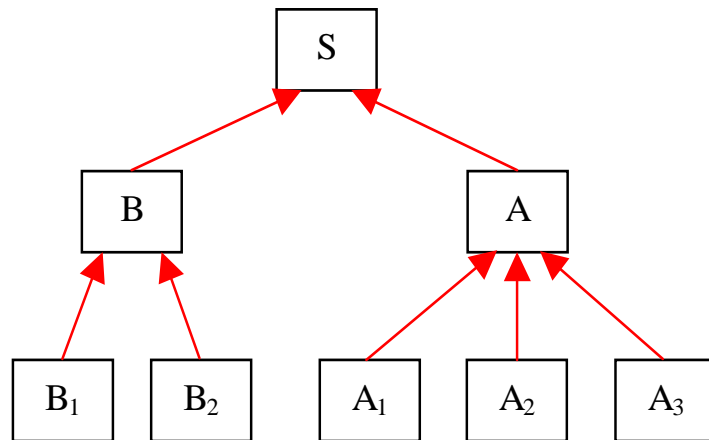
# Relation Algebra: Example



Set of *Entities* E:

$$E = \{S, A, A_1, A_2, A_3, B, B_1, B_2\}$$

# Relation Algebra: Example



*Part-of* relation P:

$$P = \{ \langle B, S \rangle, \langle A, S \rangle, \\ \langle B_1, B \rangle, \langle B_2, B \rangle \\ \langle A_1, A \rangle, \langle A_2, A \rangle, \langle A_3, A \rangle \\ \}$$

A part-of relation:

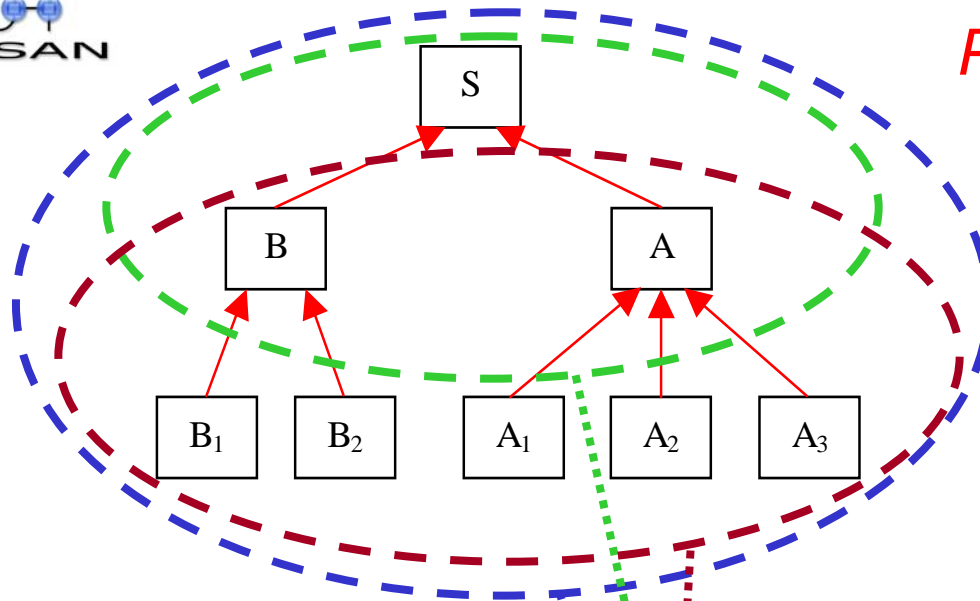
- describes the decomposition tree;
- is both: *functional* and *a-cyclic*.



# Relation Algebra: Example

*Part-of* relation P:

$$P = \{ \langle B, S \rangle, \langle A, S \rangle, \langle B_1, B \rangle, \langle B_2, B \rangle, \langle A_1, A \rangle, \langle A_2, A \rangle, \langle A_3, A \rangle \}$$



Domain of P:

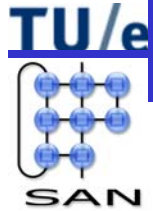
$$\text{dom}(P) = \{ B_1, B_2, B, A_1, A_2, A_3, A \}.$$

Range of P:

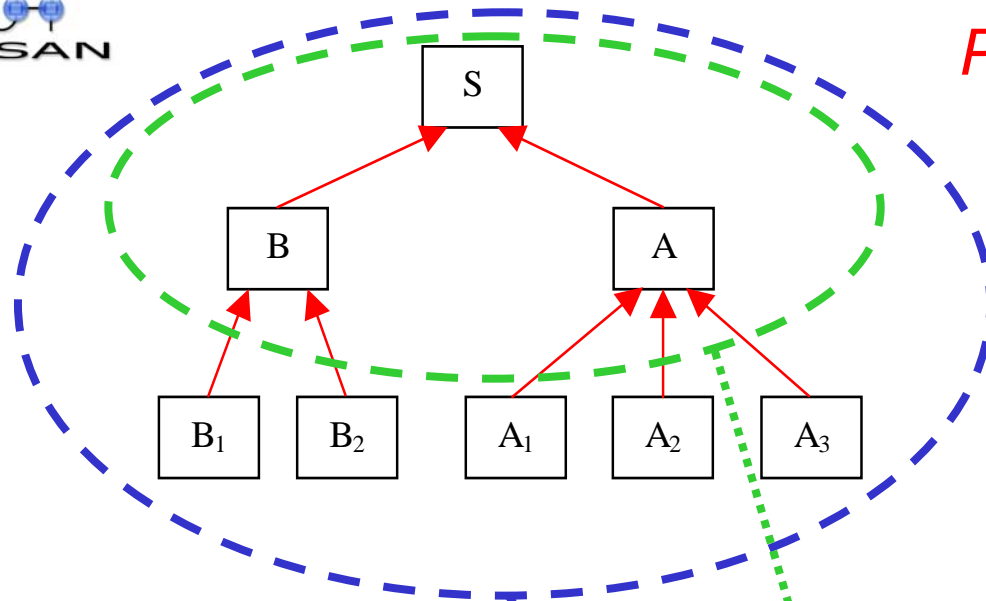
$$\text{ran}(P) = \{ B, A, S \}.$$

Carrier of P:

$$\text{car}(P) = \{ B_1, B_2, B, A_1, A_2, A_3, B, A, S \} = E.$$



# Relation Algebra: Example

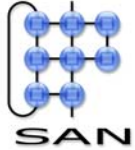


*Part-of* relation P:

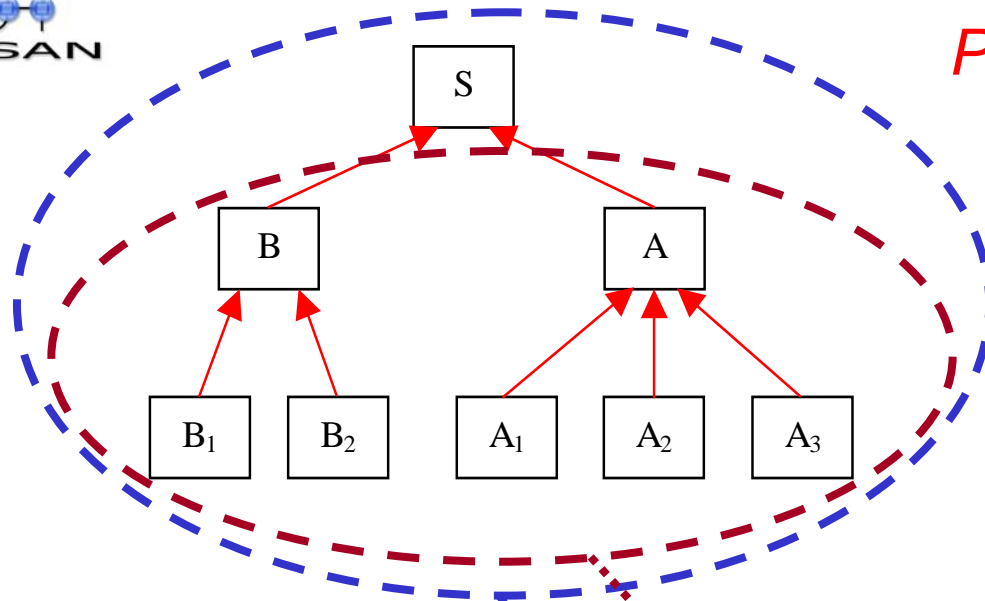
$$P = \{ \langle B, S \rangle, \langle A, S \rangle, \\ \langle B_1, B \rangle, \langle B_2, B \rangle, \\ \langle A_1, A \rangle, \langle A_2, A \rangle, \langle A_3, A \rangle \}$$

**Question:** How to express the leafs of the decomposition tree in relation algebra using P?

**Answer:**  $\text{leafs}(E) = \text{car}(P) - \text{ran}(P) = \{ B_1, B_2, A_1, A_2, A_3 \}$ .



# Relation Algebra: Example



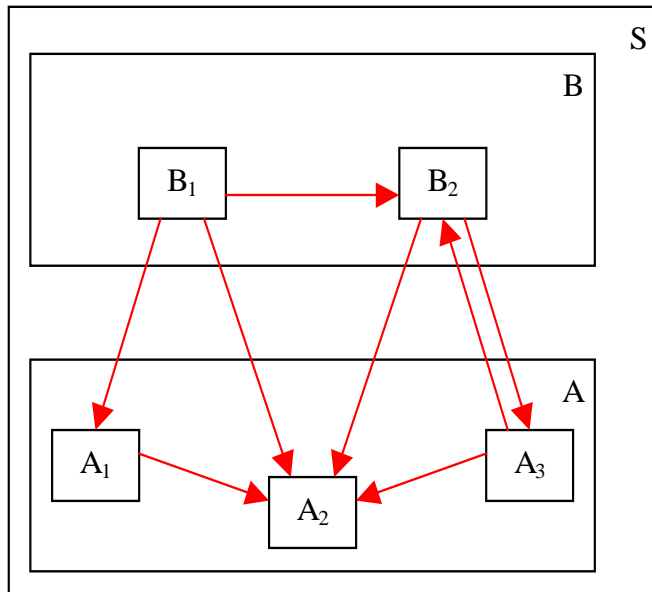
*Part-of* relation P:

$$P = \{ \langle B, S \rangle, \langle A, S \rangle, \\ \langle B_1, B \rangle, \langle B_2, B \rangle, \\ \langle A_1, A \rangle, \langle A_2, A \rangle, \langle A_3, A \rangle \}$$

**Question:** How to express the root of the decomposition tree in relation algebra using P?

**Answer:**  $\text{root}(E) = \text{car}(P) - \text{dom}(P) = \{ S \}$ .

# Relation Algebra: Example



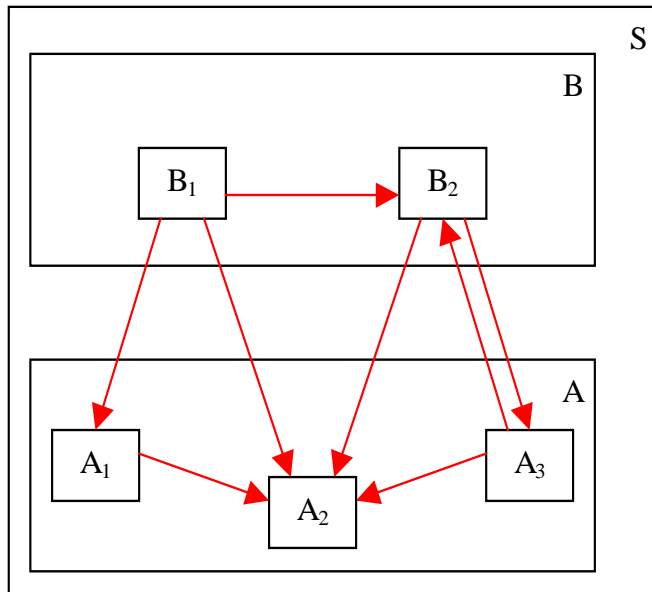
*Uses* relation  $U$ :

$$U = \{ \langle A_1, A_2 \rangle, \langle A_3, A_2 \rangle, \\ \langle B_1, A_1 \rangle, \langle B_1, A_2 \rangle, \langle B_1, B_2 \rangle, \\ \langle B_2, A_2 \rangle, \langle B_2, A_3 \rangle, \\ \langle A_3, B_2 \rangle \\ \}$$

**Question:** How to express the entities that use, but are not used by, entities in relation algebra ?

**Answer:**  $\text{use\_only} = \text{car}(U) - \text{ran}(U) = \{ B_1 \}$ .

# Relation Algebra: Example



*Uses* relation  $U$ :

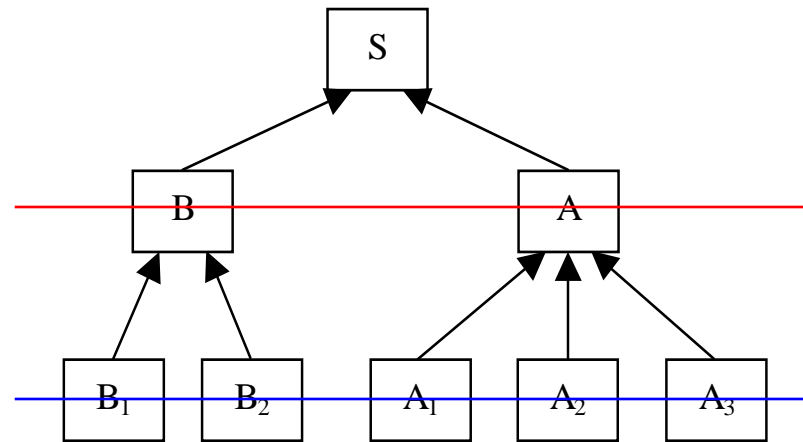
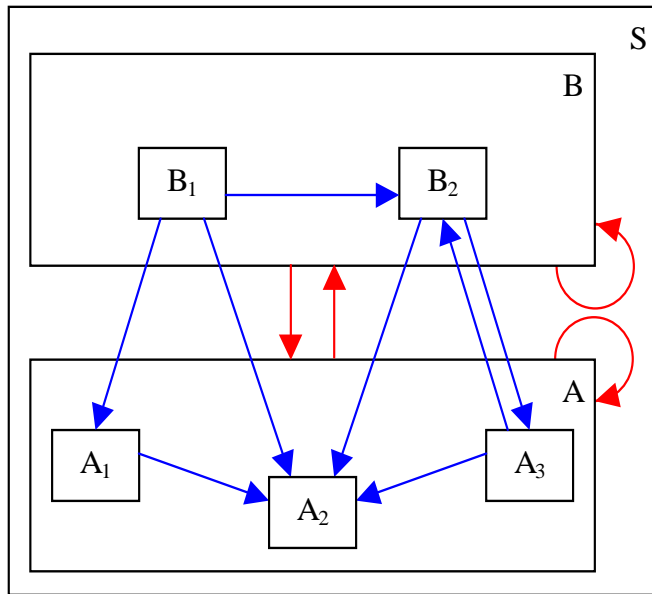
$$U = \{ \langle A_1, A_2 \rangle, \langle A_3, A_2 \rangle, \\ \langle B_1, A_1 \rangle, \langle B_1, A_2 \rangle, \langle B_1, B_2 \rangle, \\ \langle B_2, A_2 \rangle, \langle B_2, A_3 \rangle, \\ \langle A_3, B_2 \rangle \\ \}$$

**Question:** How to express the leaf entities that neither use nor are used by entities in relation algebra ?

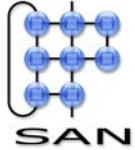
**Answer:** isolated = leafs( $E$ ) – car( $U$ ) =  $\emptyset$ .



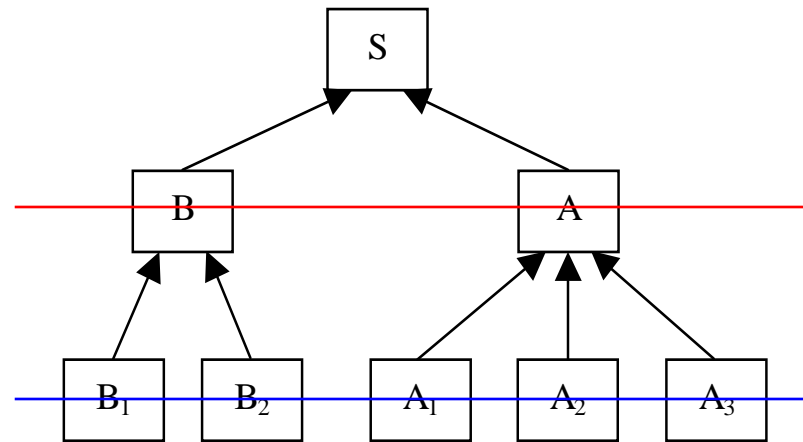
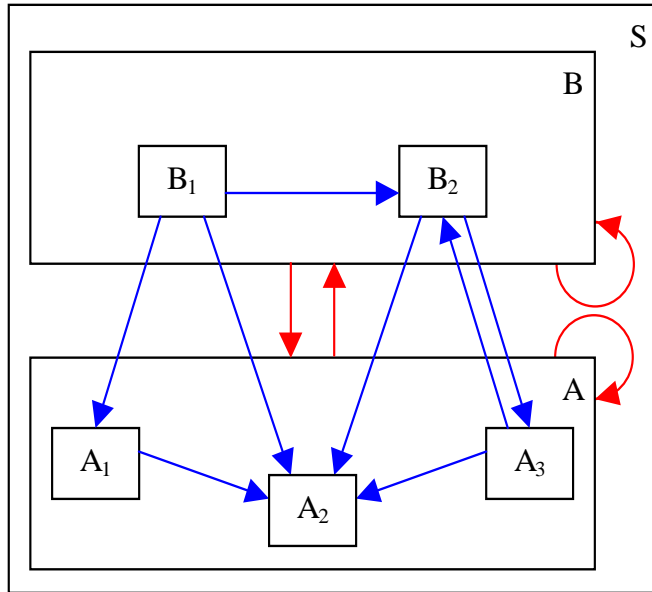
# Relation Algebra: Example



Transform a relation to a higher level, i.e. *replace both the source and the destination of each arrow by its enclosing entity.*

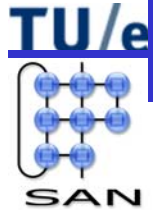


# Relation Algebra: Example

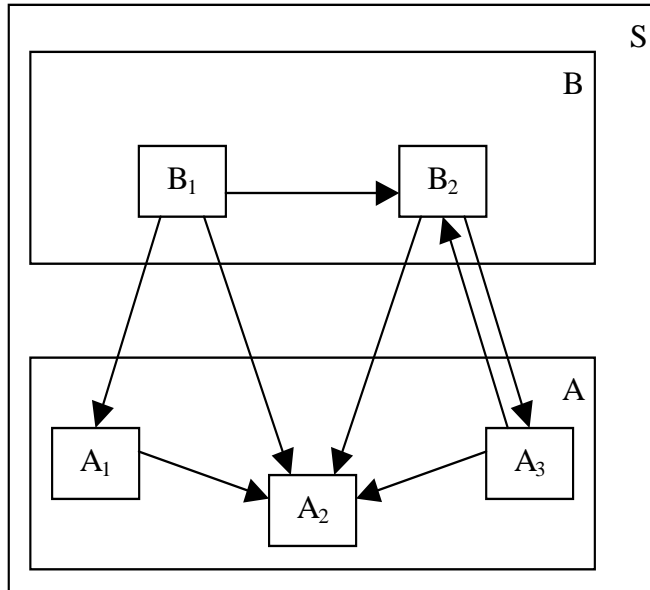


Example:  $\langle B_1, A_1 \rangle$

- *replace source*:  $B_1$  by  $B$ ;
- *replace destination*:  $A_1$  by  $A$ .



# Relation Algebra: Example



## *Lifting:*

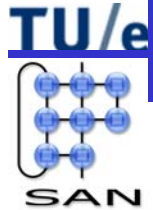
def.:  $U \uparrow P \equiv P^{-1} ; U ; P$

ex.:  $\langle B_1, B \rangle \in P \Leftrightarrow \langle B, B_1 \rangle \in P^{-1};$

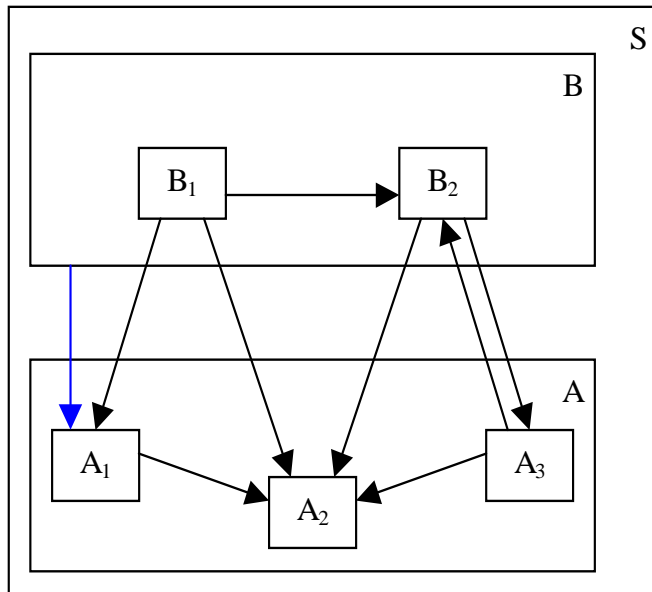
$\langle B_1, A_1 \rangle \in U;$

$\langle A_1, A \rangle \in P;$

For  $U \uparrow P$ ,  $P$  must be a part-of relation



# Relation Algebra: Example



## *Lifting:*

def.:  $U \uparrow P \equiv P^{-1} ; U ; P$

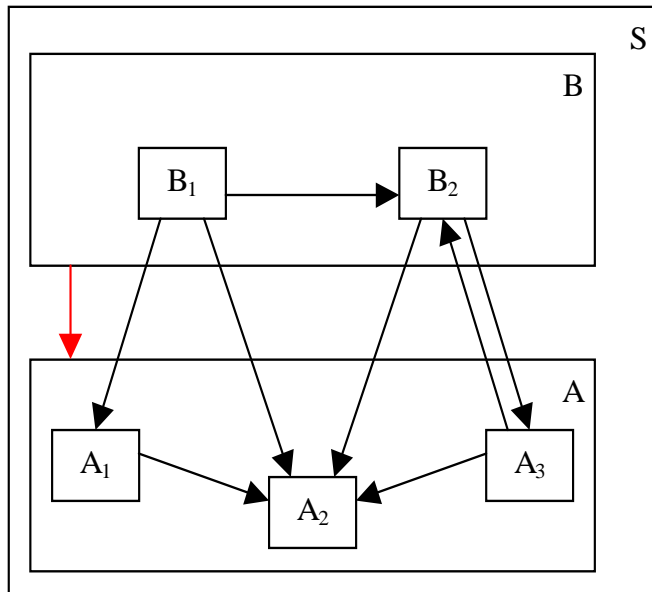
ex.:  $\langle B_1, B \rangle \in P \Leftrightarrow \langle B, B_1 \rangle \in P^{-1};$

$\langle B_1, A_1 \rangle \in U;$

$\langle A_1, A \rangle \in P;$

$\langle B, A_1 \rangle \in P^{-1} ; U$

# Relation Algebra: Example



## *Lifting:*

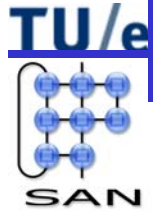
def.:  $U \uparrow P \equiv P^{-1} ; U ; P$

ex.:  $\langle B_1, B \rangle \in P \Leftrightarrow \langle B, B_1 \rangle \in P^{-1};$

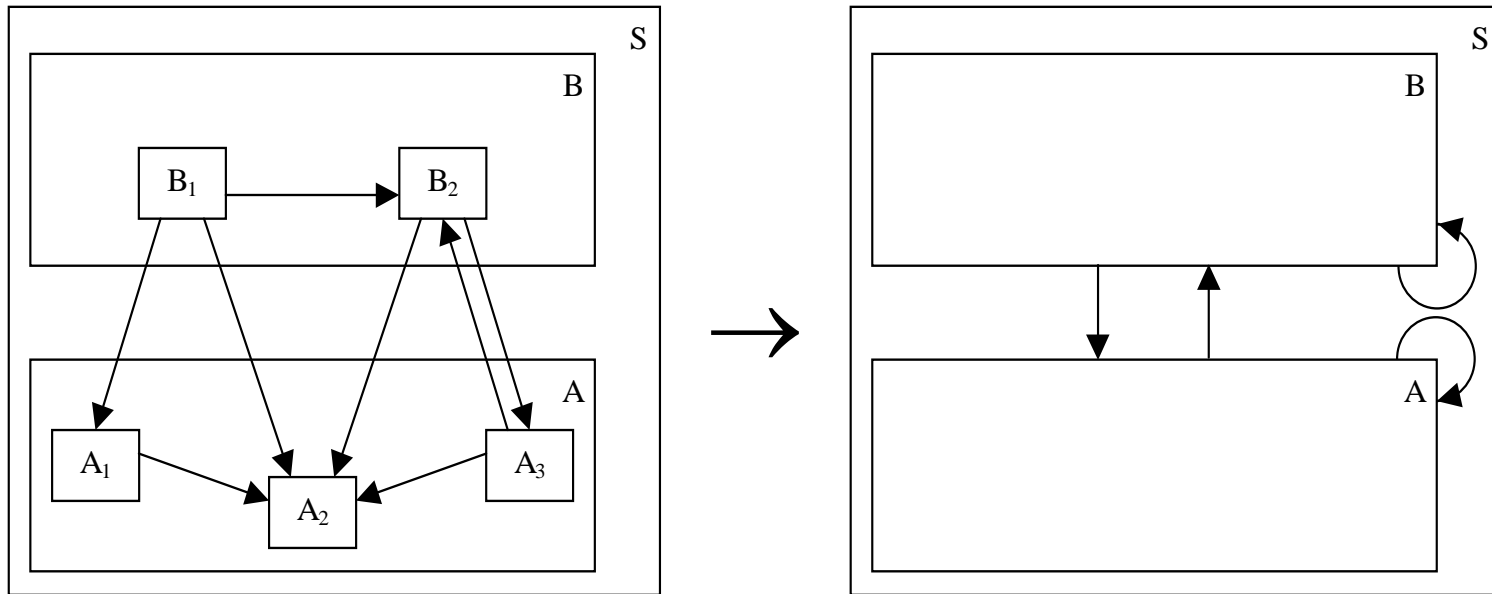
$\langle B_1, A_1 \rangle \in U;$

$\langle A_1, A \rangle \in P;$

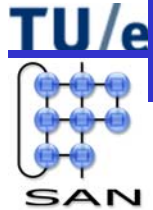
$\langle B, A \rangle \in P^{-1} ; U ; P$



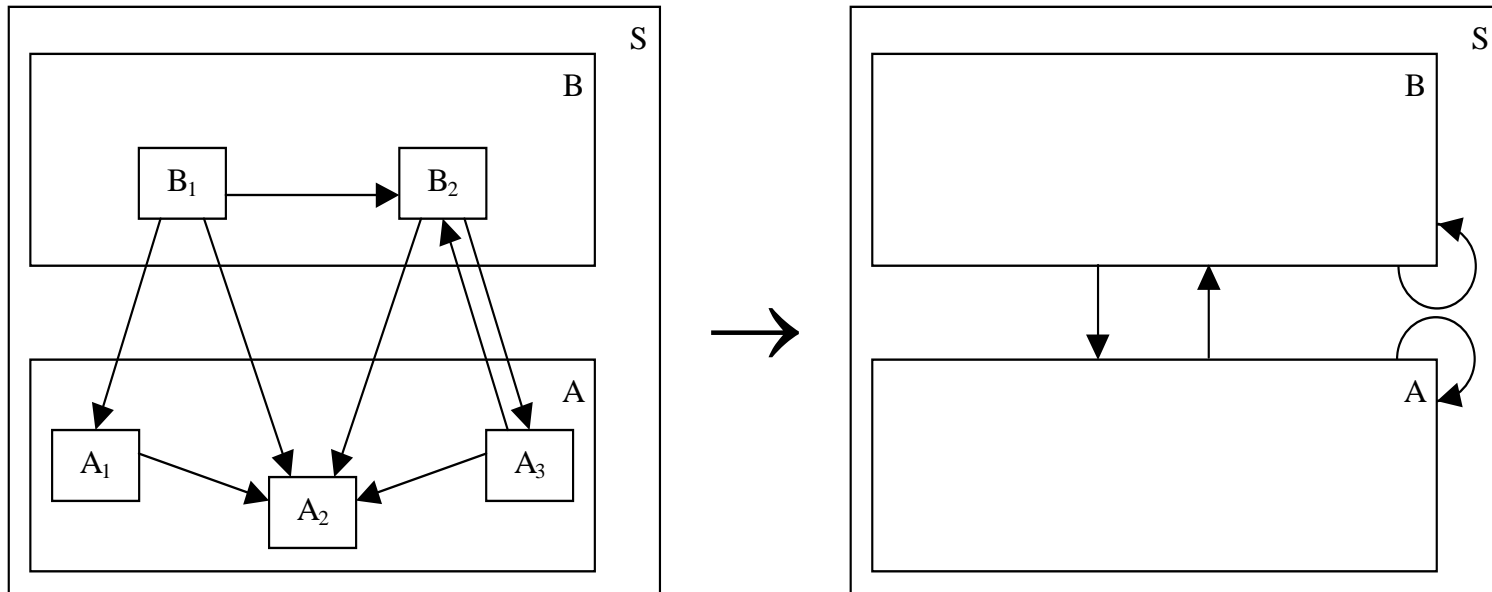
# Relation Algebra: Example



Hiding the decomposition structure  
of both  $A$  and  $B$

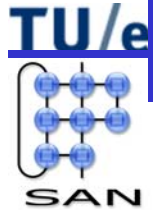


# Relation Algebra: Example

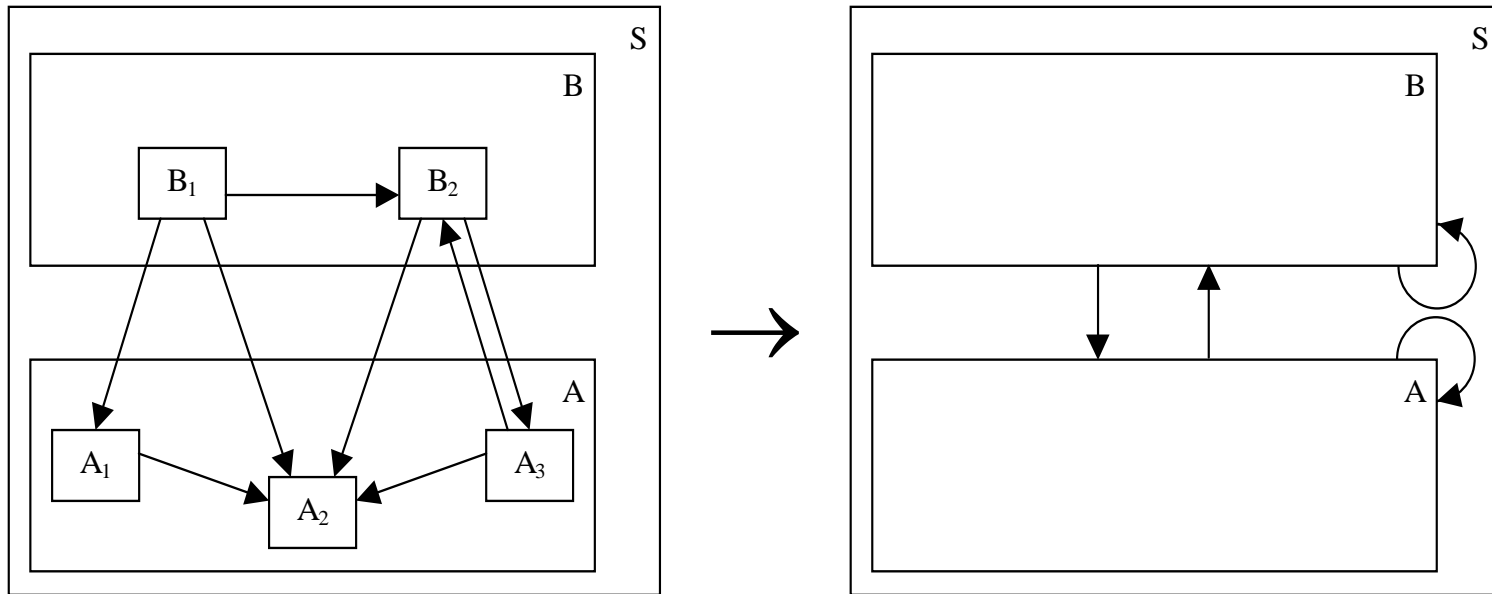


**Question:** How to express the set of entities  $E'$  after hiding in relation algebra using  $P$ ?

**Answer:**  $E' = \text{ran}(P) = \{ S, A, B \}$ .



# Relation Algebra: Example

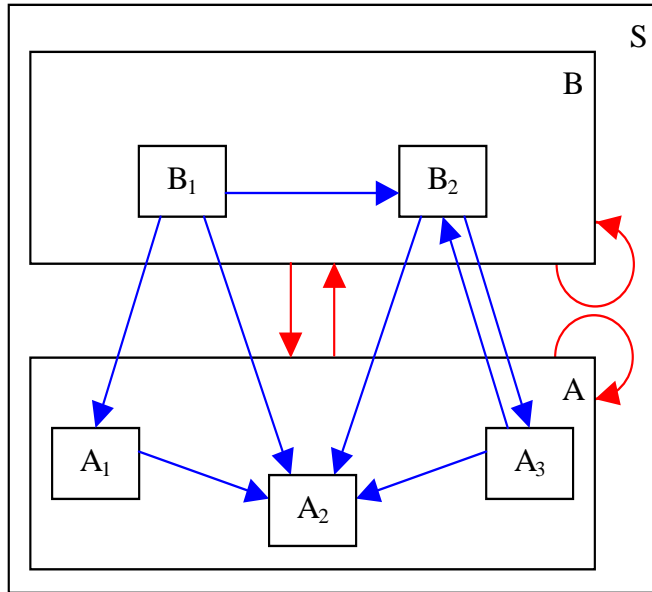


**Question:** How to express the part-of relation  $P'$  after hiding in relation algebra ?

**Answer:**  $P' = P \upharpoonright_{\text{car}} E' = \{ S, A, B \}$ .



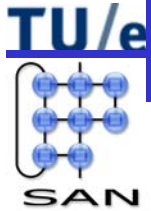
# Relation Algebra: Example



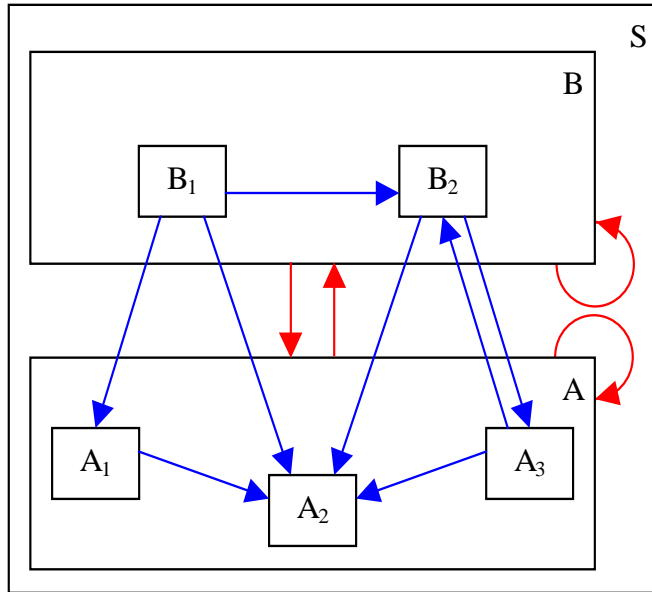
*Lowering:*

$$\text{def.: } U \downarrow P \equiv P ; U ; P^{-1}$$

For  $U \downarrow P$ ,  $P$  must be a part-of relation



# Relation Algebra: Example



## Lowering:

def.:  $U \downarrow P \equiv P ; U ; P^{-1}$

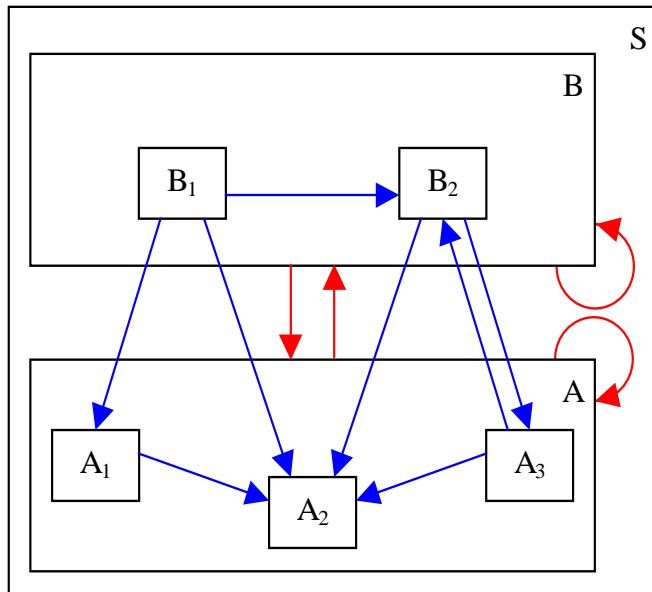
ex.:  $\{\langle A, B \rangle\} \downarrow P = \{\langle A_1, B_1 \rangle, \langle A_1, B_2 \rangle$   
 $, \langle A_2, B_1 \rangle, \langle A_2, B_2 \rangle$   
 $, \langle A_3, B_1 \rangle, \langle A_3, B_2 \rangle$   
 $\}$

$\{\langle A, B \rangle\} \downarrow P \cap U = \{\langle A_3, B_2 \rangle\}$

Layering rule:

$$\{\langle A, B \rangle\} \downarrow P \cap U = \emptyset$$

# Relation Algebra: Example

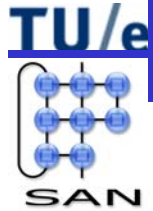


*Size-oriented lifting:*

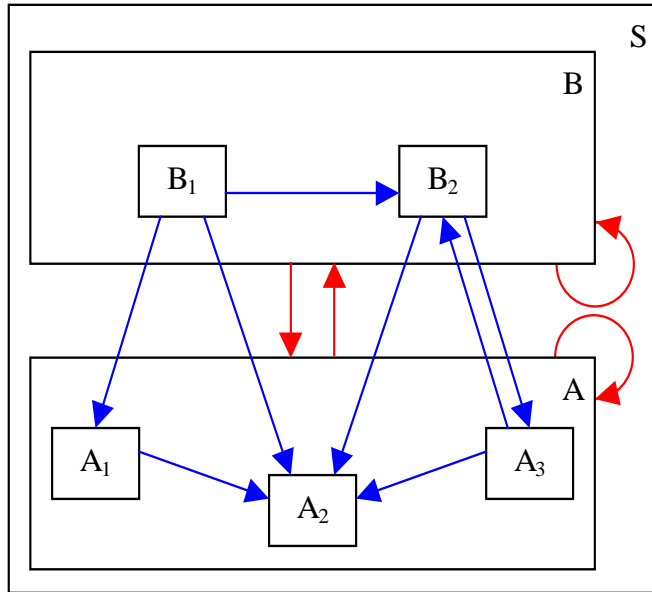
def.:  $U_M \uparrow P \equiv [P^{-1}] ; U_M ; [P]$

where  $U_M = [U]$

ex.:  $\langle B, A \rangle$  4 times in  $U_M \uparrow P$



# Relation Algebra: Example



*Fan-out-oriented lifting:*

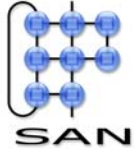
$$U_M \uparrow \triangleright P \equiv \lceil P^{-1} \rceil ; \lceil \lfloor U_M \rfloor \rceil ; P$$

$\langle B, A \rangle$  2 times in  $U_M \uparrow \triangleright P$

*Fan-in-oriented lifting:*

$$U_M \uparrow \triangleleft P \equiv \lceil P^{-1} \rceil ; \lfloor U_M \rfloor ; \lceil P \rceil$$

$\langle B, A \rangle$  3 times in  $U_M \uparrow \triangleleft P$

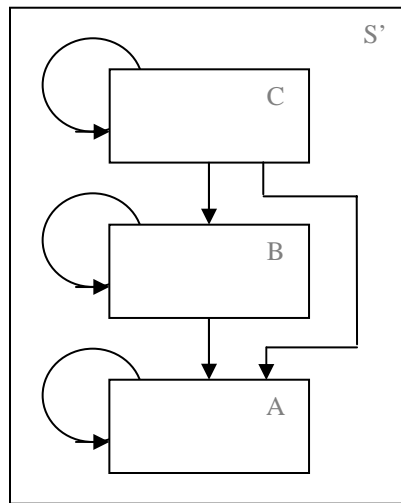


# Relation Algebra: Application

- Analytical techniques:
  - lifting,
  - checking of rules,
  - impact analysis,
  - finding unused and unavailable components,
  - identification of top and bottom layers,
  - study of alternative component groupings,
  - improvement of presentation by suppressing elements,
  - improvement of locality,
  - checking of linearity,
  - analysis of cycles,
  - improvement of presentation by transitive reduction.

[Feijs et al 98]

# Verification



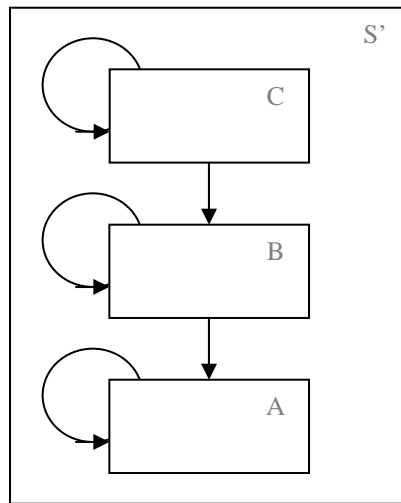
System  $S'$  consists of 3 levels, and is *layered*.

**Question:** How to express layering rule LR for  $S'$  in relation algebra?

**Answer:** Layering rule LR for  $S'$ :

$$LR(S'): \{ \langle A, B \rangle, \langle A, C \rangle, \langle B, C \rangle \} \downarrow P \cap U = \emptyset$$

# Verification



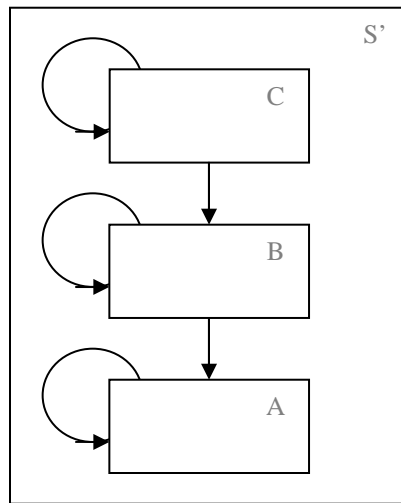
System  $S'$  consists of 3 levels, and is *strictly* layered.

**Question:** How to express strictly layering rule SLR for  $S'$  in relation algebra?

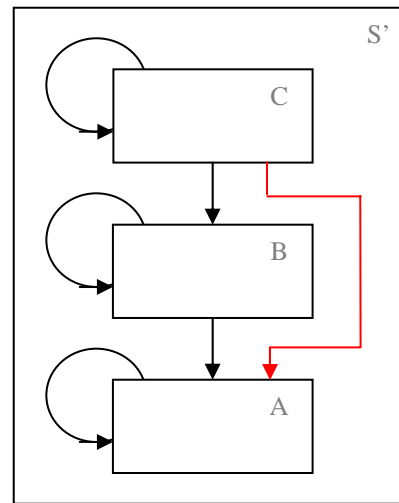
**Answer:** Strictly layering rule SLR for  $S'$ :

$$\text{SLR}(S'): \{ \langle A, B \rangle, \langle A, C \rangle, \langle B, C \rangle, \langle C, A \rangle \} \downarrow P \cap U = \emptyset$$

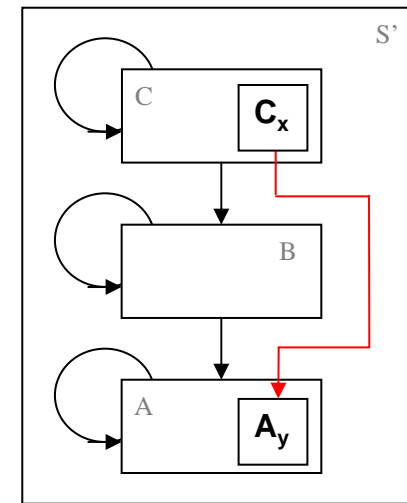
# Verification



Intended



Derived

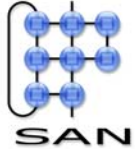


**Question:** How to express the exception to the strictly layering rule SLR(S') for S' in relation algebra?

**Answer:** Delete specific uses relation between modules !

$$\{\langle A, B \rangle, \langle A, C \rangle, \langle B, C \rangle, \langle C, A \rangle\} \downarrow P \cap U - \{\langle C_x, A_y \rangle\} = \emptyset$$





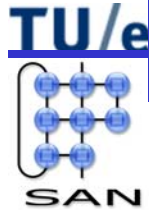
# Conclusion

## Goals revisited

- Student understands:
  - the need for module architecture control;
    - diversion of intended and derived architecture over time
  - how module architecture control can be performed;
    - describe the architecture in terms of rules, check compliance of the derived architecture, and adapt
  - the basics of relation algebra.
- Student can apply relation algebra on a simple example.



# References



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- [Boehm 1995] *Engineering context (for software architecture), invited talk, 1<sup>st</sup> Int. Workshop on Architecture for Software Systems, Seattle, Washington, April 1995.*
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