$(0,\cdots,0,1,p-1),(0,0,\cdots,0,1)$. The corresponding weight sequence $W(H)$ ends by r unit vectors. Therefore, if $t=(p-1)/2$, $C=C_{k,d}^{N}$ and $q=k-d+1=p$ any of p^r subcodes $C(W(H),g)$, $g\epsilon F_p^r$, contains a systematic code with redundancy $r=\lceil\log_p(2tN+1)\rceil$; moreover, if $2tN+1=p^r$, then the code C is partitioned into p^r perfect codes $C(W(H),g)$ capable of correcting single peak-shifts of size t, each of which is a systematic code with redundancy $r=\log_p(2tN+1)$.

Example: Let $t=1$, $p=3$, $q=k-d+1=3$, and $N=13$. Then, $2tN+1=p^3$ and, thus, $r=3$.
Then,

$$H=\begin{array}{ccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ \{0 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 1 & 0\} \\ 0 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 0 & 1 & 0 & 2 & 1 \end{array}$$

and

$$W(H)=\begin{array}{ccccccccccccc} 0 & 2 & 1 & 0 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ \{0 & 0 & 2 & 2 & 2 & 1 & 0 & 1 & 2 & 1 & 0 & 1 & 0\} \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}$$

give rise to systematic (d,k)-codes of reduced length 13 and of redundancy 3, which are perfect codes capable of correcting single peak-shifts of size 1 for any d and k such that $k-d=2$.

VI. CONCLUSION

We introduce a definition of arbitrary (d,k)- and perfect (d,k)-codes capable correcting single peak-shifts of given size t. For the construction of perfect codes we use a general combinatorial method connected with finding "good" weight sequences in Abelian groups and introduce the concept of perfect t-shift N-designs. We give explicit constructions of such designs for $t=1$, $t=2$, and $t=(p-1)/2$, where p is a prime. Our construction is not only effective, but also universal in the sense that it does not depend on the (d,k)-constraints. It also allows to correct automatically those peak-shifts that violate (d,k)-constraints and to determine the beginning of the next codeword.

For an ideal multibit peak-shift channel, decoding errors that do not occur in the Nth substring do not propagate to subsequent blocks, as the length of the codeword does not change. However, if a decoding error occurs in the Nth substring, the first symbol of the next block is in error. If $\pm tw_1$ is a valid syndrome, we make a decoding error in this block. Only if again in the Nth substring a decoding error occurs, we may speak of error propagation. By appropriate choice of w_1 we may avoid this phenomenon.

Catastrophic error propagation occurs whenever random errors are involved. These errors completely ruin the structure of the codewords. They insert new phrases or delete existing phrases in a codeword and thus synchronization regarding the beginning of the first symbol of a codeword is completely lost. One way to solve this problem is to fix the length of the codeword to a certain value. We can construct codes with a fixed binary length L by considering the union of all codewords of binary length L belonging to the (d,k)-codes of reduced length N, $L/(k+1)\leq N\leq L/(d+1)$. The codewords of fixed binary length start with d zeros and end with a symbol equal to 1. These codewords can be stored without merging digits.

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An Updated Table of Minimum-Distance Bounds for Binary Linear Codes

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***Abstract*—Tables with lower and upper bounds for $d_{\max}(n,k)$, the maximum possible minimum distance of a binary linear code with word length n and dimension k are shown.**

***Index Terms*—Binary linear codes, lower and upper bounds on minimum distance**

I. NEW TABLE OF BOUNDS

We present some corrections and a further update on [64]. The update consists of new Tables I (Bounds), II (Index), and V (Statistics), and new lists of Labels and References. We also add one more condition to the list of Formal Invariance Conditions (see below). We adhere to the terminology used in [64], but here the references that appear in the table of bounds will be called *labels* (to avoid confusion with regular references).

The following corrections have been incorporated. The [69, 12, 29] code (labeled MS) from [40] has been withdrawn since it is not linear [67]. The [73, 49, 9] code (labeled V) from [35] has been withdrawn since it was in fact a [73, 46, 9] code (cf. [44]). Part of the damage done to the table by this withdrawal is repaired using a [64, 40, 9] Goppa code and a [68, 42, 10] code found by Shearer. It is quite

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possible that better codes are known. By the way, Table III (Formal Invariance Conditions) contains several typographical errors, to be corrected as follows: P3-lower should read

$$Lb[n-1,k] \geq Lb[n,k]-1.$$

P3-upper should read

$$Ub[n+1,k] \leq Ub[n,k]+1.$$

B-lower should read

$$Lb[n-s,k-s+1] \geq Lb[n,k].$$

where $s = Ub[n,n-k]$, and E should read

$$Ub[n+2\cdot Ub[n,k]+1,k+1] \leq 2\cdot Ub[n,k].$$

The new B2-lower and B2-upper (cf. Proposition 1) read

$$Lb[n-s,k-s+2j+1] \geq Lb[n,k]-2j$$

and

$$Ub[n+s,k+s-2j-1] \leq Ub[n,k]+2j,$$

for $Ub[n+s,n-k+1+2j] \leq s$ and $0 \leq j < (s-1)/2$. Wiseman [70] tells us that various codes already occur in his 1981 thesis; this led to a change of attribution for 3 codes.

The following changes have been made. Labels C1 and C2 have been merged into Ch, because the latter covers both (some of these labels have moved up or down one entry). Label C3 has been renamed to CS. Label HY has been replaced by DH since the latter is based on an extension (of the report supporting the former), which contains more results and is more readily accessible. Labels J and K have been replaced by Jo1 and Jo2 (simple Johnson bound [31] and full Johnson bound [32]), except that the claim $Ub[126,113] \leq 4$, which is not a consequence of the Johnson bound but follows from a simple improvement (cf. [6]) has been labeled Jo+. Label S has been included in the list of Labels (to give an earlier reference for the existence of a [89, 11, 40] code). Various other labels have been changed so as to evoke more easily the name(s) of the corresponding author(s).

This update is based on the codes and nonexistence results reported in [6, 7, 11, 13, 14, 15, 17, 12, 19, 22, 23, 28, 29, 50, 51, 61, 62, 63, 67, 68, 71] (and constructions B2, B2x, Q, QC defined here), of which 400 "survived." As a consequence, 622 lower bounds and 3030 upper bounds have been improved ([6] alone yielded 2823 improved upper bounds), establishing $d_{\max}$ for 210 more parameter pairs (in comparison to the previous table *without* the now withdrawn MS and V codes). The former labels HC, MS, Pa, Q and R disappeared. It should be noted that $d_{\max}$ is now completely determined for length at most 27.

Further improvements are still welcome and can be sent to A. E. Brouwer (aeb@cwi.nl) or Henk van Tilborg at the same address. We accept only results accompanied by complete proofs, explicit constructions, or generator or parity check matrices. Additional information on new codes, like weight enumerator or minimum distance of dual code, is also much appreciated.

II. A GENERALIZATION OF CONSTRUCTION Y1

Proposition 1: Suppose a binary $[n,k,d]$-code C exists, and let s be an integer not smaller than the minimum distance of the dual code $C^{\perp}$. Then a $[n-s,k-s+2j+1,d-2j]$-code exists for each j with $0 \leq j < (s-1)/2$.

Proof: Let w be a word of nonzero weight $t \leq s$ in $C^{\perp}$, and let W be its support. Let I and J be disjoint sets of coordinate positions such that $W \subseteq I \cup J$, $|I| = s-2j-1$, $|J| = 2j+1$ and $|W \cap J|$ is odd. Then the code D obtained from C by shortening at the positions of I and puncturing at the positions of J has word length $n-2$, dimension (at least) $k-s+2j+1$ and minimum distance (at least) $d-2j$. □

This simple observation yields a number of improvements in the table, if we take $s = Ub(n,n-k)$, the smallest known upper bound for the minimum distance of $[n,n-k]$-codes. For example, there exist codes with parameters [85, 10, 38], [114, 22, 38], [115, 28, 33], [116, 13, 48], [117, 11, 51], [117, 27, 36], [121, 14, 51], [122, 12, 54], found using $s = 2j+2 = 4$, 8, 12, 6, 4, 10, 6, 4, respectively. (The required bounds $Ub(122,99) \leq 8$ and $Ub(127,99) \leq 10$ follow from the Johnson bound). Since it was known already (Corollary 3.4 [29]) that $Ub(85,10) \leq 38$, it follows that $d_{\max}(85,10) = 38$.

III. LABELS

No explicit label is given for 1) trivial codes $[n,1,n]$ and $[n,n,1]$, 2) codes obtained by adding a parity check [40, p. 27], 3) codes obtained by puncturing (truncation) [40, p. 28], 4) codes obtained by shortening [40, p. 29].

A	Residual code construction [25] and [40, p. 593]. When this reference appears on the lower bound at (n,k), it refers back to the lower bound at $(n+2\cdot Lb[n,k],k+1)$.
AEB	Improved linear programming bound for linear codes [6].
AI, AI2	Algebraically punctured codes and incidence matrix codes [1], and code extension by construction XX [2].
B	Construction Y1 of [53] followed by repeated shortening [25] (i.e., case $j = 0$ of Proposition 1). When this reference appears on the lower bound at (n,k), it refers back to the lower bound at $(n+s,k+s-1)$ and the upper bound at $(n+s,n-k+1)$ for some s with $s \geq Ub[n+s,n-k+1]$. When it appears on the upper bound at (n,k), it refers back to the upper bounds at $(n,n-k)$ and $(n-s,k-s+1)$ where $s = Ub[n,n-k]$.
B2	Shortening and puncturing, using (an upper bound for) the minimum distance of the dual code. (Proposition 1.)
B2x	Idem, but using specific information on the minimum distance $d^{\perp}$ of the dual code (not derived from the table). For the moment two such ingredients are used: the LC [55, 16, 19]-code has $d^{\perp} = 6$, and the SW [117, 36, 32]-code has $d^{\perp} = 9$ (The old construction Y1 is a special case of this.)
BCH	Primitive Bose–Ray Chaudhuri–Hocquenguem codes [41, pp. 166–167], [36].
BM	Improvement on Griesmer bound [3].
C	Concatenation [40, p. 76]. When this reference appears on the lower bound at (n,k), it refers back to lower bounds at (m,k) and $(n-m,k)$ for some $m,k \leq m < n$.
CDJ	A [60, 17, 20] code [7].
Ch	Extended and concatenated codes [9].
CLS	A [45, 13, 16] code [11].
CS	Codes from symmetry groups [10].
D	$(u\|u+v)$ construction [52], [40, p. 76]. When this reference appears on the lower bound at $(2n,k)$, it

	refers back to lower bounds at (n, j) and $(n, k-j)$ for some j, $j < k$.
Das	Nonexistence of a [66, 13, 28] code [13].
DEI	Nonexistence of certain codes [14].
DH	Bounds on eight-dimensional codes [15].
DJ	Construction of [45, 16, 13] and [51, 25, 11] codes [17].
DK	Nonexistence of certain codes [12].
DM	Improvement on Griesmer bound [16].
E	One-step Griesmer upper bound (converse of A) [20] and [25]. When this reference appears on the upper bound at (n, k), it refers back to the upper bound at $(n - Ub[n, k] - 1, k - 1)$.
FP	No [12, 5, 5] codes exist [18].
G	A [64, 40, 9] Goppa code [40, p. 345].
GG, GG1	Construction of various codes by Groneick and Grosse [19, 5]
Gr	Griesmer codes [20].
Gu2	Construction of certain codes [22].
GuB	Systematic quasi-cyclic codes [23].
Hg	Alternant codes [27].
HP	Computer search [24].
HT	Nonexistence of certain codes [29].
HY2	A [33, 8, 14] code [28].
Je	Row-cyclic codes [30].
Jo1	Johnson upper bound [31].
Jo2	Johnson improved upper bound [32].
Jo+	Improved version of the Johnson bound for linear codes [6].
Ka	Construction using BCH codes [34].
L	Karlin circulant codes [33].
LC	A [55, 16, 19] Goppa code [38].
LP	Ruled out by Delsarte's Linear Programming bound (as computed by Berntzen and Kemper and others).
Lv	Improvement on Griesmer bound [39].
N	Cyclic codes up to length 63 [8].
O	Adryanov–Saskovets construction [4, p. 333]
Pi, Pi2	Codes derived from cyclic codes [42], and a quasi-cyclic [27, 10, 9] code [43].
PT	Cyclic codes of length 69 to 99 [44].
PTX	Idem, combined with construction X of [53]. (The constructions of [88, 17, 32] and [95, 55, 14] were pointed out by L. Tolhuizen.)
Pu, Pu2	Codes constructed using Wiseman's method [45, p. 72], and no [26, 13, 8] codes exist [46].
Q	Construct a $[3n, 2k, 2d]$ binary code from a $[n, k, d]$ quaternary code. Here applied to quaternary [28, 4, 20] and [40, 5, 28] codes. (For these, see [21] and [37]).
QC	Construction of binary [102, 37, 24] and [105, 39, 24] codes as generalized concatenated codes, using quaternary [26, 11, 12] and [27, 12, 12] codes and a binary [24, 14, 6] code (and a 3–row all-1 vector).
Ro	Shortened Goppa codes [48].
RR	Rao code and shortened Rao code [47].
S	Cyclic codes [4, p. 433].
Sh, Sh2	Computer search [50].
Si	No [25, 15, 6] codes exist [51].
SRC	Sloane–Reddy–Chen [53].
SS	Algebraically punctured cyclic codes [54].
Su	Constructions using Goppa codes [55].
SW	Constructions of cyclic codes [49].
vT1	Quasi-cyclic codes [56].
vT2	Codes meeting the Griesmer bound [57].
vT3	Bounds on seven-dimensional codes [58].
vT4	No [55, 7, 26] code exists [59].
To, To2, To3	Construction of various generalized concatenated codes [60, p. 50], [62], [63].
Wa	Wagner [65].
We	Concatenated codes [66].
Wi	Concatenated codes [69, 70].
Wz	Constructions using geometric Goppa and other codes [67], inspired by [9].
Wz2	A [44, 9, 18] code found by simulated annealing [68].
X	Non-primitive BCH codes, their duals, and shortened BCH codes [26].
YH1	There is no [25, 8, 10] code [71].
ZL	Generalized cascade codes [73].
Zv	Lengthened codes [72].

IV. ADDED IN PROOF

Since the previous text was written, four more codes have been withdrawn. The [79, 9, 35] code (labeled Pu) from [45] has been withdrawn since it was in fact a [79, 9, 33] code. The [66, 14, 25] and [88, 30, 24] codes (labeled L) from [33] have been withdrawn; we find only [65, 14, 23] and [88, 30, 23] codes in the way indicated. The [66, 24, 17] code (labeled SRC) from [53] has been withdrawn by one of its authors.

Many new codes were found in the meantime. Groneick and Gross [19] continued with a steady stream of constructions and found codes with parameters[81, 31, 20], [83, 17, 29], [85, 13, 34], [85, 22, 26], [87, 13, 35], [87, 36, 20], [88, 15, 34], [90, 15, 35], [93, 13, 38], [94, 24, 28], [95, 13, 39], [95, 23, 31], [96, 15, 38], [97, 18, 33], [98, 15, 39], [98, 24, 29], [98, 26, 27], [98, 26, 25], [101, 24, 31], [102, 31, 25], [105, 18, 38], [105, 23, 33], [109, 23, 35], [111, 9, 52], [111, 12, 48], [114, 9, 53], [124, 10, 57], and [127, 10, 59].

Farkaš and Brühl [74] constructed [37, 9, 15], [42, 12, 15], and [48, 16, 15] codes.

Gulliver (in a personal communication) constructed [45, 10, 18], [85, 9, 39], and [105, 11, 46] codes.

These updates have already been incorporated in Tables I and II. See the following pages for Tables I, II, and V.

TABLE I, Column 1
Bounds on Minimum-Distance

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k/n
1	1																									1
2	2	1																								2
3	3	2	1																							3
4	4	A 2	2	1																						4
5	5	3	2	2	1																					5
6	6	4	3	2	2	1																				6
7	7	A 4	4	3	2	2	1																			7
8	8	C 5	4	D 4	B 2	2	2	1																		8
9	9	6	A 4	4	3	2	2	2	1																	9
10	10	A 6	5	4	4	3	2	2	2	1																10
11	11	C 7	6	5	4	4	3	2	2	2	1															11
12	12	8	A 6	D 6	FP 4	4	4	3	2	2	2	1														12
13	13	A 8	7	6	5	4	4	4	3	2	2	2	1													13
14	14	C 9	8	7	6	5	4	4	4	3	2	2	2	1												14
15	15	10	8	8	7	6	5	4	4	4	3	2	2	2	1											15
16	16	A 10	A 8	8	D 8	A 6	6	5	4	4	D 4	B 2	2	2	2	1										16
17	17	C 11	C 9	8	8	7	6	6	N 5	4	4	3	2	2	2	2	1									17
18	18	12	10	A 8	8	8	7	6	6	4	4	4	3	2	2	2	2	1								18
19	19	A 12	A 10	9	8	8	8	7	6	B 5	4	4	4	3	2	2	2	2	1							19
20	20	C 13	C 11	10	Gr 9	8	8	8	7	6	5	4	4	4	3	2	2	2	2	1						20
21	21	14	12	A 10	10	E 8	8	8	8	7	6	5	4	4	4	3	2	2	2	2	1					21
22	22	A 14	12	11	10	9	8	8	8	8	7	6	5	4	4	4	3	2	2	2	2	1				22
23	23	C 15	A 12	12	11	10	HP 9	8	8	8	8	N 7	6	Mn 5	4	4	4	3	2	2	2	2	1			23
24	24	16	C 13	12	D 12	E 10	10	8	8	8	8	8	6	6	4	4	4	4	3	2	2	2	2	1		24
25	25	A 16	14	A 12	12	11	10	YH1 9	8	8	8	8	6	6	Si 5	4	4	4	4	3	2	2	2	2	1	25
26	26	C 17	A 14	13	12	12	11	10	9	8	8	8	Pu2 7	6	6	5	4	4	4	4	3	2	2	2	2	26
27	27	18	C 15	14	13	12	A 12	10	10	Pi2 9	8	8	8	L 7	6	6	5	4	4	4	4	3	2	2	2	27
28	28	A 18	16	A 14	D 14	BM 12	12	DM 11	10	10	8-9	8	8	8	6-7	6	6	5	4	4	4	4	3	2	2	28
29	29	C 19	16	15	14	13	12	12	11	10	9-10	8-9	8	8	7-8	6-7	6	6	5	4	4	4	4	3	2	29
30	30	20	A 16	16	15	14	12	12	12	11	10	9-10	8-9	8	8	7-8	6-7	6	6	5	4	4	4	4	3	30
31	31	A 20	C 17	16	16	15	vT3 13	12	12	12	BCH 11	10	9-10	8-9	8	8	7-8	6-7	6	6	5	4	4	4	4	31
32	32	C 21	18	16	16	D 16	14	13	12	12	12	10	Sh 10	8-10	B 8	8	CS 8	6-8	6-7	6	6	5	4	4	4	32
33	33	22	A 18	A 16	16	16	14	HY2 14	12-13	12	12	HT 11	10	9-10	8-9	8	8	7-8	6-8	6-7	6	6	5	4	4	33
34	34	A 22	C 19	C 17	16	16	vT3 15	14	13-14	12-13	12	Sh 12	10-11	10	9-10	8-9	8	8	7-8	6-8	6	6	Sh 6	4-5	4	34
35	35	C 23	20	18	A 16	16	16	15	Fi 14	12-14	12	12	11-12	10-11	10	9-10	8-9	8	8	7-8	AEB 6-7	6	6	5-6	4-5	35
36	36	24	20	A 18	17	16	16	DH 16	14	13-14	HT 12-13	12	B 12	10-12	10-11	Sh 10	8-10	8-9	8	8	7-8	6-7	6	6	5-6	36
37	37	A 24	A 20	C 19	18	Gr 17	16	16	FB HT 15	14	O 13-14	12-13	12	11-12	10-12	10-11	9-10	8-10	8-9	8	8	7-8	6-7	6	6	37
38	38	C 25	C 21	20	A 18	18	E 16	16	16	14-15	14	12-14	12-13	12	11-12	10-12	10-11	9-10	8-10	8-9	8	Sh 8	6-8	6-7	6	38
39	39	26	22	20	C 19	18	vT3 17	16	16	Ev 15-16	14-15	12-14	12-14	12-13	12	11-12	10-12	10-11	9-10	8-10	8-9	8	7-8	6-8	6-7	39
40	40	A 26	A 22	A 20	20	BM 18	18	16-17	16	16	14-16	13-14	12-14	12-14	12-13	12	11-12	10-12	10-11	9-10	8-10	8-9	8	7-8	6-8	40
41	41	C 27	C 23	C 21	20	19	18	17-18	E 16	16	15-16	HT 14-15	13-14	12-14	12-14	12	12	11-12	10-12	10-11	N 9-10	8-10	8-9	8	7-8	41
42	42	28	24	22	A 20	20	vT1 19	DM 18	16-17	16	16	FB 15-16	14-15	13-14	12-14	AEB 12-13	12	12	11-12	10-12	10	8-10	8-10	8-9	8	42
43	43	A 28	24	A 22	21	20	20	18	17-18	16-17	16	16	14-16	14-15	N 13-14	12-14	12-13	12	12	11-12	HT 10-11	9-10	8-10	8-10	8-9	43
44	44	C 29	A 24	C 23	22	21	20	DM 19	18	17-18	16-17	16	15-16	14-16	14-15	12-14	12-14	12-13	12	12	11-12	10-11	9-10	8-10	8-10	44
45	45	30	C 25	24	A 22	SS 22	E 20	DM 20	18-19	Gu3 18	16-18	16-17	CLS 16	14-16	14-16	DJ 13-14	12-14	12-14	12-13	12	12	11-12	10-11	9-10	8-10	45
46	46	A 30	26	24	23	22	21	20	GG1 19-20	18-19	GG1 17-18	16-18	16-17	15-16	14-16	DR 14-15	12-14	12-14	12-14	12-13	12	12	11-12	10-11	8-10	46
47	47	C 31	A 26	24	24	23	22	21	20	18-20	18-19	16-18	16-18	16	15-16	14-16	13-15	12-14	12-14	12-14	12-13	12	12	N 11-12	9-11	47
48	48	32	C 27	A 24	24	D 24	E 22	DM 22	E 20	19-20	18-20	17-18	16-18	B 16-17	16	FB 15-16	Jo 14-16	12-15	12-14	12-14	12-14	12-13	12	12	10-12	48
49	49	A 32	28	C 25	24	24	23	22	20-21	20	B2x 19-20	HT 18-19	B2x 17-18	16-18	16-17	16	14-16	13-16	12-15	12-14	12-14	12-14	12-13	12	10-12	49
50	50	C 33	28	26	A 24	24	24	23	20-22	20-21	20	18-20	18-19	16-18	16-18	16-17	15-16	14-16	13-16	12-15	12-14	12-14	12-14	12-13	10-12	50
51	51	34	A 28	A 26	25	24	24	N 24	21-22	20-22	20-21	19-20	18-20	17-19	16-18	16-18	N 16-17	14-16	N 14-16	12-16	12-15	12-14	12-14	12-14	DJ 11-13	51
52	52	A 34	C 29	C 27	26	Gr 25	24	24	22	Pu 21-22	20-22	20-21	19-20	18-20	17-19	16-18	16-18	15-17	14-16	13-16	12-16	12-15	12-14	12-14	12-14	52
53	53	C 35	30	28	A 26	26	24	24	DEI 22-23	22	20-22	20-22	E 20	19-20	18-20	17-19	16-18	16-18	15-17	14-16	13-16	12-16	12-15	12-14	12-14	53
54	54	36	A 30	28	27	26	24	24	23-24	22-23	GG1 21-22	20-22	20-21	20	19-20	18-20	16-19	16-18	16-18	15-17	14-16	13-16	12-16	12-15	12-14	54
55	55	A 36	C 31	A 28	28	27	Ev vT4 25	24	24	Ev 23-24	22-23	20-22	20-22	20-21	20	LC 19-20	16-20	16-18	16-18	16-18	N 15-17	14-16	GG1 13-16	12-16	12-15	55
56	56	C 37	32	C 29	28	D 28	26	24-25	24	24	22-24	21-23	20-22	20-22	20-21	20	16-20	AEB 16-19	16-18	16-18	16-18	14-17	14-16	12-16	12-16	56
57	57	38	32	30	A 28	28	26	25-26	24-25	24	SBC 23-24	22-24	21-22	20-22	20-22	20	17-20	16-20	16-19	16-18	16-18	15-18	14-17	13-16	12-16	57
58	58	A 38	A 32	A 30	29	28	A vT3 27	DH 26	24-26	B 24	24	22-24	GG1 HT 22-23	20-22	20-22	AEB 20-21	18-20	16-20	16-20	16-19	16-18	16-18	15-18	14-17	13-16	58
59	59	C 39	C 33	C 31	30	29	28	26-27	25-26	24-25	24	23-24	22-24	21-23	20-22	20-22	19-21	17-20	16-20	16-20	16-19	16-18	16-18	15-18	B 14-16	59
60	60	40	34	32	A 30	D 30	28	26-28	26	Ch 25-26	24-25	24	23-24	22-24	21-23	20-22	CDJ 20-22	18-21	17-20	16-20	16-20	16-19	16-18	16-18	15-17	60
n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k\n

TABLE I, COLUMN 1 *(CONTINUED)*

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k/n
		A	A				Lv		DEI																	
61	61	40	34	32	31	30	29	27-28	26-27	26	24-26	24-25	24	23-24	22-24	21-22	20-22	19-22	18-21	17-20	16-20	16-20	16-19	16-18	16-18	61
		C	C					E								AEB										
62	62	41	35	32	32	31	30	28	27-28	26-27	25-26	24-26	24-25	24	23-24	22-23	21-22	20-22	18-22	18-21	17-20	16-20	16-20	16-19	16-18	62
				A						BCH	E					BCH		BCH	E		E					
63	63	42	36	32	32	32	31	28-29	28	27-28	26-27	24-26	24-26	24-25	24	23-24	22-23	21-22	19-22	18-22	18-21	16-20	16-20	16-20	16-19	63
		A		C			D																			
64	64	42	36	33	32	32	32	29-30	28	28	26-28	24-27	24-26	24-26	24	24	22-24	22-23	20-22	18-22	18-22	16-21	16-20	16-20	16-20	64
		C	A					DH	DEI		Ch				B											
65	65	43	36	34	32	32	32	30	28-29	28	27-28	24-28	24-26	24-26	24-25	24	23-24	22-24	20-23	19-22	18-22	17-22	16-21	16-20	16-20	65
			C	A	A								Das					O								
66	66	44	37	34	32	32	32	30	29-30	28-29	28	25-28	24-27	24-26	24-26	24-25	24	23-24	20-24	20-23	19-22	18-22	17-22	16-21	16-20	66
		A		C	C			Su DH		O																
67	67	44	38	35	33	32	32	31	30	29-30	28-29	26-28	25-28	24-27	24-26	24-26	24-25	24	20-24	20-24	20-23	19-22	18-22	17-22	16-21	67
		C	A			A													Wi							
68	68	45	38	36	34	32	32	32	30-31	30	28-30	27-29	26-28	25-28	24-27	24-26	24-26	24-25	21-24	20-24	20-24	20-23	19-22	18-22	17-22	68
			C		A								E													
69	69	46	39	36	34	33	32	32	31-32	30	28-30	28-30	27-28	26-28	25-28	24-27	24-26	24-26	22-25	20-24	20-24	20-24	20-23	19-22	18-22	69
		A		A	C		Gr			Su NT			GB2			O										
70	70	46	40	36	35	34	33	32	32	31	28-30	28-30	28-29	26-28	26-28	25-28	24-27	24-26	22-26	21-25	20-24	20-24	20-24	20-23	19-22	70
		C		C		A		E											Wi		Wi					
71	71	47	40	37	36	34	34	32	32	32	29-31	28-30	28-30	27-29	26-28	26-28	24-28	24-26	23-26	22-26	21-25	20-24	20-24	20-24	20-23	71
			A								GB2			A			Ka	AEB								
72	72	48	40	38	36	35	34	32-33	32	32	30-32	28-31	28-30	28-30	26-28	26-28	25-28	24-27	24-26	22-26	22-26	20-25	20-24	20-24	20-24	72
		A	C	A	A	SS									AEB											
73	73	48	41	38	36	36	34	32-34	32-33	32	30-32	29-32	28-31	28-30	27-29	26-28	26-28	24-28	24-27	23-26	22-26	21-26	20-24	20-24	20-24	73
		C		C	C		HP vT2	DH		E			Wi2			SEC		A12			Wi		Wi AEB			
74	74	49	42	39	37	36	35	33-34	32-34	32	31-32	30-32	29-32	28-30	28-30	27-29	26-28	25-28	24-28	24-27	23-26	22-26	21-25	20-24	20-24	74
			A			A					To2			NT												
75	75	50	42	40	38	36	36	34-35	32-34	32-33	32	30-32	30-32	28-31	28-30	28-30	26-29	26-28	24-28	24-28	24-27	22-26	22-26	20-25	20-24	75
		A	C		A	C			Ka								Ka									
76	76	50	43	40	38	37	36	34-36	33-35	32-34	32-33	31-32	30-32	29-32	28-31	28-30	27-30	26-29	24-28	24-28	24-28	23-26	22-26	21-26	20-25	76
		C			C		E	DH		B												AEB	Wi		Wi	
77	77	51	44	40	39	38	36	35-36	34-36	32-34	32-34	32-33	31-32	30-32	29-32	28-30	28-30	26-30	24-29	24-28	24-28	24-27	23-26	22-26	21-26	77
				A		A	HP						To			GC AEB		A12								
78	78	52	44	40	40	38	37	36	34-36	33-35	32-34	32-34	32-33	30-32	30-32	29-31	28-30	27-30	24-30	24-28	24-28	24-28	24-27	22-26	22-26	78
		A	A	C		C				Wz									GC	AEB						
79	79	52	44	41	40	39	38	36-37	34-36	34-36	32-35	32-34	32-34	31-33	30-32	30-32	28-31	28-30	25-30	24-29	24-28	24-28	24-28	23-27	22-26	79
		C	C				E	vT1	E					Pi	E										Wi	
80	80	53	45	42	40	40	38	37-38	35-36	34-36	32-36	32-35	32-34	32-34	30-32	30-32	28-32	28-31	26-30	24-30	24-29	24-28	24-28	24-28	23-27	80
				A	A		Su																			
81	81	54	46	42	40	40	39	38	36-37	35-36	33-36	32-36	32-34	32-34	30-33	30-32	28-32	28-32	26-31	25-30	24-30	24-29	24-28	24-28	24-28	81
		A	A	C	C					GC1			NT							Pi						
82	82	54	46	43	41	40	40	38	36-38	36-37	34-36	32-36	32-35	32-34	31-34	30-33	28-32	28-32	26-32	26-30	24-30	24-30	24-29	24-28	24-28	82
		C	C			A		DH								GC	GC			AEB						
83	83	55	47	44	42	40	40	39	37-38	36-38	35-37	33-36	32-36	32-35	32-34	31-34	29-33	28-32	27-32	26-31	25-30	24-30	24-30	24-28	24-28	83
					A	C		Q		B2	GB2													AEB		
84	84	56	48	44	42	41	40	40	38-39	37-38	36-38	34-37	33-36	32-36	32-35	32-34	30-34	28-33	28-32	27-32	26-31	25-30	24-30	24-29	24-28	84
		A		A	C				Gu3				GC1 E							PT		GC				
85	85	56	48	44	43	42	40	40	39-40	38	36-38	34-38	34-36	32-36	32-36	32-34	30-34	28-34	28-32	28-32	26-32	26-31	24-30	24-30	24-29	85
		C	A	C		A	vT2			NT						AEB		Su	AEB							
86	86	57	48	45	44	42	41	40	40	38-39	37-38	35-38	34-37	33-36	32-36	32-35	30-34	29-34	28-33	28-32	27-32	26-32	25-31	24-30	24-30	86
			C			C	A1						GC1													
87	87	58	49	46	44	43	42	40-41	40	39-40	38-39	36-38	35-38	34-37	33-36	32-36	31-35	30-34	28-34	28-33	28-32	27-32	26-32	24-30	24-30	87
		A		A	A			GC							GC1		PTX							AEB		
88	88	58	50	46	44	44	42	41-42	40-41	40	39-40	36-39	36-38	34-38	34-36	32-36	32-36	30-35	28-34	28-34	28-33	28-32	27-32	24-31	24-30	88
		C	A	C	C		vT2			B	S				AEB								X			
89	89	59	50	47	45	44	43	42	40-42	40	40	36-40	36-38	35-38	34-37	32-36	32-36	30-36	29-34	28-34	28-34	28-33	28-32	24-32	24-31	89
			C			A	D						NT		GC1				LP							
90	90	60	51	48	46	44	44	42	40-42	40-41	40	37-40	36-39	36-38	35-38	32-37	32-36	31-36	30-35	29-34	28-34	28-34	28-33	25-32	24-32	90
		A			A			DH				GB2				GC									GC	
91	91	60	52	48	46	45	44	42-43	40-42	40-42	40-41	38-40	36-40	36-39	36-38	33-38	32-37	32-36	31-36	30-35	29-34	28-34	28-34	26-32	25-32	91
		C			C		SS																	AEB		
92	92	61	52	48	47	46	45	43-44	41-43	40-42	40-42	38-41	37-40	36-40	36-38	34-38	32-38	32-36	32-36	31-36	30-34	29-34	28-34	26-33	26-32	92
			A	A		A		A1	GC1				GC1		AEB			AEB			AEB					
93	93	62	52	48	48	46	46	44	42-44	40-42	40-42	38-42	38-40	36-40	36-39	34-38	32-38	32-37	32-36	32-36	31-35	30-34	29-34	27-34	26-33	93
		A	C	C						DEI			DK											GC		
94	94	62	53	49	48	47	46	44	42-44	40-43	40-42	39-42	38-41	37-40	36-40	34-39	32-38	32-38	32-37	32-36	32-36	31-35	30-34	28-34	26-34	94
		C						DH					GC1				GC						GC		GC	
95	95	63	54	50	48	48	47	45	43-44	41-44	40-42	40-42	39-42	38-41	37-40	35-40	33-39	32-38	32-38	32-37	32-36	32-36	31-35	28-34	27-34	95
			A	A			D	vT1	D	Wz	AEB				GC1	We										
96	96	64	54	50	48	48	48	46	44-45	42-44	40-43	40-42	40-42	38-42	38-40	36-40	34-40	32-39	32-38	32-38	32-36	32-36	32-36	28-35	28-34	96
		A	C	C	A					E					AEB			GC			AEB					
97	97	64	55	51	48	48	48	46	44-46	42-44	41-44	40-43	40-42	39-42	38-41	36-40	34-40	33-40	32-38	32-38	32-37	32-36	32-36	28-36	28-34	97
		C			C			DH							GC1		GC		AEB					GC	AEB	
98	98	65	56	52	49	48	48	47	45-46	43-45	42-44	40-44	40-43	40-42	39-42	36-41	35-40	34-40	32-39	32-38	32-38	32-37	32-36	29-36	28-35	98
						A		DH	GB		GB															
99	99	66	56	52	50	48	48	48	46	44-46	43-45	41-44	40-44	40-43	40-42	36-42	36-40	34-40	33-40	32-39	32-38	32-38	32-37	30-36	28-36	99
		A	A	A	A				AEB				GC1			Ch	AEB			Ch						
100	100	66	56	52	50	49	48	48	46-47	44-46	44-46	42-45	41-44	40-44	40-42	37-42	36-41	34-40	34-40	33-40	32-38	32-38	32-38	30-37	28-36	100
		C	C	C	C		Gr								AEB						AEB			GC		
101	101	67	57	53	51	50	49	48	47-48	44-46	44-46	42-46	42-45	40-44	40-43	38-42	36-42	35-41	34-40	34-40	32-39	32-38	32-38	31-38	29-36	101
						A			D	DK				E										GC1	AEB	
102	102	68	58	54	52	50	50	48	48	44-47	44-46	43-46	42-46	41-44	40-44	38-43	36-42	36-42	35-41	34-40	32-40	32-39	32-38	32-38	30-37	102
		A	A	A				E					GC1		GC1											
103	103	68	58	54	52	51	50	48	48	45-48	44-47	44-46	43-46	42-45	41-44	39-44	37-42	36-42	36-42	35-40	33-40	32-40	32-39	32-38	30-38	103
		C	C	C	A											We	AEB			We AEB						
104	104	69	59	55	52	52	51	48-49	48	46-48	45-48	44-47	44-46	42-46	42-44	40-44	38-43	37-42	36-42	36-41	34-40	33-40	32-40	32-39	31-38	104
					C		SS	HP			Gu3				AEB			GC1					GC			
105	105	70	60	56	53	52	52	49-50	48-49	46-48	46-48	44-48	44-46	43-46	42-45	40-44	38-44	38-43	36-42	36-42	34-41	34-40	33-40	32-40	32-38	105
		A				A		E					AEB		GC1										AEB	
106	106	70	60	56	54	52	52	50	48-50	47-49	46-48	44-48	44-47	44-46	43-46	40-44	39-44	38-44	37-43	36-42	34-42	34-41	34-40	32-40	32-39	106
		C	A		A											AEB										
107	107	71	60	56	54	53	52	50-51	49-50	48-50	47-48	45-48	44-48	44-47	44-46	40-45	40-44	39-44	38-44	37-42	35-42	34-42	34-41	32-40	32-40	107
			C	A	C				GB		GB2 B	GB				Ch				AEB				Ch		
108	108	72	61	56	55	54	53	51-52	50-51	48-50	48-49	46-48	44-48	44-48	44-46	41-46	40-45	40-44	39-44	38-43	36-42	35-42	34-42	33-40	32-40	108
		A		C		A	SS	DH					GC1		AEB								GC	AEB		
109	109	72	62	57	56	54	54	52	50-52	48-51	48-50	46-49	45-48	44-48	44-47	42-46	40-46	40-45	40-44	39-44	36-43	36-42	35-42	34-41	32-40	109
		C	A					E		GB	E									Pi						
110	110	73	62	58	56	55	54	52	51-52	49-52	48-50	47-50	46-49	44-48	44-48	42-46	40-46	40-46	40-45	40-44	36-44	36-43	36-42	34-42	32-41	110
			C	A					GC			GC				AEB										
111	111	74	63	58	56	56	55	53	52	50-52	48-51	48-50	46-50	44-49	44-48	43-47	41-46	40-46	40-46	40-44	37-44	36-44	36-43	35-42	32-42	111
		A		C	A		D	vT1						GC1		We				AEB				We		
112	112	74	64	59	56	56	56	54	52-53	50-52	48-52	48-50	46-50	45-50	44-48	44-48	42-47	41-46	40-46	40-45	38-44	37-44	36-44	36-42	33-42	112
		C			C			E		E		AEBGC1			AEB			GC1				GC1		AEB		
113	113	75	64	60	57	56	56	54	52-54	51-52	48-52	48-51	47-50	46-50	44-49	44-48	42-48	42-47	40-46	40-46	38-45	38-44	36-44	36-43	34-42	113
			A			A			GC1	Q					GC1											
114	114	76	64	60	58	56	56	55	53-54	52-53	49-52	48-52	48-51	46-50	45-50	44-48	43-48	42-48	41-47	40-46	39-46	38-44	37-44	36-44	35-43	114
		A	C	A	A			DH								AEB						AEB				
115	115	76	65	60	58	57	56	56	54	52-54	50-53	49-52	48-52	46-51	46-50	44-49	44-48	43-48	42-48	41-46	40-46	39-45	38-44	37-44	36-44	115
		C		C	C		Gr		DEI			GC1				Ch				AEB						
116	116	77	66	61	59	58	57	56	54-55	53-54	50-54	50-52	48-52	47-52	46-51	45-50	44-49	44-48	43-48	42-47	41-46	40-46	39-45	38-44	37-44	116
			A			A			GB	Q		AEB		GC1						X						
117	117	78	66	62	60	58	58	56	55-56	54-55	51-54	50-53	48-52	48-52	46-52	46-50	44-50	44-49	44-48	43-48	42-47	41-46	40-46	39-45	38-44	117
		A	C	A																		GC1		GC1		
118	118	78	67	62	60	59	58	56	56	54-56	52-54	51-54	49-53	48-52	47-52	46-50	44-50	44-50	44-49	44-48	42-48	42-46	40-46	40-46	38-45	118
		C		C	A			DH			AEBGC1					AEB B2						AEB				
119	119	79	68	63	60	60	59	56-57	56	55-56	52-55	52-54	50-54	49-53	48-52	47-51	45-50	44-50	44-50	44-48	43-48	42-47	41-46	40-46	39-46	119
					C		D	A1		Q						We				AEB						
120	120	80	68	64	61	60	60	57-58	56-57	56	53-56	52-54	51-54	50-54	48-53	48-52	46-51	44-50	44-50	44-49	44-48	43-48	42-47	41-46	40-46	120
n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k\n

TABLE I, Column 1 *(Continued)*

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k/n
121	121	A 80	A 68	64	62	A 60	60	58	56-58	E 56	54-56	B2 AEB 53-55	52-54	B2 51-54	49-54	48-52	46-52	45-50	44-50	44-50	44-48	44-48	43-48	42-47	41-46	121
122	122	C 81	C 69	64	A 62	61	60	DH 58-59	56-58	56-57	54-56	54-56	52-55	52-54	50-54	48-52	47-52	AEB 46-51	45-50	44-50	AEB 44-49	44-48	44-48	43-48	42-47	122
123	123	82	70	A 64	C 63	62	61	59-60	57-58	56-58	55-57	54-56	53-56	52-55	51-54	AEB 48-53	48-52	47-52	46-51	45-50	44-50	44-49	44-48	44-48	43-48	123
124	124	A 82	A 70	C 65	64	A 62	D 62	60	AEB 58-59	GG 57-58	56-58	55-56	54-56	53-56	52-54	Ch 49-54	48-53	48-52	47-52	46-51	45-50	44-50	44-49	44-48	44-48	124
125	125	C 83	C 71	66	64	63	62	Lv 61	58-60	58-59	56-58	AEB 56-57	55-56	54-56	AEB 53-55	50-54	48-54	48-52	48-52	47-52	46-50	45-50	44-50	44-49	44-48	125
126	126	84	72	A 66	64	64	63	62	59-60	58-60	56-58	56-58	56-57	55-56	54-56	50-54	48-54	AEB 48-53	48-52	48-52	AEB 47-51	46-50	44-50	44-50	44-48	126
127	127	A 84	A 72	C 67	A 64	64	64	D 63	E 60	GG 59-60	AEB 56-59	56-58	56-58	56-57	BCH 55-56	Ho AEB 51-55	48-54	48-54	48-53	48-52	48-52	BCH 47-51	44-50	44-50	AEB 44-49	127
n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	k\n

TABLE I, COLUMN 2

n\k	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	k/n
26	1																									26
27	2	1																								27
28	2	2	1																							28
29	2	2	2	1																						29
30	2	2	2	2	1																					30
31	3	2	2	2	2	1																				31
32	D 4	B 2	2	2	2	2	1																			32
33	4	3	2	2	2	2	2	1																		33
34	4	4	3	2	2	2	2	2	1																	34
35	4	4	4	3	2	2	2	2	2	1																35
36	4-5	4	4	4	3	2	2	2	2	2	1															36
37	5-6	4-5	4	4	4	3	2	2	2	2	2	1														37
38	6	5-6	4	4	4	4	3	2	2	2	2	2	1													38
39	6	6	NT 5	4	4	4	4	3	2	2	2	2	2	1												39
40	6-7	6	6	5	4	4	4	4	3	2	2	2	2	2	1											40
41	6-8	6-7	6	6	5	4	4	4	4	3	2	2	2	2	2	1										41
42	7-8	6-8	6-7	6	6	5	4	4	4	4	3	2	2	2	2	2	1									42
43	8	7-8	6-8	6-7	6	6	5	4	4	4	4	3	2	2	2	2	2	1								43
44	8-9	8	7-8	6-8	6	6	6	5	4	4	4	4	3	2	2	2	2	2	1							44
45	8-10	8-9	8	7-8	AEB 6-7	6	6	6	5	4	4	4	4	3	2	2	2	2	2	1						45
46	8-10	8-10	8	8	7-8	6-7	6	6	6	5	4	4	4	4	3	2	2	2	2	2	1					46
47	8-10	8-10	B 8-9	8	8	7-8	6-7	6	6	6	5	4	4	4	4	3	2	2	2	2	2	1				47
48	9-11	8-10	8-10	8-9	8	RR 8	6-8	6-7	6	6	Sh 6	4-5	4	4	4	4	3	2	2	2	2	2	1			48
49	Sh 10-12	8-11	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	1		49
50	10-12	9-12	8-11	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	1	50
51	10-12	10-12	9-12	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	51
52	10-13	10-12	10-12	AEB 9-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	52
53	11-14	10-13	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	53
54	12-14	11-14	10-13	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	54
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64	16-18	16-18	16-18	14-17	14-16	12-16	12-16	12-15	12-14	12-14	12-14	10-13	10-12	10-12	C 9-11	8-10	8-10	8-10	8-9	8	8	6-8	6-7	6	6	64
65	AEB 16-19	16-18	16-18	15-18	14-17	13-16	12-16	12-16	12-15	12-14	12-14	11-14	10-13	10-12	10-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	65
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69	AEB 17-21	16-20	16-20	16-20	16-19	16-18	16-18	15-18	14-17	13-16	12-16	12-16	12-14	12-14	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	Sh 8	69
70	18-22	17-21	16-20	16-20	16-20	16-19	16-18	16-18	15-18	14-17	13-16	12-16	AEB 12-15	12-14	11-14	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	Jo+ 9	70
71	19-22	18-22	SRC 17-21	16-20	16-20	16-20	16-19	16-18	16-18	15-18	14-17	12-16	12-16	12-15	12-14	Sv 11-14	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-9	71
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73	AEB 20-23	PT 20-22	18-22	17-22	16-21	16-20	16-20	16-20	16-18	16-18	PT 16-18	AEB 14-17	Ka 13-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	9-12	LP 8-11	8-10	8-10	73
74	20-24	20-23	18-22	18-22	17-22	16-21	16-20	16-20	AEB 16-19	16-18	16-18	14-18	14-17	12-16	12-16	12-16	12-15	To 11-14	10-14	10-14	10-13	10-12	9-12	8-11	8-10	74
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76	20-24	20-24	PTX 20-24	18-23	18-22	18-22	17-22	16-20	16-20	16-20	16-19	15-18	14-18	13-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	LP 10-13	10-12	10-12	Ka 9-12	76
77	20-25	20-24	20-24	18-24	18-23	18-22	18-22	AEB 17-21	16-20	16-20	16-20	16-19	15-18	14-18	13-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-13	10-12	10-12	77
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79	21-26	20-26	20-25	20-24	19-24	AEB 18-23	18-22	18-22	18-22	17-21	16-20	16-20	16-20	16-19	L 15-18	13-18	12-18	12-16	12-16	12-16	12-16	10-15	10-14	10-14	10-13	79
80	22-26	Mi 21-26	20-26	20-25	20-24	19-24	18-23	18-22	18-22	18-22	17-21	16-20	16-20	16-20	16-19	Ms 14-18	12-18	LP 12-17	12-16	12-16	12-16	11-16	10-14	10-14	10-14	80
81	22-27	22-26	20-26	20-26	20-24	GC 20-24	18-24	18-23	18-22	18-22	18-22	17-21	16-20	16-20	16-20	14-19	13-18	12-18	12-17	12-16	12-16	12-16	LP 11-15	10-14	10-14	81
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83	24-28	Mi 23-27	AEB 22-26	21-26	20-26	20-25	20-24	19-24	18-24	18-23	18-22	18-22	18-22	17-21	16-20	15-20	14-20	14-18	13-18	12-18	12-17	12-16	12-16	12-16	11-15	83
84	24-28	24-28	22-27	Jo 22-26	20-26	20-26	20-25	20-24	19-24	18-24	18-23	18-22	18-22	18-22	17-20	16-20	15-20	LP 14-19	14-18	13-18	12-18	12-17	12-16	12-16	12-16	84
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n/k	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	k\n

TABLE I, COLUMN 2 (*CONTINUED*)

n\k	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	k/n
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89	24-30	24-30	24-30	24-29	24-28	Mi 23-28	20-28	20-27	20-26	20-26	AEB 20-25	20-24	19-24	18-24	18-23	18-22	18-22	18-22	18-21	L 17-20	14-20	14-20	14-19	14-18	13-18	89
90	24-31	24-30	24-30	24-30	24-29	24-28	20-28	20-28	20-27	20-26	20-26	20-25	20-24	19-24	18-24	18-23	18-22	18-22	18-22	18-21	14-20	14-20	14-20	14-18	14-18	90
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92	24-32	24-32	24-31	24-30	24-30	AEB 24-29	22-28	21-28	20-28	AEB 20-27	20-26	20-26	20-26	20-25	20-24	19-24	18-24	18-23	18-22	18-22	15-22	14-21	14-20	14-20	14-19	92
93	24-32	24-32	24-32	24-30	24-30	24-30	22-29	PT 22-28	20-28	20-28	20-27	20-26	20-26	20-26	20-25	20-24	19-24	18-24	18-23	18-22	16-22	15-22	14-21	14-20	14-20	93
94	25-33	24-32	24-32	AEB 24-31	24-30	24-30	23-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	20-25	20-24	19-24	18-24	18-23	16-22	16-22	Ch 15-22	14-20	14-20	94
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97	26-34	26-34	AEB 25-33	24-32	24-32	24-32	24-31	24-30	23-30	22-30	21-29	20-28	20-28	20-28	20-26	20-26	20-26	AEB 20-25	20-24	20-24	19-24	18-23	17-22	16-22	15-22	97
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101	28-36	27-36	26-35	26-34	25-34	24-34	24-33	24-32	24-32	24-31	24-30	23-30	22-30	AEB 22-29	20-28	20-28	20-28	20-27	20-26	20-26	20-26	20-25	20-24	20-24	19-24	101
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106	32-38	31-38	30-38	29-37	28-36	27-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	22-30	22-30	20-29	20-28	20-28	20-28	20-26	20-26	20-26	106
107	32-39	32-38	31-38	30-38	29-36	28-36	27-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	Ch 23-31	22-30	22-30	21-30	20-28	20-28	20-28	LP 20-27	20-26	20-26	107
108	32-40	32-39	B2x 32-38	30-38	B2x AEB 30-37	28-36	B2x 28-36	26-36	26-35	25-34	24-34	24-34	24-32	24-32	24-32	24-32	22-30	22-30	22-30	LP 21-29	20-28	20-28	20-28	20-27	20-26	108
109	32-40	32-40	32-39	31-38	30-38	29-37	28-36	27-36	26-36	26-35	25-34	24-34	AEB 24-33	24-32	24-32	24-32	AEB 23-31	22-30	22-30	22-30	21-29	20-28	20-28	20-28	20-27	109
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111	32-41	32-40	32-40	AEB 32-39	32-38	31-38	30-38	29-37	28-36	27-36	26-36	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	22-30	22-30	21-29	20-28	20-28	111
112	32-42	32-40	32-40	32-40	32-39	32-38	31-38	30-38	29-36	28-36	27-36	AEB 26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	22-30	22-30	21-29	20-28	112
113	33-42	AEB 32-41	32-40	32-40	32-40	32-39	32-38	31-38	AEB 30-37	29-36	28-36	26-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	22-30	22-30	21-29	113
114	34-42	33-42	32-41	32-40	32-40	32-40	32-39	32-38	31-38	30-37	29-36	27-36	26-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	22-30	22-30	114
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116	AEB 36-43	35-42	34-42	32-42	32-41	32-40	32-40	32-40	32-38	32-38	31-38	28-37	28-36	27-36	26-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	24-32	23-31	22-30	116
117	37-44	36-43	35-42	33-42	32-42	32-40	32-40	32-40	AEB 32-39	32-38	SN 32-38	29-38	28-37	28-36	27-36	26-36	SN 26-34	24-34	24-34	24-34	24-32	24-32	24-32	SN 24-32	22-30	117
118	GC1 38-44	36-44	GC1 36-43	34-42	32-42	AEB 32-41	32-40	32-40	32-40	32-39	32-38	30-38	28-38	28-36	28-36	27-36	AEB 26-35	25-34	24-34	24-34	LP 24-33	24-32	24-32	24-32	LP 22-31	118
119	38-44	37-44	36-44	35-43	33-42	32-42	32-41	32-40	32-40	32-40	32-39	31-38	28-38	AEB 28-37	28-36	28-36	27-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	Mi2 23-32	119
120	AEB 39-45	38-44	37-44	36-44	34-43	33-42	32-42	32-41	32-40	32-40	32-40	SN 32-39	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	24-34	24-34	24-33	24-32	24-32	120
121	40-46	39-45	38-44	37-44	35-44	34-42	33-42	32-42	32-41	32-40	32-40	32-40	30-39	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	24-34	24-34	24-33	24-32	121
122	41-46	40-46	39-45	38-44	36-44	AEB 35-43	34-42	33-42	32-42	32-41	32-40	32-40	31-40	30-38	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	24-34	24-34	24-33	122
123	42-46	41-46	40-46	39-45	36-44	36-44	35-43	34-42	33-42	32-42	32-40	32-40	32-40	AEB 31-39	30-38	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	24-34	24-34	123
124	AEB 43-47	42-46	41-46	40-46	36-44	36-44	36-44	35-43	34-42	33-42	AEB 32-41	32-40	32-40	32-40	31-39	30-38	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	24-34	124
125	44-48	43-47	42-46	41-46	AEB 36-45	36-44	36-44	36-44	35-43	34-42	33-42	32-41	32-40	32-40	32-40	31-39	30-38	29-38	28-38	28-37	28-36	28-36	27-36	26-35	25-34	125
126	44-48	44-48	43-47	42-46	36-46	36-45	36-44	36-44	36-44	35-42	34-42	32-42	32-41	32-40	32-40	32-40	31-39	30-38	28-38	28-38	28-37	28-36	28-36	27-36	26-34	126
127	44-48	44-48	44-48	BCH 43-47	36-46	36-46	36-45	36-44	36-44	AEB 36-43	SN 35-42	32-42	32-42	32-41	32-40	32-40	32-40	BCH 31-39	28-38	28-38	28-38	28-37	28-36	28-36	BCH LP 27-35	127
n/k	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	k\n

TABLE I, COLUMN 3

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51	1																									51
52	2	1																								52
53	2	2	1																							53
54	2	2	2	1																						54
55	2	2	2	2	1																					55
56	2	2	2	2	2	1																				56
57	3	2	2	2	2	2	1																			57
58	4	3	2	2	2	2	2	1																		58
59	4	4	3	2	2	2	2	2	1																	59
60	4	4	4	3	2	2	2	2	2	1																60
61	4	4	4	4	3	2	2	2	2	2	1															61
62	Jo+ 4	4	4	4	4	3	2	2	2	2	2	1														62
63	5	4	4	4	4	4	3	2	2	2	2	2	1													63
64	6	5	4	4	4	4	D 4	B 2	2	2	2	2	2	1												64
65	6	6	K 5	4	4	4	4	3	2	2	2	2	2	2	1											65
66	6	6	6	4-5	4	4	4	4	3	2	2	2	2	2	2	1										66
67	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1									67
68	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1								68
69	6-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1							69
70	7-8	6-8	6-8	6-7	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1						70
71	8	7-8	6-8	6-8	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1					71
72	8-9	8	7-8	6-8	LP 6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1				72
73	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1			73
74	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	1		74
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76	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	2	76
77	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	2	77
78	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	2	78
79	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	2	79
80	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	2	80
81	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	3	2	81
82	10-14	10-14	10-13	10-12	9-12	8-12	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	Sh 6	4-6	4-5	4	4	4	4	3	82
83	10-14	10-14	10-14	10-13	10-12	9-12	LP 8-11	8-10	8-10	8-10	8-10	Jo+ 8	8	7-8	6-8	6-7	6	6	5-6	4-6	4-5	4	4	4	4	83
84	11-15	10-14	10-14	10-14	10-13	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	8	7-8	6-8	6-7	6	6	5-6	4-6	4-5	4	4	4	84
85	12-16	11-15	10-14	10-14	10-14	10-13	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	8	7-8	6-8	6-7	6	6	5-6	4-6	4-5	4	4	85
86	12-16	12-16	11-15	10-14	10-14	10-14	Jo+ 10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	8	7-8	6-8	6-7	6	6	5-6	4-6	4-5	4	86
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89	12-18	12-17	12-16	12-16	LP 12-15	PT 11-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	Sh 8	6-8	6-8	6-7	6	6	5-6	89
90	13-18	12-18	12-16	12-16	12-16	12-15	10-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	90
91	PT 14-18	12-18	Jo2 12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	10-12	Hg 9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	Jo+ 6	6	91
92	14-18	13-18	12-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	10-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	92
93	14-19	14-18	13-18	12-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	93
94	14-20	14-19	14-18	13-18	12-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	9-12	8-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	94
95	14-20	14-20	14-19	To4 14-18	12-18	12-18	12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	9-12	8-12	8-10	8-10	8-10	8-10	8-9	8	7-8	95
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97	14-21	14-20	14-20	14-20	14-19	13-18	12-18	12-18	12-17	12-16	12-16	12-16	11-14	10-14	10-14	LP 10-13	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-10	8-9	97
98	15-22	14-21	14-20	14-20	14-20	14-19	13-18	12-18	12-18	12-17	12-16	12-16	LP 12-15	11-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-10	98
99	16-22	15-22	14-21	14-20	14-20	14-20	14-19	13-18	12-18	12-18	12-17	12-16	12-16	12-15	Ro 11-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	8-10	8-10	99
100	17-22	16-22	14-22	14-21	14-20	14-20	14-20	14-18	13-18	12-18	12-18	12-17	12-16	12-16	12-15	10-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	8-10	100
101	18-23	17-22	15-22	14-22	14-21	14-20	14-20	Jo2 14-19	14-18	Ro 13-18	12-18	12-18	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	8-10	101
102	19-24	18-23	16-22	15-22	14-22	14-20	14-20	14-20	14-19	14-18	12-18	12-18	LP 12-17	12-16	12-16	12-16	11-15	10-14	10-14	10-14	10-13	10-12	10-12	9-12	8-11	102
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n/k	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	k\n

TABLE I, COLUMN 3 *(CONTINUED)*

n\k	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	k/n
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112	20-28	20-28	20-27	20-26	20-26	19-26	18-25	17-24	16-24	16-24	16-23	16-22	15-22	14-21	14-20	14-20	14-20	14-20	14-18	13-18	12-18	LP 12-17	12-16	12-16	12-16	112
113	20-28	20-28	20-28	20-27	20-26	20-26	19-26	18-25	17-24	16-24	16-24	16-22	16-22	15-22	14-21	14-20	14-20	14-20	Jo2 14-19	14-18	13-18	12-18	12-17	12-16	12-16	113
114	21-29	20-28	20-28	20-28	20-27	20-26	20-26	19-26	18-24	17-24	16-24	LP 16-23	16-22	16-22	15-22	14-21	14-20	14-20	14-20	14-19	14-18	13-18	12-18	12-17	12-16	114
115	22-30	21-29	20-28	20-28	20-28	20-27	20-26	20-26	LP 19-25	18-24	17-24	16-24	16-23	16-22	16-22	15-22	14-21	14-20	14-20	14-20	14-19	14-18	13-18	12-18	12-17	115
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117	22-30	22-30	22-30	21-28	20-28	20-28	LP 20-27	20-26	20-26	20-26	19-25	18-24	17-24	16-24	16-23	16-22	16-22	15-22	14-21	14-20	14-20	14-20	14-19	14-18	13-18	117
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119	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	19-25	18-24	17-24	16-24	16-23	16-22	16-22	15-22	14-21	14-20	14-20	14-20	14-19	119
120	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	19-25	18-24	17-24	16-24	16-23	16-22	16-22	15-22	14-20	14-20	14-20	14-20	120
121	23-32	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	19-25	18-24	17-24	16-24	16-22	16-22	16-22	LP 15-21	14-20	14-20	14-20	121
122	24-32	23-32	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	19-25	18-24	17-24	LP 16-23	16-22	16-22	16-22	15-21	14-20	14-20	122
123	24-33	24-32	23-32	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-27	20-26	20-26	20-26	19-25	18-24	17-24	16-23	16-22	16-22	16-22	15-21	14-20	123
124	24-34	24-33	24-32	23-32	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	20-28	20-26	20-26	20-26	20-26	19-24	18-24	17-24	16-23	16-22	16-22	16-22	15-21	124
125	24-34	24-34	24-33	24-32	23-32	22-32	22-31	22-30	22-30	22-30	22-29	21-28	20-28	LP 20-27	20-26	20-26	20-26	LP 20-25	19-24	18-24	17-24	16-23	16-22	16-22	16-22	125
126	24-34	24-34	24-34	24-32	24-32	23-32	22-32	22-31	22-30	22-30	22-30	22-28	21-28	20-28	20-27	20-26	20-26	20-26	20-25	19-24	18-24	16-24	16-23	16-22	16-22	126
127	24-34	24-34	24-34	LP 24-33	24-32	24-32	BCH 23-32	22-32	22-31	22-30	22-30	LP 22-29	22-28	BCH 21-28	20-28	20-27	20-26	20-26	20-26	20-25	BCH 19-24	16-24	16-24	16-23	16-22	127
n/k	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	k\n

TABLE I, Column 4

n\k	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	k/n
76	1																									76
77	2	1																								77
78	2	2	1																							78
79	2	2	2	1																						79
80	2	2	2	2	1																					80
81	2	2	2	2	2	1																				81
82	2	2	2	2	2	2	1																			82
83	3	2	2	2	2	2	2	1																		83
84	4	3	2	2	2	2	2	2	1																	84
85	4	4	3	2	2	2	2	2	2	1																85
86	4	4	4	3	2	2	2	2	2	2	1															86
87	4	4	4	4	3	2	2	2	2	2	2	1														87
88	4-5	4	4	4	4	3	2	2	2	2	2	2	1													88
89	4-6	4	4	4	4	4	3	2	2	2	2	2	2	1												89
90	5-6	LP 4-5	4	4	4	4	4	3	2	2	2	2	2	2	1											90
91	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1										91
92	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1									92
93	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1								93
94	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1							94
95	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1						95
96	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1					96
97	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1				97
98	8-9	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1			98
99	8-10	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1		99
100	8-10	LP 8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1	100
101	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	101
102	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	102
103	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	103
104	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	104
105	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	105
106	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	2	106
107	10-12	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	3	107
108	LP 10-13	10-12	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	4	108
109	10-14	10-13	10-12	10-12	10-12	9-12	8-11	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	4	109
110	10-14	10-14	10-13	10-12	10-12	10-12	9-12	Jo+ 8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	4	110
111	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	4	111
112	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	4	112
113	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	6-7	6	6	6	5-6	4-5	113
114	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-8	Jo+ 6	6	6	6	5-6	114
115	12-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	6	115
116	12-17	12-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	6	116
117	12-18	12-17	12-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8-9	8	7-8	6-8	6-7	6	6	117
118	13-18	12-18	12-17	12-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-10	8	8	7-8	6-8	6-7	6	118
119	14-18	13-18	12-18	12-17	12-16	12-16	12-16	12-15	11-14	10-14	10-14	10-13	10-12	10-12	10-12	9-11	8-10	8-10	8-10	LP 8-9	8	8	7-8	6-8	6-7	119
120	14-19	14-18	13-18	12-18	12-17	12-16	12-16	12-16	12-14	11-14	10-14	10-14	10-12	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	8	7-8	6-8	120
121	14-20	14-19	14-18	13-18	12-18	12-17	12-16	12-16	LP 12-15	12-14	11-14	10-14	LP 10-13	10-12	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	8	7-8	121
122	14-20	14-20	14-19	14-18	13-18	12-18	Jo+ 12-16	12-16	12-16	12-15	12-14	11-14	10-14	10-13	10-12	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	8	122
123	14-20	14-20	14-20	14-18	14-18	13-18	12-17	12-16	12-16	12-16	12-15	12-14	11-14	10-14	10-13	10-12	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-9	8	123
124	14-20	14-20	14-20	LP 14-19	14-18	14-18	13-18	12-17	12-16	12-16	12-16	12-15	12-14	11-14	10-14	10-13	10-12	10-12	10-12	10-12	9-11	8-10	8-10	8-10	8-9	124
125	15-21	14-20	14-20	14-20	14-19	14-18	14-18	13-18	12-17	12-16	12-16	12-16	12-15	12-14	11-14	10-14	10-13	10-12	10-12	10-12	10-12	9-10	8-10	8-10	8-10	125
126	16-22	15-21	14-20	14-20	14-20	14-19	14-18	14-18	13-18	12-17	12-16	12-16	12-16	12-15	12-14	11-14	10-14	10-13	10-12	10-12	10-12	LP 10-11	9-10	8-10	8-10	126
127	16-22	16-22	BCH 15-21	14-20	14-20	14-20	14-19	14-18	14-18	BCH 13-18	12-17	12-16	12-16	12-16	12-15	12-14	BCH 11-14	10-14	10-13	10-12	10-12	10-12	10-11	BCH 9-10	8-10	127
n/k	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	k\n

TABLE I, COLUMN 5

n\k	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	k/n
101	1																									101
102	2	1																								102
103	2	2	1																							103
104	2	2	2	1																						104
105	2	2	2	2	1																					105
106	2	2	2	2	2	1																				106
107	2	2	2	2	2	2	1																			107
108	3	2	2	2	2	2	2	1																		108
109	4	3	2	2	2	2	2	2	1																	109
110	4	4	3	2	2	2	2	2	2	1																110
111	4	4	4	3	2	2	2	2	2	2	1															111
112	4	4	4	4	3	2	2	2	2	2	2	1														112
113	4	4	4	4	4	3	2	2	2	2	2	2	1													113
114	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1												114
115	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1											115
116	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1										116
117	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1									117
118	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1								118
119	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1							119
120	6-7	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1						120
121	6-8	6-7	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1					121
122	7-8	6-8	6-7	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1				122
123	8	7-8	6-8	6-7	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1			123
124	8	8	7-8	6-8	6-7	6	6	6	6	5-6	4-5	4	4	4	4	4	3	2	2	2	2	2	2	1		124
125	8-9	8	8	7-8	6-8	6-7	6	6	6	6	5-6	4	4	4	4	4	4	3	2	2	2	2	2	2	1	125
126	8-10	8-9	8	8	7-8	6-8	6-7	6	6	6	6	LP 5	4	4	4	4	4	4	3	2	2	2	2	2	2	126
127	8-10	8-10	8-9	8	8	BCH 7-8	6-8	6-7	6	6	6	6	BCH 5	4	4	4	4	4	4	D 3	2	2	2	2	2	127
n/k	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	k\n

TABLE II*

INDEX OF EXTERNAL LABELS

L	87	7	42	Al
L	93	8	44	Al
L	120	8	57	Al
L	74	18	25	Al2
L	78	18	27	Al2
L	84	10	37	B2
L	119	17	45	B2
L	121	12	53	B2
L	121	14	51	B2
L	49	11	19	B2x
L	49	13	17	B2x
L	108	28	32	B2x
L	108	30	30	B2x
L	108	32	28	B2x
L	31	11	11	BCH
L	63	10	27	BCH
L	63	16	23	BCH
L	63	18	21	BCH
L	63	30	13	BCH
L	63	36	11	BCH
L	127	15	55	BCH
L	127	22	47	BCH
L	127	29	43	BCH
L	127	43	31	BCH
L	127	50	27	BCH
L	127	57	23	BCH
L	127	64	21	BCH
L	127	71	19	BCH
L	127	78	15	BCH
L	127	85	13	BCH
L	127	92	11	BCH
L	127	99	9	BCH
L	127	106	7	BCH
L	127	113	5	BCH
L	60	17	20	CDJ
L	60	10	25	Ch
L	65	11	27	Ch
L	94	48	15	Ch
L	95	33	23	Ch
L	95	35	21	Ch
L	100	16	37	Ch
L	100	20	33	Ch
L	100	39	21	Ch
L	107	41	23	Ch
L	108	16	41	Ch
L	108	24	33	Ch
L	116	16	45	Ch
L	124	16	49	Ch
L	45	13	16	CLS
L	32	17	8	CS
L	36	8	16	DH
L	58	8	26	DH
L	65	8	30	DH
L	74	8	33	DH
L	77	8	35	DH
L	99	8	48	DH
L	109	8	52	DH
L	115	8	56	DH
L	45	16	13	DJ
L	51	25	11	DJ
L	42	8	18	DM
L	45	8	20	DM
L	48	8	22	DM
L	37	9	15	FB
L	42	12	15	FB
L	48	16	15	FB
L	64	40	9	G
L	99	9	46	GB
L	99	11	43	GB
L	108	9	50	GB
L	108	12	46	GB
L	110	10	49	GB
L	117	9	55	GB
L	70	13	28	GB2
L	72	11	30	GB2
L	84	11	36	GB2
L	91	12	38	GB2
L	108	11	48	GB2
L	78	16	29	GG
L	79	19	25	GG
L	81	31	20	GG
L	83	16	31	GG
L	83	17	29	GG
L	85	22	26	GG
L	87	36	20	GG
L	88	8	41	GG
L	91	16	33	GG
L	91	25	25	GG
L	94	24	28	GG
L	95	17	33	GG
L	95	23	31	GG
L	95	25	27	GG
L	97	18	33	GG
L	98	17	35	GG
L	98	24	29	GG
L	98	26	27	GG
L	98	29	25	GG
L	101	24	31	GG
L	102	31	25	GG
L	105	23	33	GG
L	109	23	35	GG
L	111	9	52	GG
L	111	12	48	GG
L	124	10	57	GG
L	127	10	59	GG
L	46	9	19	GG1
L	46	11	17	GG1
L	54	11	21	GG1
L	55	23	13	GG1
L	58	13	22	GG1
L	82	10	36	GG1
L	85	13	34	GG1
L	87	13	35	GG1
L	88	15	34	GG1
L	90	15	35	GG1
L	93	9	42	GG1
L	93	13	38	GG1
L	95	13	39	GG1
L	96	15	38	GG1
L	98	15	39	GG1
L	100	13	41	GG1
L	102	25	30	GG1
L	102	27	28	GG1
L	103	13	43	GG1
L	103	15	41	GG1
L	105	18	38	GG1
L	106	15	43	GG1
L	109	13	45	GG1
L	112	14	45	GG1
L	113	13	47	GG1
L	113	18	42	GG1
L	113	22	38	GG1
L	114	9	53	GG1
L	114	15	45	GG1
L	116	12	50	GG1
L	117	14	48	GG1
L	118	22	42	GG1
L	118	24	40	GG1
L	118	26	38	GG1
L	118	28	36	GG1
L	119	12	52	GG1
L	20	5	9	Gr
L	37	6	17	Gr
L	52	6	25	Gr
L	70	7	33	Gr
L	101	7	49	Gr
L	116	7	57	Gr
L	45	10	18	Gu3
L	85	9	39	Gu3
L	105	11	46	Gu3
L	91	64	9	Hg
L	23	7	9	HP
L	74	7	35	HP
L	78	7	37	HP
L	105	8	49	HP
L	33	8	14	HY2
L	48	17	14	Je
L	84	29	22	Je
L	72	17	25	Ka
L	73	38	13	Ka
L	76	9	33	Ka
L	76	17	27	Ka
L	76	50	9	Ka
L	27	14	7	L
L	79	40	15	L
L	89	45	17	L
L	103	52	19	L
L	107	54	19	L
L	55	16	19	LC
L	17	9	5	N
L	23	12	7	N
L	41	21	9	N
L	43	15	13	N
L	47	24	11	N
L	51	8	24	N
L	51	17	16	N
L	51	19	14	N
L	55	21	15	N
L	63	11	26	N
L	63	19	19	N
L	63	21	18	N
L	63	28	15	N
L	63	46	7	N
L	65	53	5	N
L	37	11	13	O
L	66	18	23	O
L	67	10	29	O
L	67	39	11	O
L	70	16	25	O
L	35	9	14	Pi
L	80	14	32	Pi
L	82	20	26	Pi
L	110	20	40	Pi
L	27	10	9	Pi2
L	73	27	20	PT
L	73	36	16	PT
L	85	20	28	PT
L	87	31	22	PT
L	89	56	11	PT
L	91	51	14	PT
L	93	33	22	PT
L	76	28	20	PTX
L	88	17	32	PTX
L	52	10	21	Pu
L	84	8	40	Q
L	114	10	52	Q
L	117	10	54	Q
L	120	10	56	Q
L	102	37	24	QC
L	105	39	24	QC
L	99	65	11	Ro
L	101	60	13	Ro
L	105	57	15	Ro
L	48	31	8	RR
L	89	11	40	S
L	32	13	10	Sh
L	34	12	12	Sh
L	34	23	6	Sh
L	36	16	10	Sh
L	38	22	8	Sh
L	48	36	6	Sh
L	49	26	10	Sh
L	55	31	10	Sh
L	69	50	8	Sh
L	82	68	6	Sh
L	89	69	8	Sh
L	96	75	8	Sh
L	68	42	10	Sh2
L	57	11	23	SRC
L	71	28	17	SRC
L	74	16	27	SRC
L	45	6	22	SS
L	73	6	36	SS
L	92	7	45	SS
L	105	7	52	SS
L	109	7	54	SS
L	67	8	31	Su
L	70	10	31	Su
L	81	7	39	Su
L	86	18	29	Su
L	117	36	32	SW
L	117	42	26	SW
L	117	49	24	SW
L	120	37	32	SW
L	127	36	35	SW
L	74	43	11	To
L	78	13	32	To
L	75	11	32	To2
L	95	54	14	To4
L	42	7	19	vT1
L	80	8	37	vT1
L	96	8	46	vT1
L	112	8	54	vT1
L	39	7	17	vT3
L	23	14	5	Wa
L	96	16	36	We
L	104	16	40	We
L	104	20	36	We
L	112	16	44	We
L	112	24	36	We
L	120	16	48	We
L	127	16	51	We
L	68	19	21	Wi
L	71	19	23	Wi
L	71	21	21	Wi
L	74	21	23	Wi
L	74	23	21	Wi
L	77	23	23	Wi
L	77	25	21	Wi
L	80	25	23	Wi
L	80	27	21	Wi
L	83	27	23	Wi
L	89	31	23	Wi
L	74	13	29	Wi2
L	98	35	23	Wi2
L	119	50	23	Wi2
L	79	10	34	Wz
L	80	41	14	Wz
L	96	10	42	Wz
L	96	27	26	Wz
L	105	32	26	Wz
L	110	36	26	Wz
L	89	23	28	X
L	105	43	21	X
L	117	20	43	X
L	78	46	11	ZL
L	39	10	15	Zv
L	55	7	25	Zv
L	55	10	23	Zv
L	71	41	11	Zv
U	35	21	7	AEB
U	42	16	13	AEB
U	45	30	7	AEB
U	52	29	11	AEB
U	56	18	19	AEB
U	58	16	21	AEB
U	58	42	7	AEB
U	59	39	9	AEB
U	62	16	23	AEB
U	63	39	11	AEB
U	65	26	19	AEB
U	68	40	13	AEB
U	69	26	21	AEB
U	70	38	15	AEB
U	72	18	27	AEB
U	73	15	29	AEB
U	73	26	23	AEB
U	73	37	17	AEB
U	74	23	25	AEB
U	74	34	19	AEB
U	77	22	27	AEB
U	77	33	21	AEB
U	78	16	31	AEB
U	79	20	29	AEB
U	79	31	23	AEB
U	82	30	25	AEB
U	83	20	31	AEB
U	83	27	27	AEB
U	84	24	29	AEB
U	85	40	21	AEB
U	86	16	35	AEB
U	86	19	33	AEB
U	87	38	23	AEB
U	88	24	31	AEB
U	89	15	37	AEB
U	89	36	25	AEB
U	92	24	33	AEB
U	92	31	29	AEB
U	92	35	27	AEB
U	93	15	39	AEB
U	93	18	37	AEB
U	93	21	35	AEB
U	94	29	31	AEB
U	96	11	43	AEB
U	97	15	41	AEB
U	97	21	37	AEB
U	97	28	33	AEB
U	97	43	25	AEB
U	98	19	39	AEB
U	98	25	35	AEB
U	98	40	27	AEB
U	100	9	47	AEB
U	100	17	41	AEB
U	100	34	31	AEB
U	101	15	43	AEB
U	101	21	39	AEB
U	101	39	29	AEB
U	102	25	37	AEB
U	103	33	33	AEB
U	104	17	43	AEB
U	104	20	41	AEB
U	104	30	35	AEB
U	105	15	45	AEB
U	106	13	47	AEB
U	106	25	39	AEB
U	107	16	45	AEB
U	108	20	43	AEB
U	108	30	37	AEB
U	109	15	47	AEB
U	109	24	41	AEB
U	109	38	33	AEB
U	109	42	31	AEB
U	111	16	47	AEB
U	111	29	39	AEB
U	112	20	45	AEB
U	112	37	35	AEB
U	113	12	51	AEB
U	113	15	49	AEB
U	113	24	43	AEB
U	113	27	41	AEB
U	113	34	37	AEB
U	115	16	49	AEB
U	115	22	45	AEB
U	116	20	47	AEB
U	116	26	43	AEB
U	117	12	53	AEB
U	117	34	39	AEB
U	118	31	41	AEB
U	118	42	35	AEB
U	119	11	55	AEB
U	119	16	51	AEB
U	119	22	47	AEB
U	119	39	37	AEB
U	120	20	49	AEB
U	120	26	45	AEB
U	121	12	55	AEB
U	122	18	51	AEB
U	122	21	49	AEB
U	122	31	43	AEB
U	123	16	53	AEB
U	123	39	39	AEB
U	124	9	59	AEB
U	124	26	47	AEB
U	124	36	41	AEB
U	125	12	57	AEB
U	125	15	55	AEB
U	125	30	45	AEB
U	126	18	53	AEB
U	126	21	51	AEB
U	127	11	59	AEB
U	127	16	55	AEB
U	127	25	49	AEB
U	127	35	43	AEB
U	28	6	12	BM
U	40	6	18	BM
U	66	13	27	Das
U	53	9	23	DEI
U	61	9	27	DEI
U	65	9	29	DEI
U	94	10	43	DEI
U	116	9	55	DEI
U	67	8	31	DH
U	83	8	39	DH
U	91	8	43	DH
U	95	8	45	DH
U	119	8	57	DH
U	122	8	59	DH
U	46	16	15	DK
U	94	13	41	DK
U	102	10	47	DK
U	28	8	11	DM
U	44	8	19	DM
U	98	8	47	DM
U	12	5	4	FP
U	33	12	11	HT
U	36	11	13	HT
U	37	9	15	HT
U	39	28	5	HT
U	41	12	15	HT
U	43	21	11	HT
U	49	12	19	HT
U	58	13	23	HT
U	70	10	31	HT
U	75	14	31	HT
U	82	13	35	HT
U	86	10	39	HT
U	90	13	39	HT
U	62	51	4	Jo+
U	70	50	8	Jo+
U	83	62	8	Jo+
U	86	57	12	Jo+
U	91	74	6	Jo+
U	110	83	10	Jo+
U	114	96	6	Jo+
U	122	82	16	Jo+
U	91	53	17	Jo2
U	101	58	19	Jo2
U	113	69	19	Jo2
U	72	55	7	LP
U	73	48	11	LP
U	76	47	13	LP
U	80	43	17	LP
U	81	48	15	LP
U	83	57	11	LP
U	84	43	19	LP
U	89	55	15	LP
U	90	19	35	LP
U	90	77	5	LP
U	91	49	19	LP
U	95	49	21	LP
U	96	46	23	LP
U	96	69	11	LP
U	97	66	13	LP
U	98	63	15	LP
U	100	77	9	LP
U	102	63	17	LP
U	103	56	21	LP
U	105	54	23	LP
U	107	48	27	LP
U	107	52	25	LP
U	108	45	29	LP
U	108	76	13	LP
U	109	73	15	LP
U	111	63	21	LP
U	112	72	17	LP
U	114	62	23	LP
U	115	59	25	LP
U	117	57	27	LP
U	118	46	33	LP
U	118	50	31	LP
U	118	54	29	LP
U	119	95	9	LP
U	121	72	21	LP
U	121	84	15	LP
U	121	88	13	LP
U	122	69	23	LP
U	124	79	19	LP
U	125	64	27	LP
U	125	68	25	LP
U	126	97	11	LP
U	126	112	5	LP
U	127	50	35	LP
U	127	54	33	LP
U	127	62	29	LP
U	61	7	29	Lv
U	125	8	61	Lv
U	26	13	7	Pu2
U	25	15	5	Si
U	74	7	35	vT2
U	86	7	41	vT2
U	89	7	43	vT2
U	31	7	13	vT3
U	34	7	15	vT3
U	58	7	27	vT3
U	55	7	25	vT4
U	25	8	9	YH1

TABLE V
STATISTICS

Label Counts				Gap Counts	
Reference	Lower	Upper	Total	Gap Size	Count
implicit	7698	7720	15418	0	2914
A	3	0	3	1	686
B	1	16	17	2	1017
B2	12	0	12	3	473
B2x	5	0	5	4	886
C	119	0	119	5	419
D	22	0	22	6	641
E	0	176	176	7	265
External	268	216	484	8	541
				9	168
				10	118
				total	8128
				Nonzero	5214
				Max Gap	10
				Weight	22499

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The Linear Programming Bound for Binary Linear Codes

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***Abstract*—Combining Delsarte's linear programming bound with the information that certain weights cannot occur, new upper bounds for $d_{\min}(n, k)$, the maximum possible minimum distance of a binary linear code with given word length n and dimension k, are derived.**

***Index Terms*—Binary linear code, upper bound.**

I. INTRODUCTION

Let C be a code (i.e., nonempty subset) in a distance-regular graph Γ. Let the *inner distribution* a of C be the vector defined by

$$a_i = \frac{1}{|C|}\#\{(x, y) \in C \times C | d(x, y) = i\},$$

and let Q be the dual eigenmatrix of Γ. Then Delsarte's linear programming bound states that the entries of the MacWilliams transform $b := aQ$ of a are nonnegative. In the particular case of linear codes in the Hamming graph $H(n, q)$, a_i and b_i equal the number of words of weight i in C and its dual $C^{\perp}$, respectively.

Our aim is to show nonexistence of binary linear codes with given word length n, dimension k, and minimum distance d, using a known lower bound dd for the minimum distance $d^{\perp}$ of the dual code, and a known set I of indexes such that no words of weight $w \in I$ occur. Indeed, given this information we can try to solve the system

$$\begin{cases} a_0 = 1 \\ a_i = 0 \ (1 \le i \le d-1) \\ a_i \ge 0 \ (d \le i \le n) \\ a_i = 0 \ (i \in I) \\ (aQ)_0 = 1 \\ (aQ)_j = 0 \ (1 \le j \le dd-1) \\ (aQ)_j \ge 0 \ (dd \le j \le n) \end{cases} \tag{LP}$$

for the variables a_i, and if there is no feasible solution then no such code exists. In some cases a contradiction is first obtained after adding additional constraints to the system (LP), like (in the binary case, if $d = 2e$)

$$\frac{1}{A(n, d, e)} a_{n-e} + a_{n-e+1} + \cdots + a_n \le 1 \tag{1}$$

and

$$\frac{1}{A(n, d, e+f)} a_{n-e-f} + \left(1 - \frac{A(n-e+f, d, e+f)}{A(n, d, e+f)}\right) a_{n-e+f} + a_{n-e+f+1} + \cdots + a_n \le 1, \tag{2}$$

for $1 \le f \le e$.

We find dd by inspecting known upper bounds (as found, for example, in Verhoeff's tables [15], [16], updated by Hill and Traynor [9]) for the maximum possible minimum distance of binary linear codes. Indeed, if no $[n-w, k-w+1, d]$-code exists, then $C^{\perp}$ does not have words of weight w, and if this holds for $1 \le w \le dd-1$, then $d^{\perp} \ge dd$. Also, if no $[n-w, k-1, d-\lfloor w/2 \rfloor]$-code exists, then C does not have words of weight w. Thus, we can take I to be the set of all w for which this is known.

References for the linear programming bound are Delsarte [7] and MacWilliams and Sloane [13, 17, section 4]. Our approach was inspired by Hill and Traynor [9], who use all equalities, but only the first few inequalities of the system (LP). Our work was much facilitated by the availability of Verhoeff's software so that the table lookup required to determine dd and I could be done automatically.

II. THE JOHNSON BOUND

In Verhoeff's tables, and in Helgert and Stinaff's table on which they were based, there are entries labeled "K" or "J" or "R" explained as "Johnson upper bound" and "Improvement of Johnson bound" with a reference for the latter to private communications by Johnson and by McEliece and Welch. We recomputed the Johnson bound as given by Johnson in [11], and found that all entries labeled "K" or "J" are special cases of this bound, except for the single entry affirming that no [126, 113, 5] code exists. However, we shall see below that this entry follows from a sharpening of the Johnson bound. Moreover, all entries labeled "R" follow from known bounds. Thus, there is no need anymore for references to these two private communications. In fact, the bounds in Johnson [11] rule out many codes still permitted by Verhoeff's table.

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