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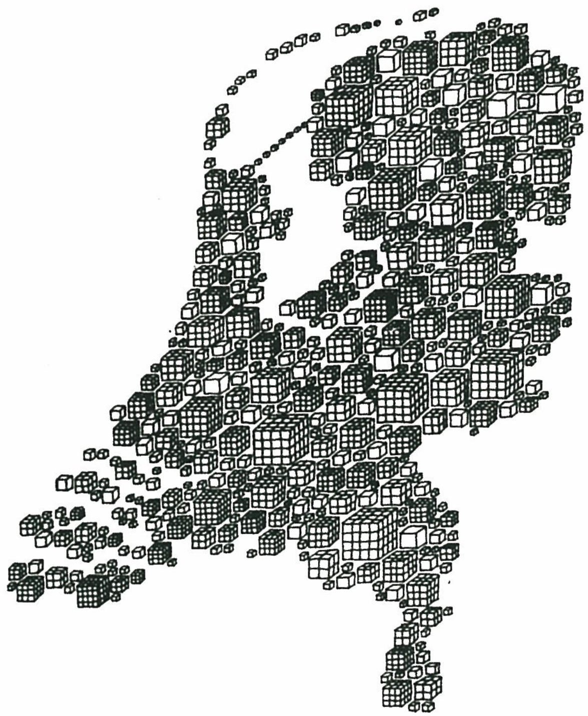


Heemskerkstraat 9 6662 AL ELST The Netherlands

MAGIC, AND IS NHO MAGIC

by Tom Verhoeff

"Wonder, en is gheen wonder"
Simon Stevin (1548-1620)

I bought my Rubik's MAGIC the first week it was available on the East coast of the United States. That was back in October 1986. It looked like another neat toy, but the solution to the advertized problem -- linking the rings -- turned out to be easy. What now? Throw it away, complain to Rubik? No, wait: Let's make up some new problems. For instance, what about a computer program to simulate the moves of this puzzle? For Rubik's CUBE that is almost trivial, even if you don't know how to solve it. And, how many configurations are possible with MAGIC? With a little help from Group Theory one can calculate that for the cube, but what about MAGIC? Obviously these questions only make sense if you can agree on what constitute the legal transformations (moves) and legal configurations (states). Straightening this out may seem easy but is far from trivial, even if you know how to solve MAGIC. Once we agree upon the proper states and moves, it is still difficult to do the counting.

Stevin's intention of his motto quoted above was to convey the idea that the task of the Natural Sciences is to bring understanding without destroying excitement: Nature is "wonderful, but not incomprehensible". I think the same holds for the Sciences of the Artificial, in particular puzzle solving. (In case you wonder, the "h" in the word "nho" imitates the archaic spelling of the corresponding Dutch word "gheen" in Stevin's motto.)

This article will give you an impression of some MAGIC problems I solved. Basically it covers the same material as the talk I gave at Eindhoven University of Technology on February 27, 1987 ("De gevarieerde invarianten van Rubik's magische matje"). By the way, it will also suggest answers to some of the questions that Guus Razoux Schultz posed in the previous issue of *CFF*. There is, of course, much more to tell and explain than I will do here.

States, Moves, and Invariants

One way of capturing the legal states of a puzzle is: first, to give a set of states for which it is obvious that it contains all legal ones (and possibly many more -- hence, I call it the *superset*); second, to point out one *initial* state in this superset; and, third, to define the *moves* that generate allowed state changes. The set of legal states then consists of all states in the superset that can be reached from the initial state by zero or more moves. From this definition it is not always easy to decide whether a particular state in the superset is legal.

For example, for Sam Loyd's 15-puzzle, the superset could be the set of all 4x4 matrices with natural numbers as elements. As initial configuration is usually chosen the matrix

```
 1  2  3  4
 5  6  7  8
 9 10 11 12
13 15 14  0
```

where 0 indicates an empty square. A move is made by swapping a 0 element with any other adjacent element. (Notice that matrices in the given superset can have more than one 0 element or none.) The problem of the 15-puzzle is to find out whether

```
 1  2  3  4
 5  6  7  8
 9 10 11 12
13 14 15  0
```

is a legal state (Loyd offered $1000 for a solution in 1870).

Another way to say something about the legal states of a puzzle is to give certain *invariants*. An invariant of a puzzle is some aspect of the states that is not disturbed by the moves of the puzzle. If we know that aspect for the initial state, then we know it for all legal states. Any state in the superset for which this aspect differs from that of the initial state cannot be a legal state.

One of the invariants for the 15-puzzle is the number of occurrences of each value. This number does not change, since each move swaps 0 with an adjacent element but does not alter the value of the elements. From this invariant and the given initial configuration we conclude that all legal states will have exactly one occurrence of the values 0 through 15. Because this is also true for the second matrix, we still cannot decide whether it is a legal state. Another invariant for the 15-puzzle could be the parity of the matrix (never mind how to compute that). If I tell you that the parity of the initial state is odd and that the second matrix has even parity, then you now know for sure that the latter is not legal.

It is often desirable but not always necessary to find a set of invariants that completely characterizes the set of legal states. That is, we would like to have the following. A state in the superset is legal if and only if it satisfies all invariants of the set under consideration. Here we have used the term invariant in a slightly different sense: a state satisfies an invariant if the intended aspect of this state is the same as that of the initial state. In this sense the second invariant mentioned for the 15-puzzle would be that the parity of the matrix is odd. Note that the "only if" part always holds by definition of invariant; it is the "if" part that is interesting. It can be shown that the two invariants given above for the 15-puzzle form a complete set, although that is irrelevant for solving Loyd's problem.

I would like to make some remarks before proceeding to Rubik's MAGIC. The choice of the superset is often somewhat arbitrary. For instance, we could have chosen a smaller superset for the 15-puzzle right from the start so that the states considered would already satisfy the first invariant. But often you don't know that this is possible until after you discover the relevant invariant. One can also imagine a larger superset: the set of all matrices with natural numbers as elements. In this case one would "discover" as invariant the dimensions of the matrix; for the legal states it happens to be always 4x4. Sometimes the definition of the allowed moves requires a restricted superset (e.g., a square matrix, but not so for the 15-puzzle). Finally, I would like to draw your attention to a problem of abstraction that we have ignored so far (and would like to ignore in the future). When you buy a version of the 15-puzzle you might notice that there are also "intermediate" states, where you have moved a piece with a value printed on it, halfway into the empty position. Should these intermediate states be included in the analysis? And how about moving the whole puzzle from one place in the room to another, is that a relevant change of puzzle's state? One might model these details by a particular choice of superset and definition of allowed moves. But later on one could notice that they are irrelevant as far as the problem of the puzzle is concerned. Nevertheless one should be careful when deciding to incorporate or to ignore certain aspects.

The Invariants of Rubik's MAGIC

I assume that you all have played with Rubik's MAGIC and know about its mechanical construction. I shall use the term *piece* to denote one of its square grooved plastic panels. Each piece has two *faces* and four *edges*. When two pieces are connected by wires I call them *neighbors* of each other and the connection is also referred to as a *hinge*.

What are the legal states and moves of MAGIC? Let's consider the states first. What do we find relevant state information? We obviously abstract from (S1) its position in space. But do we care about (S2) the precise angles between pieces sharing a hinge? About (S3) the 3-dimensional shape (flat rectangle versus octagonal ring)? About (S4) the relative location of a particular piece within the 3D shape (the piece with three intersecting ring segments sits on the corner of the rectangle or somewhere else)? About (S5) the relative orientation of a piece sitting on a particular location of the 3D shape (signature next to copyright or not)? About (S6) the amount of wire stretch or the warping of pieces? About (S7) the distribution of the wires or positions of the metal wire connectors?

As far as "linking the rings" is concerned aspects S3, S4, and S5 are certainly relevant. The inclusion of S7 is left as an exercise to the reader, but I shall ignore it here. I would prefer to ignore S2 and S6. There are, however, some problems, one of them having to do with the impossibility of self-intersection (at each position in space there can be at most one piece of the puzzle, they will not move through one another). I think it is reasonable to impose the restriction that pieces should not be bent, and that the only reason for stretching wires is to compensate for the nonzero thickness of the pieces. For example, it is not legal to roll the octagonal ring inside out, as one would do with a circular rubber strip. I do not recommend trying it, since a great deal of stretch is required (I have never done it).

That brings us to the moves. We agreed to ignore (M1) movement through space as a whole. But what about (M2) rotating two pieces around their common hinge, like opening a book? Or (M3) transferring a hinge connecting two pieces, from one pair of edges to another pair, by folding the pieces face to face and unfolding them in another direction? Or (M4) bending a piece, stretching the wires, or upsetting a hinge by distortion? Or even (M5) taking off the wires? This can be done very simply by inserting a paper clip under one wire on a corner and letting the wire slip around the corner to the other side by lifting it while the paper clip acts as a lever and guide. Sorry guys, no M5 here! Besides M1 we shall also try to abstract from M2. I have not been able to capture M4 in a satisfactory way, so we shall sort of allow it and later come back to it. Obviously, M3 is the key move of the MAGIC puzzle. But if we do not allow M4, then we immediately face an enormous problem. In order to make an M3 move two adjacent pieces have to be folded face to face. In order to do so it may be necessary to rotate other hinges as well (M2) or to do some stretching (M4). The mutual restrictions imposed by the physical structure of the puzzle are very complicated.

Although we have not yet precisely defined what we consider legal states and moves, we now continue to present some invariants for MAGIC. While doing so, we also point out some of the conclusions that can be drawn from them.

(I1) The number of pieces is invariant: no reasonable move changes this; initially 8, hence, always 8. (I2) The number of hinges is invariant: we shall not "look" at the puzzle while transferring a hinge from one edge to another. There are always 8 hinges. This is one of the aspects that is not immediately obvious from the initial state in which MAGIC is sold, but a little experimenting will reveal that the initial state has indeed 8 hinges. I1 and I2 are global invariants, next follow three local ones. (I3) The set of neighbors of each piece is invariant and, hence, also the number of neighbors. That number is always 2, and always the same 2. Conclusion: the pieces always form a single ring and their order in the ring is fixed. It is a local invariant because you can determine that it is violated by inspecting only a small part of the puzzle.

From I1, I2, and I3 one can now deduce all possible flat single-layer configurations if we disregard S2, S4 and S5 for the moment. The 3 possibilities are given below. Each piece is denoted by a pair of square brackets [], hinges are indicated by parallel lines. To understand that the fourth suggestion is impossible observe that the upper left-hand corner piece can be connected to at most one other piece, whereas it should have 2 neighbors according to I3.

```
[]=[]=[]=[]     []=[]=[]     []=[]=[]     []     []
||       ||     ||    ||     ||    ||
[]=[]=[]=[]     []  []=[]    []    []     []  []  []
                ||  ||       ||    ||
                []=[]        []=[]=[]     []  []  []

 Rectangle      Boomerang    Square-      Impossible
                             with-hole

         All flat single-layer configurations
```

One can also derive all types of flat two-layer configurations where each piece of one layer sits face to face with a piece of the other layer. These are not so easy to draw, but the idea is captured by the diagram below (only one layer shown). A piece indicated by curly brackets {} is also connected to the piece that it is facing in the other layer. For simplicity's sake we ignored how such facing pieces are connected, i.e. on what edge their hinge is. (Actually, there is a subtle problem, since two facing neighbors have two "potential" hinges; think about it. I investigated them all but omit details here.)

```
{}=[]=[]={}    []=[]={}    {}=[]       {}  {}    {}={}
               ||             ||       ||  ||    ::  ::
               {}             []={}    []=[]     {}={}

   Bar         L-shape       Duck    U-Square O-Square

       All types of flat two-equal-layer configurations
```

(I4) On each piece the number of wires per channel is invariant: move M3 only exchanges wires. (I5) For each face of a piece the set of faces of neighbors that it can be folded against is invariant. In fact, every face can meet with only one and always the same face from each of its two neighbors. (I6) For each edge of a piece the set of edges with which it can share a hinge is invariant. In fact, every edge can form a hinge with only one and always the same edge of each of its two neighboring pieces. From I5 and I6 one can conclude that of the 4x4x2=32 possible ways to put two pieces edge to edge next to each other only 4 actually occur (they only "roll" around each other without slipping like interlocking gears). This also allows us to compute the number of Rectangles, Boomerangs, and Squares-with-hole when taking S4 and S5 into account. There are 8 pieces as candidate for, say, the upper left-hand corner of the shape; there are 2 directions in which we can lay down our circular chain of pieces: this choice fixes the location of all other pieces within the shape; each piece has 8 orientations (4 rotations and turning upside down), but the orientation of one piece fixes the orientations of all others; That gives us 8x2x8=128 configurations, but the Rectangle has 4 automorphisms (ways in which it can be mapped onto itself by M1 moves), the Boomerang only 2 (identity and rotation of 180 degrees around \-diagonal axis), and the Square-with-hole 8. Hence, there are 32 Rectangles, 64 Boomerangs, and 16 Squares-with-hole imaginable. One can verify that the Square-with-hole satisfies all invariants so far given; nevertheless it turns out to be an illegal configuration.

Now that we know these invariants we can also give a lower bound on the number of moves needed to solve "linking the ring". Observe that each M3 move transfers a hinge from one edge to another edge over one quarter of a turn. We use the number of "quarter turn hinge transfers" as our distance measure. Imagine that the initial Rectangle is facing you with its three disconnected rings and that the signature is in the lower right-hand corner. Let's number the pieces clockwise from 0 to 7 starting at the lower right-hand corner. It is not so difficult to discover how the pieces have to be rearranged in order to produce the desired linked-ring pattern on the back. We have the following configurations.

```
4  5  6  7      5  6  7

3  2  1  0      4  1  0

                3  2

  Initial        Final
```

Notice that the linked rings are on the backside of the final Boomerang, and that its pieces 0, 1, and 5 to 7 are lying with their initial bottom edge at the top (if this machine would allow me to, I'd have drawn these numbers upside down). The list below tells you how many quarter turns each piece has to rotate with respect to its neighbors in order to get from initial to final state. For example, piece 7 needs to roll one quarter turn onto piece 0, and that requires at least one M3 move. Summing the minimum number of moves per pair of neighbors shows that at least 10 M3 quarter turn hinge transfers are required to "link the rings". In fact, such solutions exist (but in general this analysis does not always give you a feasible number of moves). I can perform my 10-move solution in 1 to 2 seconds. It uses a very slick and smooth sequence of moves that is executed almost concurrently. I shall hint at it when discussing notations.

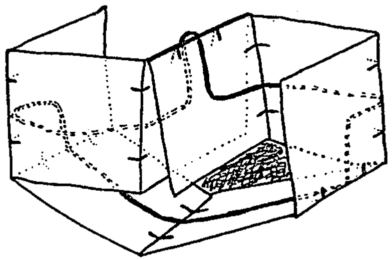
```
piece number:    0   1   2   3   4   5   6   7   0

min. # M3 moves:   2   1   0   0   1   2   2   2  .--> total of 10
```

Next follow again some global invariants. These invariants are more tricky then the previous ones, since their invariance depends on the fact that the puzzle is embedded in a three-dimensional world. In four dimensions they would not be very useful. The invariants I1 through I6 are so-called intrinsic invariants of the puzzle in that they are independent of an embedding.

(I7) The amount of *twist* is invariant. What do I mean by that? Hold MAGIC in one of its octagonal ring configurations. Now imagine cutting one hinge so that you get a long strip of 8 pieces. Restore that hinge after twisting the strip half a turn, or a full turn if you like that better. Well, actually that is impossible because the thing is not flexible enough. But you might transfer some of the other hinges by appropriate flips before restoring the hinge in question. This happens to be possible! The number of turns (not flips) made before restoring is called the amount of twist. It has a sign to indicate the direction. Half a turn of twist would give you a Moebius strip. None of the moves disturbs the amount of twist. Initially it is 0. Therefore, you can never obtain a Moebius strip configuration, or one with a full turn of twist, without cutting of course. The parity of twist (whether it is an odd or even number of half-turns) is obviously also invariant. This invariant is intrinsic, because it distinguishes one-sided from two-sided surfaces. But in four dimensions you could introduce a full turn twist without cutting!

I7 is a global invariant because you cannot check it by inspecting only a small part of the puzzle: no matter how closely you look at each piece, you cannot see the twist. It is sometimes quite difficult to determine the amount of twist of a configuration. I suggest the following method. Take a piece of string. Shrink sufficiently by drinking a MAGIC potion and step onto one face of a piece of the puzzle. Fix the string to the center of that piece. Now walk around the ring of pieces (in an arbitrary direction) while unrolling the string behind you. Once you come back to the starting face of the starting piece, you tie the two string ends together and you unfix the string from the center. If you traversed only 8 faces then the amount of twist is an even number of half turns, otherwise, if you walked over all 16 faces, it is odd. To determine the amount of twist figure out how many times the string is going around the strip of pieces (its winding number). If you can remove the string from the puzzle without untying it, then the winding number is 0. Obviously the Square-with-hole has 0 twist and has, therefore, still not been proven impossible. As another example one might look at the Kissing Couch depicted below (see Fiore, p.82). In this case the string cannot be removed and goes around the puzzle once. Therefore it has a full turn of twist and cannot be obtained from the initial configuration.



(I8) The *knot type* is invariant. Initially the puzzle is unknotted, and no reasonable move introduces knots. As with the amount of twist one might change the knot type by cutting a hinge, then tie your pet knot in the strip, and restore that hinge again. I have not been able to do so, even not after transferring some other hinges. Presumably the strip of 8 pieces is too short for that, but it should not be much of a problem to tie a knot in a longer strip. It is well known that knots disappear in four dimensional space, but I guess the whole puzzle would fall apart there as well, so don't worry. The Square-with-hole is clearly not knotted.

(I9) The total amount of *wrap* is invariant. The (total amount of) wrap of a configuration is computed by alternatingly adding and subtracting the wrap of the individual pieces while going around the ring. OK, what is the wrap of a single piece? Informally, the wrap of a piece encodes the relative position of its two neighbors with respect to each other. To be more precise, let us suppose the configuration we are dealing with is two-sided. We choose one side and a direction for a trip around the ring. Suppose we want to determine the wrap of piece A. Let Z be the neighbor that precedes A in the ring with respect to the chosen direction and let B be its succeeding neighbor. If you walk on the chosen side from Z onto A (moving through B when necessary), in which direction should you turn to enter B? If B is straight ahead of Z, we define the wrap of A as 0; if you make a left turn, then it is +1; for a right turn it is -1. And what if you turn back, that is, exit A via the same edge that you entered it on? Then there are two cases (this is probably the most important observation to be made about Rubik's MAGIC): the wrap will be either +2 or -2, but not arbitrarily. Before I tell you which is taken when, we consider in more detail the case where both neighbors are on the same edge. In order to transfer a hinge from one edge to another two pieces have to be folded face to face. Since each piece has two faces there are generally two edges that a hinge can move to: a quarter turn left or a quarter turn right. In the case we are considering, however, Z can be made to meet only one face of A, since B blocks the other face (and similarly for B). If we do the hinge transfer that corresponds to the one possible face-to-face folding of Z and A, then we are no longer in the situation where both A's neighbors are on the same edge. If A's wrap in that new situation is +1, then before the hinge transfer it was +2; if it is -1, then it was -2; of course, it cannot be 0, since a move transfers a hinge over one quarter turn only. The wrap need not grow beyond the limits -2 and +2 because of the blocking alluded to above. So the wrap of a piece can take on five values (not just four). When an M3 move (hinge transfer) is made the wrap of both pieces involved either increases or decreases by one. Remember that for one piece the outgoing edge is moving, whereas for the other piece the incoming edge is moving (the hinge between Z and A is A's incoming edge, the one between A and B its outgoing edge). Because of the alternating addition and subtraction, the total amount of wrap does not change under M3 moves. In a way, the pieces have alternating orientations like a series of revolving gears.

Let's look at some examples. The wrap of the octagonal ring configuration is probably easiest to compute. Since the tour around the ring involves no turns, it has wrap 0. Below are given the individual wraps of all pieces on the Rectangle, Boomerang, and Square-with-hole if we tour it clockwise on the side facing us. Remember that left turns are positive.

```
-1  0  0 -1     -1  0 -1      -1  0 -1

-1  0  0 -1      0 +1 -1       0     0

                -1 -1         -1  0 -1
```

Computing the alternating sums yields wrap 0 for Rectangle and Boomerang, but wrap 4 for the Square-with-hole, which thereby is dismantled as an illegal configuration. A similar argument is given by Nourse.

Notation for MAGIC Shapes

The wraps of the individual pieces give us a notation that characterizes configurations (however also see invariant I10 below). I have grown accustomed to abbreviating the wraps -2, -1, +1, and +2 by =, -, +, and # respectively. To complete the notation one must agree upon identifiers for pieces and edges. Let's use the numbering suggested above when discussing the minimum number of moves for a solution. And let's identify the edges of piece 0 by their compass reading, i.e., W(est), N(orth), E(ast), and S(outh) as they appear in the initial state. I suggest the following notation. First we write the identifier of the edge of piece 0 that connects it to piece 7, then we write the wraps of pieces 0 to 7 as computed in this order when traveling over the "disconnected rings" side. Remember that by invariants I5 and I6 this also fixes the relative orientations of all other pieces within this shape, so we don't have to specify by which edges these are connected and which faces are on what side. By computing the total wrap from a code we can determine if it is legal.

For example, the initial Rectangle has code N-00--00-; the final Boomerang has code S-+--0-0-. The codes E00000000 and N======== represent the obvious ring and star configurations respectively. It is also easy to define the allowed moves in this notation. A move consists of either increasing or decreasing by one the wrap of two neighboring pieces (remember that 0 and 7 are neighbors) under the constraint that in the resulting code all pieces have a admissable wrap (in the range -2 to +2). The following list of moves and intermediate configurations summarizes my 10-move solution to the "link the ring" problem. Notice that all minimal solutions must have the same moves, only their order can differ.

```
   code     move number

N-00--00-
      --         1
N-00-----
  --             2
N--------
     ++          3
N----00--
 ++              4
N00--00--
 -      -        5
W-0--00-=
       ++        6
W-0--000-
      --         7
W-0--0---
 -      -        8
S=0--0--=
 ++              9
S-+--0--=
       ++       10
S-+--0-0-
```

Actually, in my solution moves 1 & 2, 3 & 4, and 9 & 10 are performed concurrently. The first 4 moves form a very nice sequence: I grab the two diagonally opposite pieces 1 & 5 (without signature), right thumb on 1 and left thumb on 5, perform moves 1 & 2 without unfolding all the way (fold into U-square and flip sides up), then rotate my left hand half a turn to the left with respect to the right hand, and finally perform moves 3 & 4 (release into U-square again and unfold) to obtain the Rectangle N00--00--. For a picture of the configuration N-------- between moves 1 & 2 and 3 & 4 look at page 54 of Fiore. Guus's solution differs from mine only in the order of moves 3, 4, and 5, but this is an essential difference.

There is a certain arbitrariness to a description in my notation that can only be remedied by incorporating S2 and M2 in it. That is, it does not document preparatory M2 hinge rotations that do not transfer hinges. Usually, however, these can be inferred. Furthermore, if two neighbors are face to face then it is not clear what code to write down, since two conflicting hinges exist, at least potentially. In the latter situation I tend to delay the actual recording of a hinge transferral until the pieces are unfolded again, because only then does one know whether the hinge really moved (they could have opened the same way as they were folded together, in which case no hinge was transferred).

Finally, let me discuss what aspects are not covered by invariants I1 to I9. As far as I can tell, I1 to I9 do not form a complete set of invariants. That is, I know configurations that satisfy all 9 invariants, but that nevertheless cannot be obtained reasonably from the initial configuration. For example, try to fold #0#0=0=, a box with two pieces pointing in and two pieces sticking out.

(I10) This invariant should account for the rigidity of pieces and hinges, and the fact that self-intersection is not allowed, but I cannot express it otherwise. For example, to obtain a cube shape one has to make an M2 move that rotates all hinges (try to prove that you did not stretch their wires) and two pieces tend to collide unless it is carefully executed. The problem of rigidity has to do with the degrees of freedom of a mechanical system and is studied by a branch of mathematics known as structural topology. The degrees of freedom of a configuration depend on its 3D shape. For example, in the ring all motion is confined to a plane. It must have enough degrees of freedom to allow certain combinations of hinge rotations. Architects are interested in similar problems, but they usually want systems with few degrees of freedom, that is, structures that do not collapse.

Before proceeding to the next section I want to mention a few points that are related to I10 but of secondary interest. What about the nonzero thickness of the pieces: is that really a limitation? That is, was it intended to be a limitation? What about the friction in the hinges? I know moves from one configuration to another that are in a sense oneway. Experiment with Garage/Box ++-+00+- by "pushing in" its door/lid. Reversing this M2 move is almost impossible due to friction and inaccessibility of pieces. Inaccessibility is also illustrated painfully well by some of the cube shapes where the two "unused" pieces are sitting inside, so they can hardly be manipulated.

Counting Configurations

How many configurations are there, if we consider them to be defined by the codes introduced in the previous section? There are four edges of piece 0 that can hinge with piece 7, and each piece can have one of five wrap values. That amounts to $4 \times 5^8 = 1{,}562{,}500$ configurations. But wait a moment, the total wrap should be zero. To incorporate that into the counting, first observe (or simply accept) that the number of octuples over {-2,..,+2} with alternating sum zero is the same as the number of octuples over the same set with sum zero (no alternation). Next consider the function $f(w)=(w^{-2}+w^{-1}+w^{0}+w^{1}+w^{2})$, for which $f(1)=5$. Hence, there are as many configurations with wrap 0 as the coefficient of w^0 in $4 \times f(w)^8$, which turns out to be 152,660. We have now counted the number of configurations of MAGIC, where all pieces, faces, and edges are distinguishable. If the edges would be indistinguishable -- for example, because the faces have rotation symmetric pattern on them -- then we should divide out a factor 4.

The counting becomes more interesting if we also decide to ignore the identity of the pieces and the faces. That is, if we consider a MAGIC where all pieces have the same color and no pattern. Now we must resort to more powerful counting techniques, such as the extended Polya enumeration theorems. That also involves some group theory. The group that describes the symmetry for this single-color MAGIC is known as the Dihedral group D8, generated by a cyclic permutation of order 8 and a reflection. These permutations correspond to shifting the pieces one position through the ring and to reversing the order of traversal. We want to treat configuration codes that can be transformed into each other by cyclic permutation or reversing as equivalent and to be counted only once. For example, the Rectangles ++00++00 and +00++00+ are equivalent. Furthermore we would like to treat codes with all signs reversed as equivalent (Rectangles ++00++00 and --00--00). When this is all properly taken into account one finds that there are 1351 different shapes for the single-colored MAGIC. For the fanatics I have a list of all 1351 normalized codes. I investigated all configurations that have no pieces with wrap +2 or -2; there are only 59 such configurations.

To do the counting when taking rigidity into account seems almost hopeless to me. Be aware that one configuration code can make its appearance in a number of different 3D shapes that cannot be transformed into one another without doing some M3 moves. A good example is House N00-0000- which has four (or more?) appearances. I shall not address the "inside-out" confusion. It is also the case that different codes may look similar, but must be treated very differently in order to disentangle. A very good teaser is comparing #+--+#00 and #00#-++-, nicknamed Table Viper and Deadly Viper (see Fiore, p.79).

By the way, did you know that you can fold a cube from the initial configuration by transferring only one hinge?

Conclusion

MAGIC presents us with lots of fun and open problems: discovering new shapes, describing its state space, finding minimal solutions, counting the number of configurations under certain indistinguishability assumptions, etc. I hope that this article opens up the way to new approaches and new problems. Thanks are due to my friends Jurjen and Jelte for sharing their MAGIC experiences with me. Feel free to comment on any of my assumptions, explanations, or what have you. Please send your reactions and questions to the editor of *CFF* or directly to: *Tom Verhoeff*

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