Number Guessing with Lies

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The Game

1. Alice picks a number N in the range 0 through 15 .

2. Bob asks a series of Yes/No questions .

3. Alice answers each question, and may lie once .

4. Bob then tells the number N and which answer was a lie (if any).

How can Bob do this?

Information

Is your number one of these?

	1		3	4		6		8		10			13		15

Is your number one of these?

	1	2			5	6	8			11	12			15

Is your number one of these?

								8	9	10	11	12	13	14	15

Information

Is your number one of these?

	1	2		4			7		9	10		12			15

Is your number one of these?

				4	5	6	7					12	13	14	15

Is your number one of these?

		2	3			6	7			10	11			14	15

Is your number one of these?

1	3	5	7	9	11	13	15

Figuring it out

- Let the answers be a_i ($0 =$ No; $1 =$ Yes) for $i = 1, \ldots, 7$

- Compute

$$
\begin{aligned}
p_1 &= a_1 + a_3 + a_5 + a_7 \quad (\text{mod } 2) \\
p_2 &= a_2 + a_3 + a_6 + a_7 \quad (\text{mod } 2) \\
p_3 &= a_4 + a_5 + a_6 + a_7 \quad (\text{mod } 2)
\end{aligned}
$$

- Compute $q = p_1 + 2p_2 + 4p_3$ (each p_i is 0 or 1)

- If $q = 0$, then there was no lie

- If $q \neq 0$, then answer a_q was a lie: flip a_q (replace it by $1 - a_q$)

- Alice' secret number was $N = 8a_3 + 4a_5 + 2a_6 + a_7$

         Information

How It Works

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q_3								8	9	10	11	12	13	14	15
Q_5				4	5	6	7					12	13	14	15
Q_6		2	3			6	7			10	11			14	15
Q_7	1		3		5		7		9		11		13		15
Q_1	1		3	4		6		8		10			13		15
Q_2	1	2			5	6		8			11	12			15
Q_4	1	2		4			7		9	10		12			15

Questions Q_3, Q_5, Q_6, and Q_7 do a *Binary Search*; works *without* lie.

The three other questions help detect a single lie:

· Q_1 · Q_3 · Q_5 · Q_7

· · Q_2 Q_3 · · Q_6 Q_7

· · · · Q_4 Q_5 Q_6 Q_7

There are $8 = 2^3$ possibilities: no lie, or 7 possible single lies.

Error-Correcting Hamming(7,4) Code

Less efficient solution repeats questions Q_3, Q_5, Q_6, Q_7 three times.

We used a Hamming(7,4) code .

It has 4 data bits, 3 parity/check bits, and can correct one bit error.

The 4 data bits encode a value from 0 through 15.

Each question corresponds to the transmission of a bit.

A lie corresponds to a bit error.

Can be generalized to $2^k - k - 1$ data bits and k parity/check bits.

Variation (for kids): lie *every* time except once.