Lobke, and Other Constructions from Conical Segments

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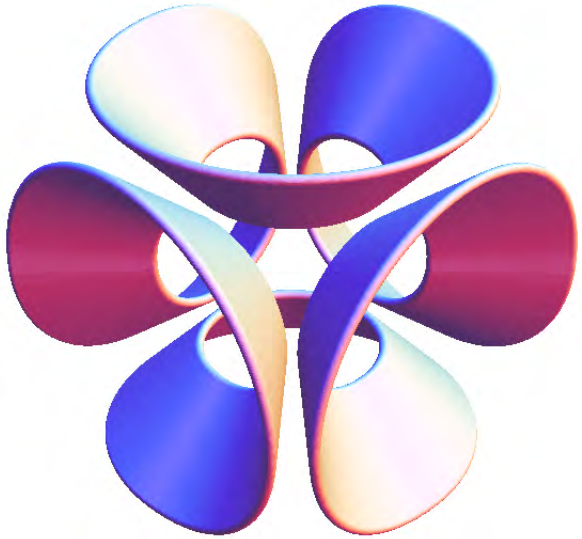
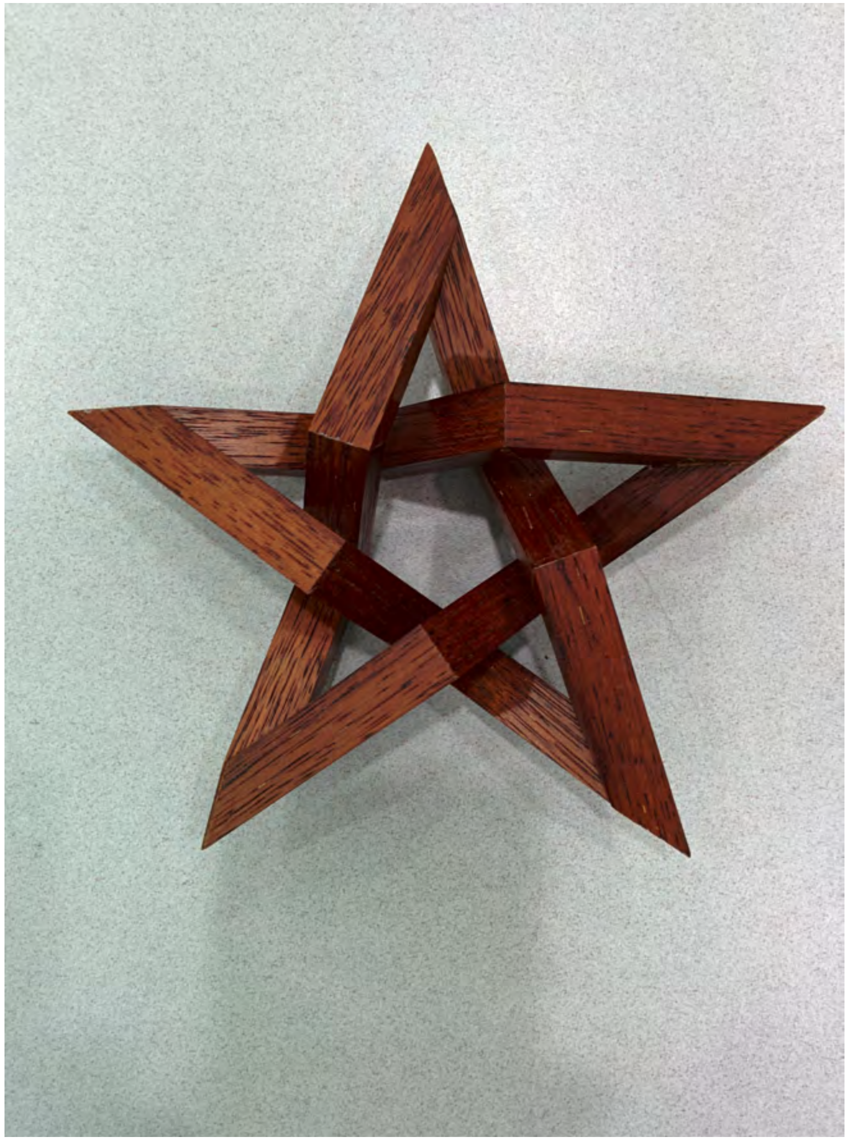


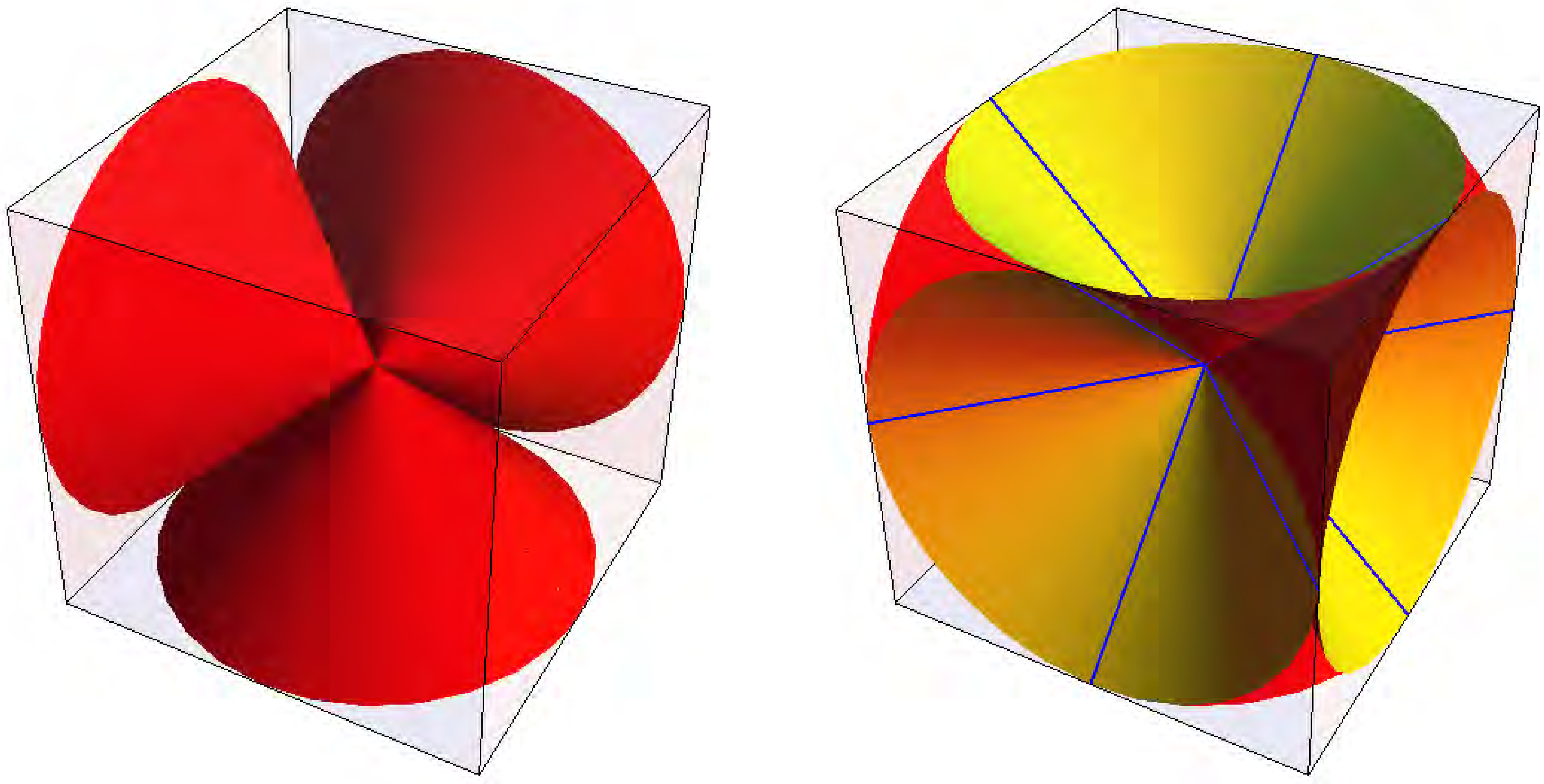
Recent Mitered Designs by Koos Verhoeff

Lobke (Koos Verhoeff, 1990s)

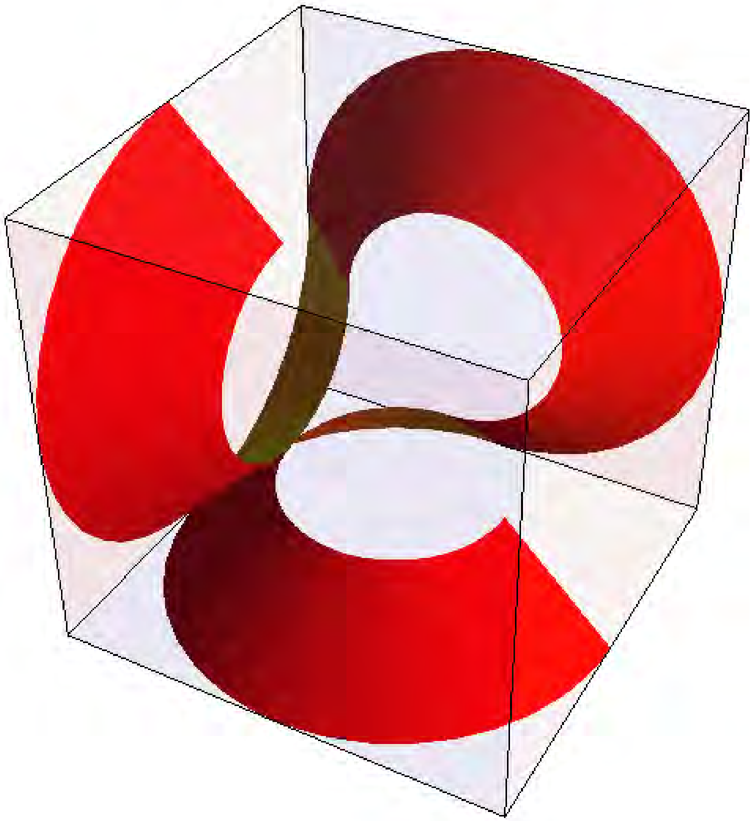
Fiberglass with polyester resin on a metal mesh (73 cm tall)

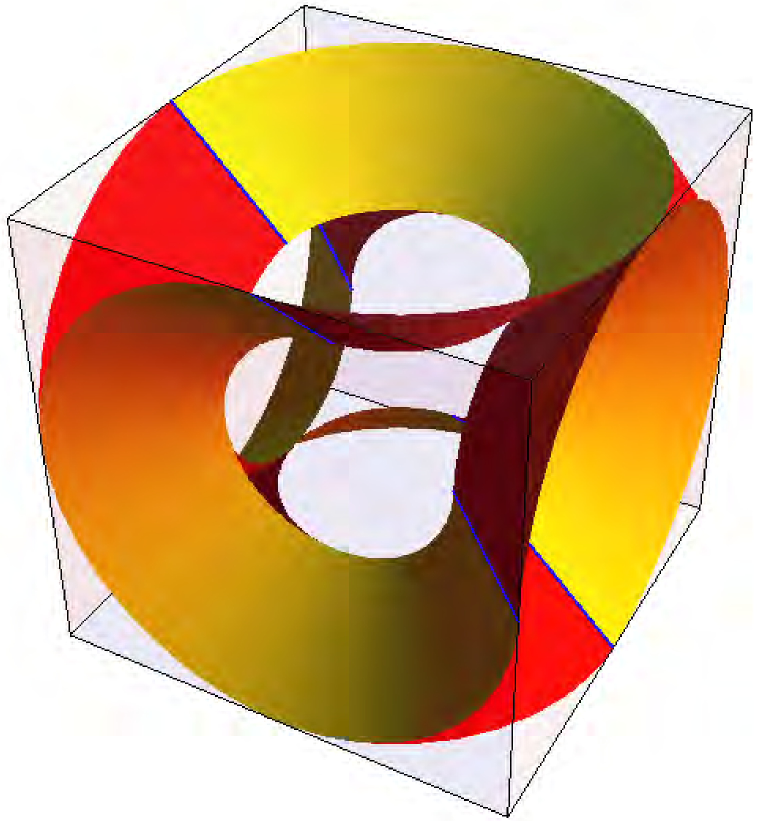
Cones with 90° Aperture in Cube



The cones touch (blue lines, on the right)

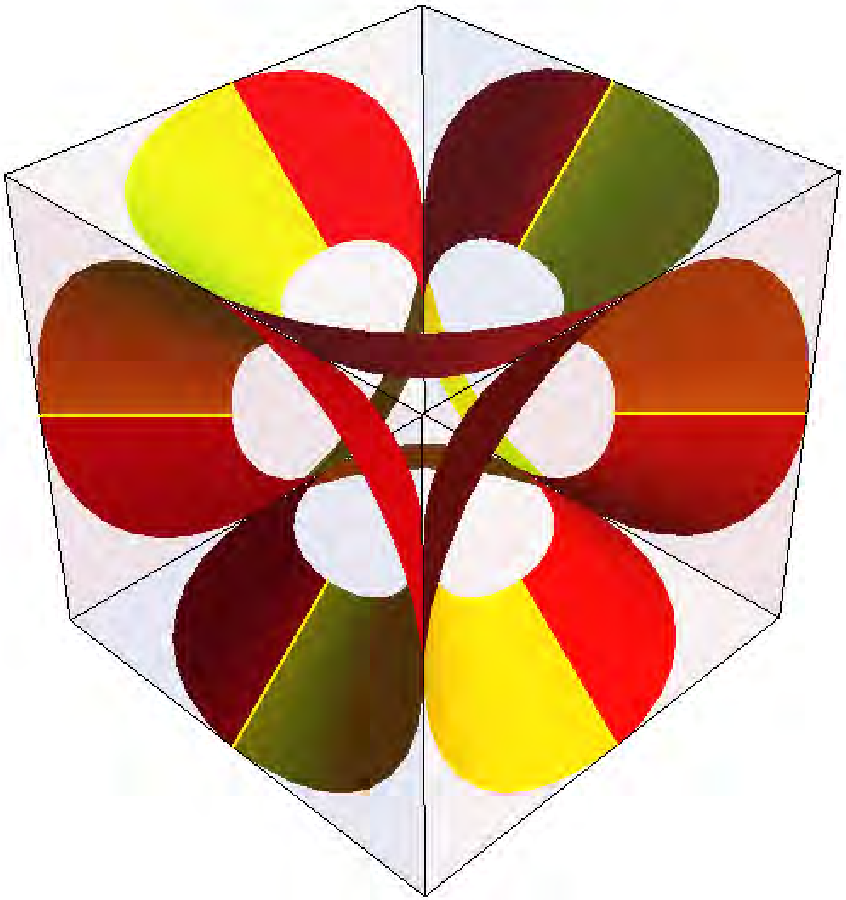
3/4 of 90° Conical Segments in Cube, Forming a Closed Strip

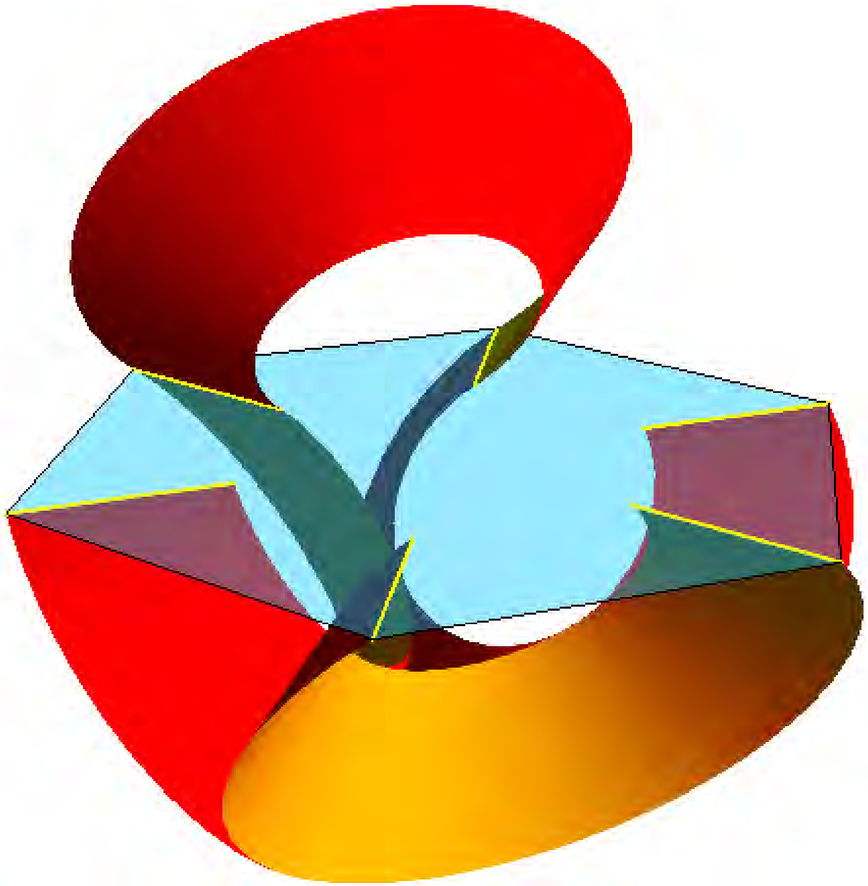




The segments touch, and connect smoothly (blue edges)

Six-fold Symmetry





Self-Intersection

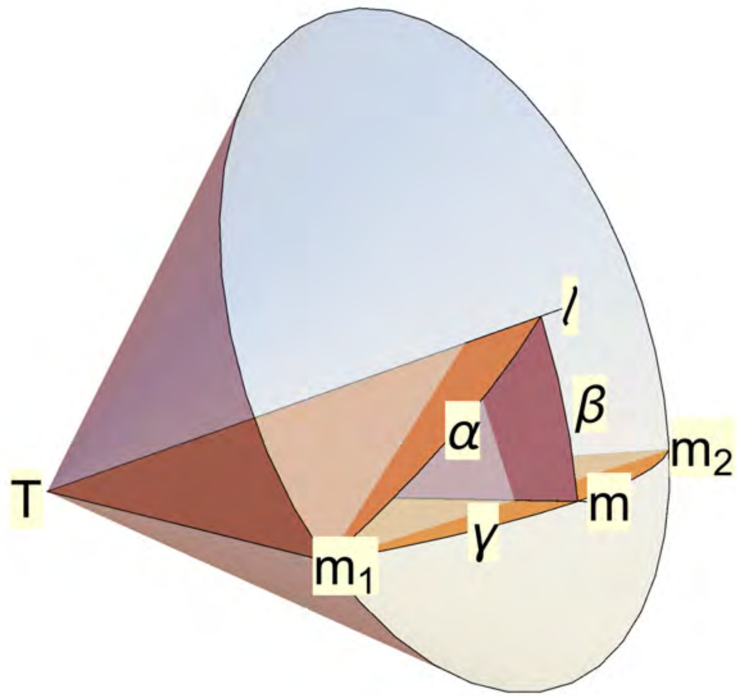
To make a sculpture, the segments must be thickened

Thickening: touching $\rightarrow$ self-intersection

Self-intersection can be avoided:

- Reduce the aperture of the cones to $< 90^\circ$
- Preserve the six-fold symmetry, i.e., the equatorial cut lines
- Preserve smooth connections
- Hence, also reduce the fraction of cone in the segments to $< 3/4$

Mathematics Involved



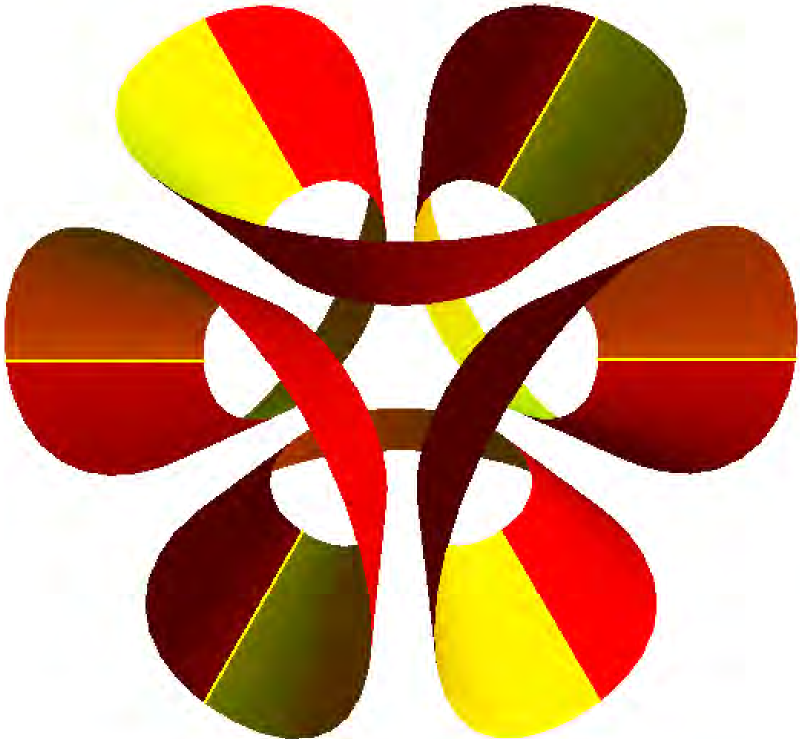
- Cone: tip T, axis ℓ, aperture 2α
- Cut by plane tilted over β
- Angle $m_1 T m_2$ is 2γ
- Pythagorean Theorem for right-angled spherical triangles:

$$\cos\alpha = \cos\beta\cos\gamma$$

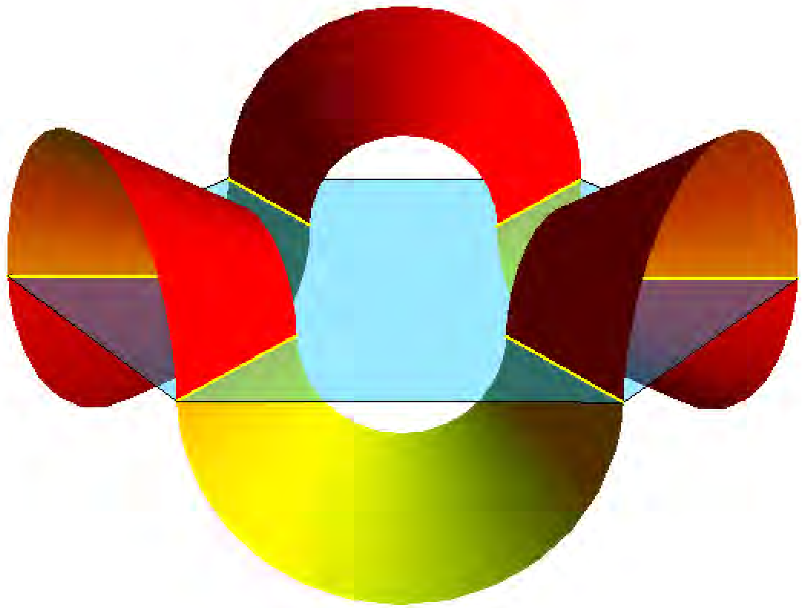
Conical Segments with Varying Aperture, Sharing Two Edges

3D Print of Conical Segments Sharing Two Edges

Reduced Aperture and Fraction

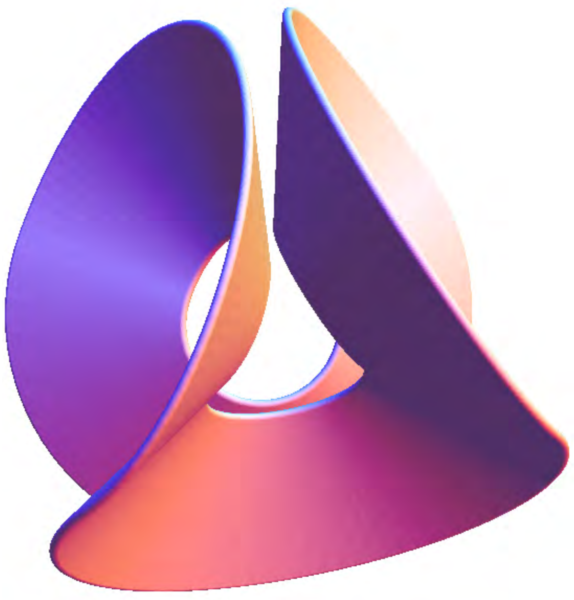


Aperture 86°
Cone Fraction 0.738



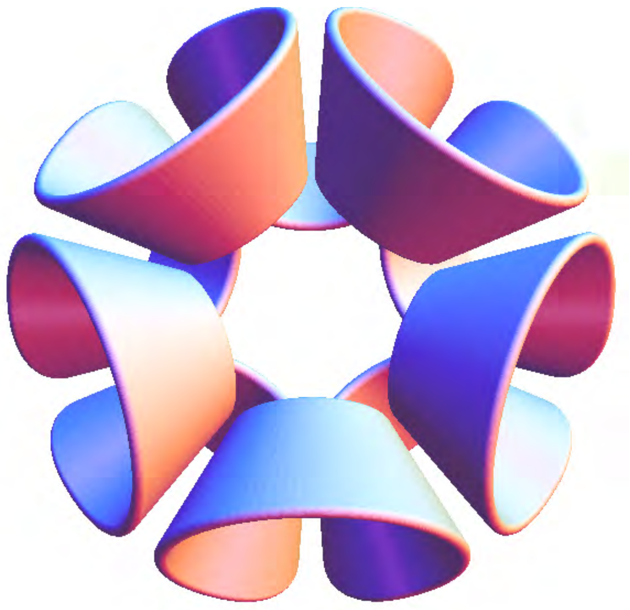
Aperture 60°
Cone Fraction 1/2

Variation 1: Vary the Number of Lobes



4 Lobes

8 Lobes



10 Lobes

Emphatic Self-Intersection

4 Lobes

6 Lobes

8 Lobes

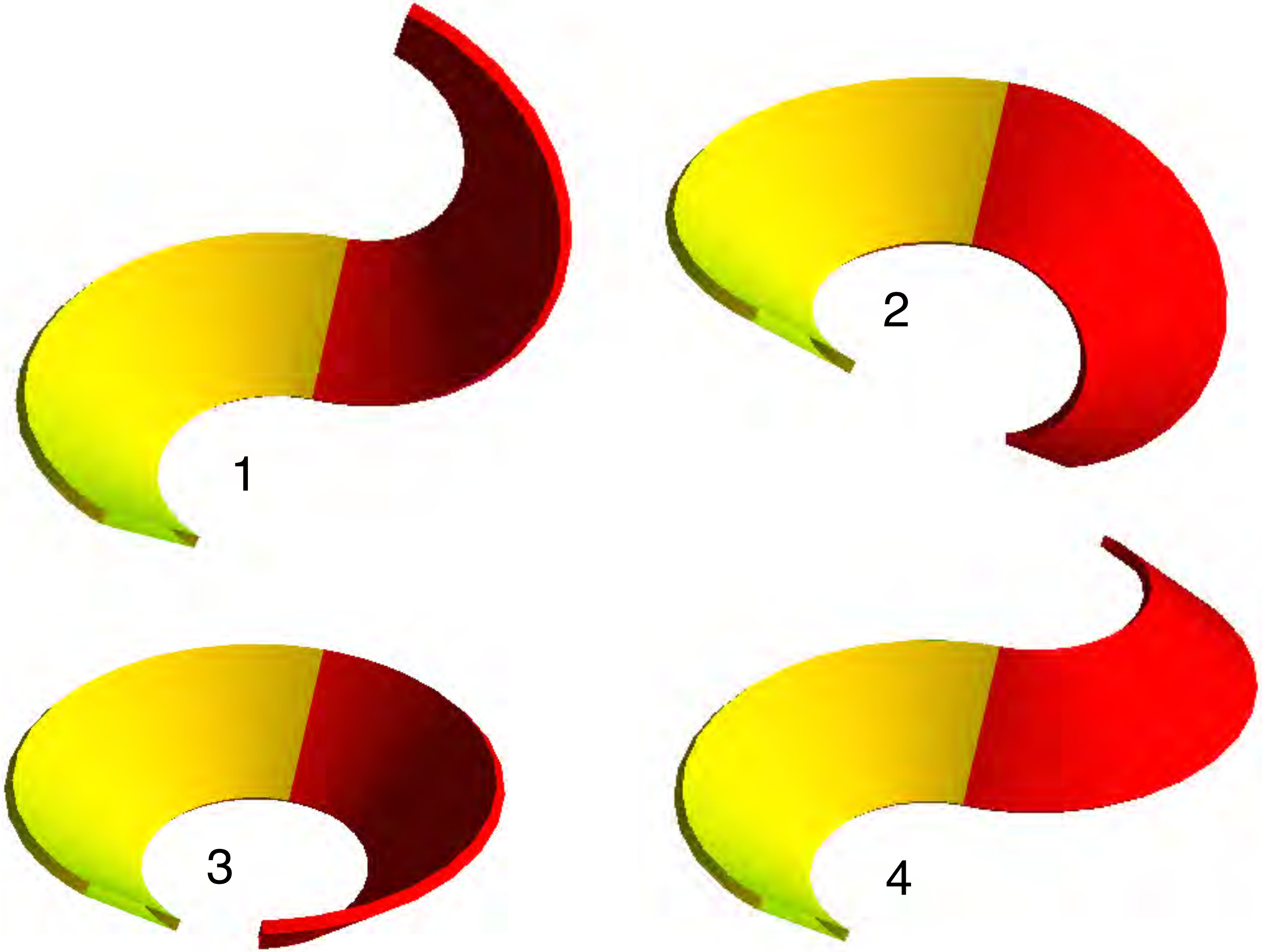
For ceramic 3D prints, self-intersection is necessary

Ceramic 3D Prints of Self-Intersecting Variants

6 Lobes

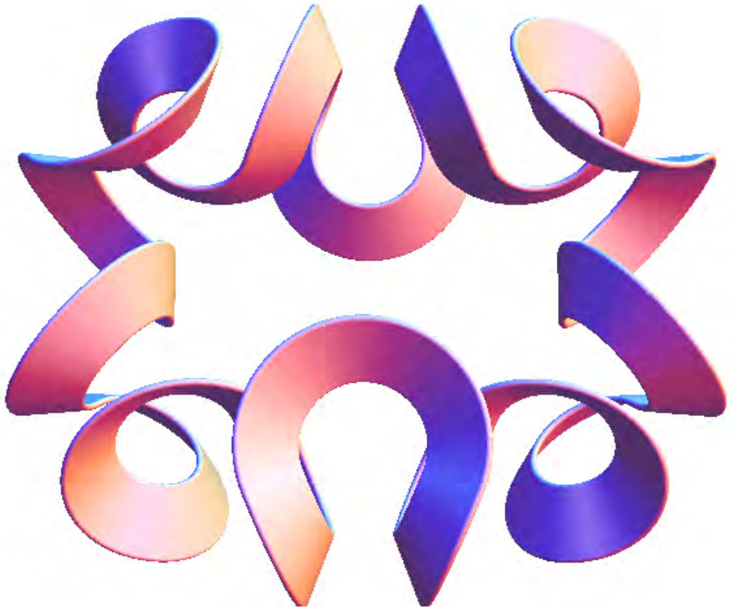
10 Lobes

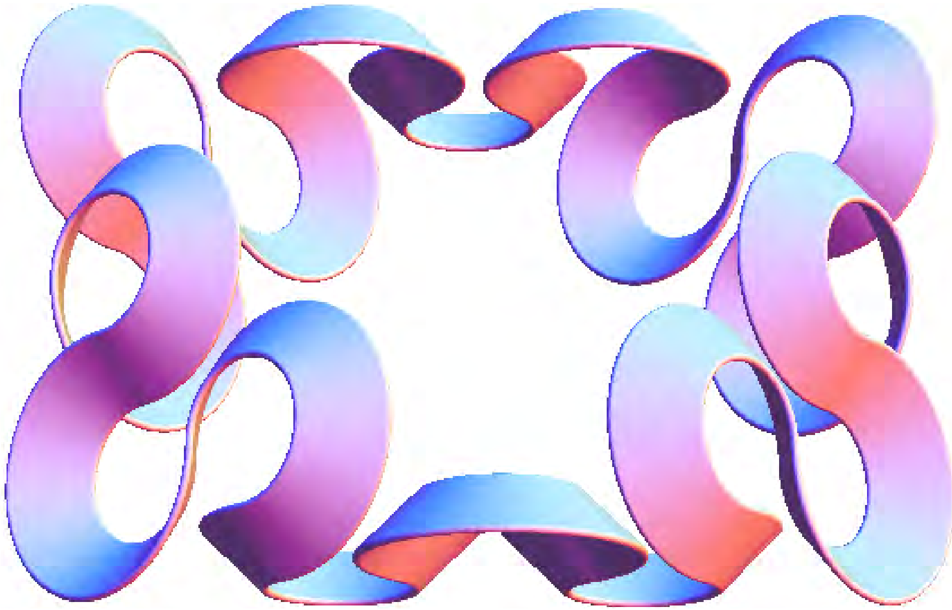
Variation 2: Vary the Connections between Segments



Problem: Create Properly Closed Smooth Strips

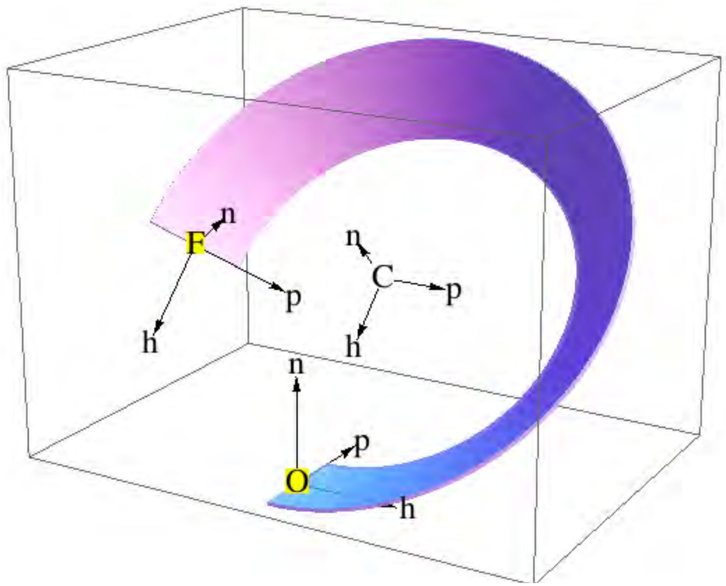
- Using just one type of conical segment
- Parameters of conical segment: aperture 2α, radius r, fraction β



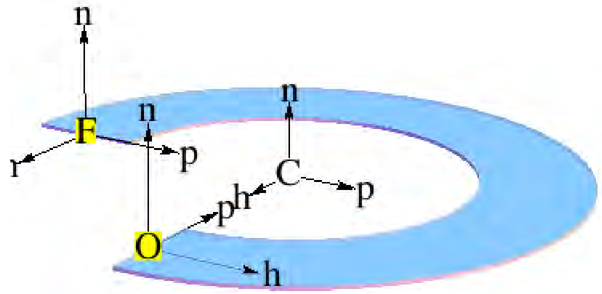


Describing Strips of Conical Segments

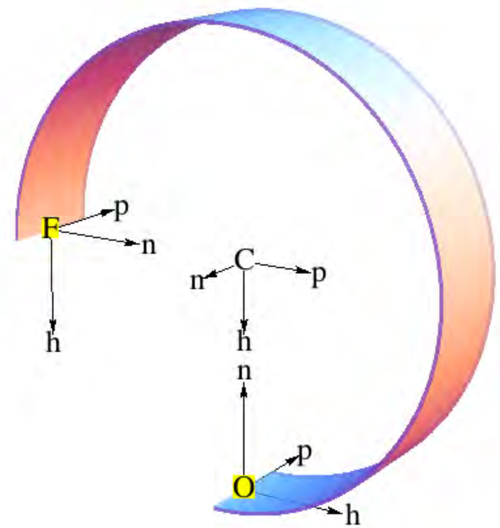
New Turtle Geometry command: $CStrip(\alpha, r, \beta)$



$CStrip(45°, 1, 270°)$



$CStrip(90°, 1, 270°)$



$CStrip(0, 1, 270°)$

Relationship to Connection Types

The following conical segments are congruent:

0. $CStrip(\alpha, r, \beta)$,

1. $CStrip(180^\circ - \alpha, r, \beta)$,

2. $CStrip(180^\circ + \alpha, r, \beta)$,

3. $CStrip(360^\circ - \alpha, r, \beta)$,

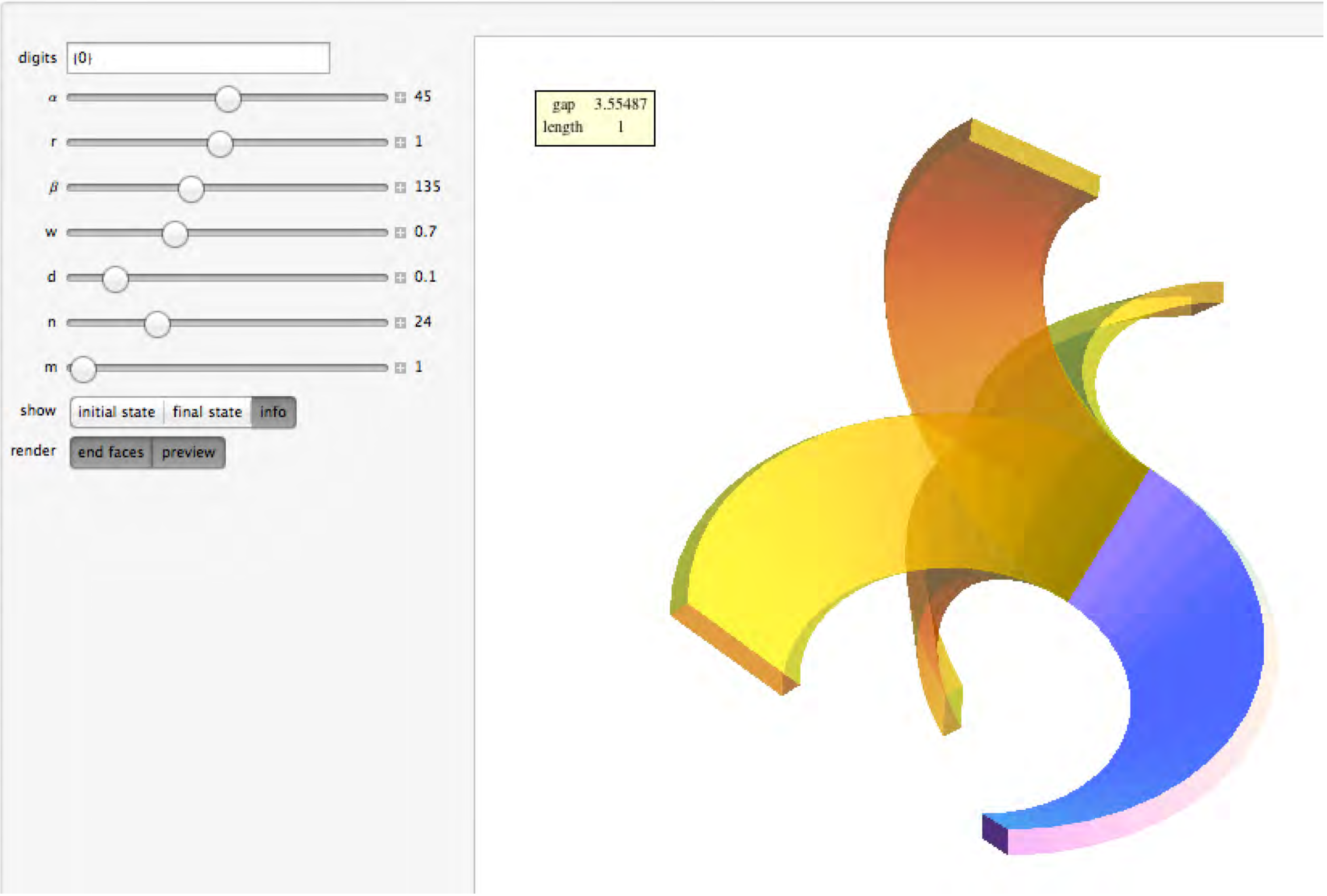
A strip is fully described by α, β, and a sequence of *indices*

Find Closed Strips by Trial and Error Elimination

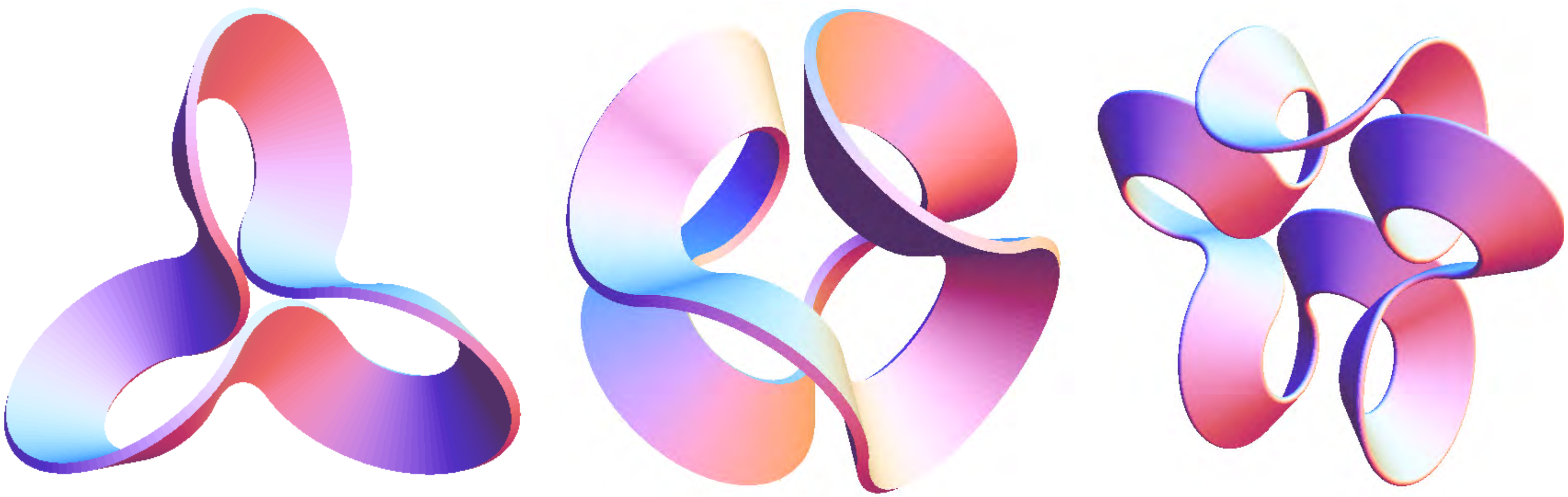
Strip generated by $\alpha = 36^\circ$, $\beta = 246 \pm 1^\circ$, sequence $(0, 1, 2, 3, 2, 1)^3$

Tweak α and/or β to obtain closure

Mathematica App to Explore Strips of Conical Segments

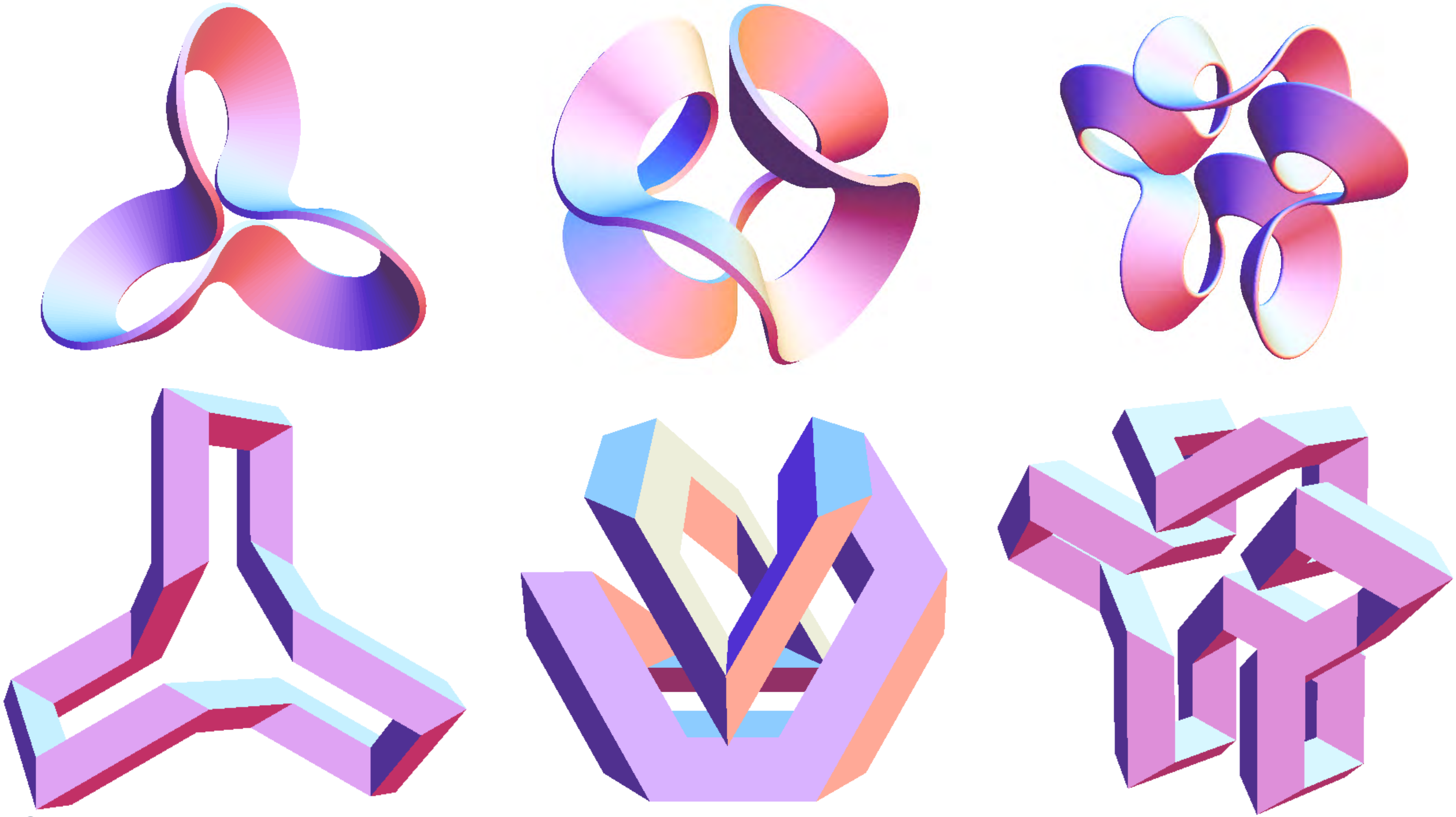


Examples of Closed Strips of Conical Segments

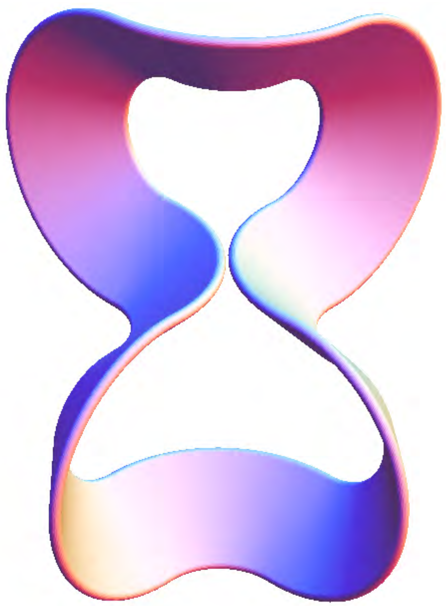


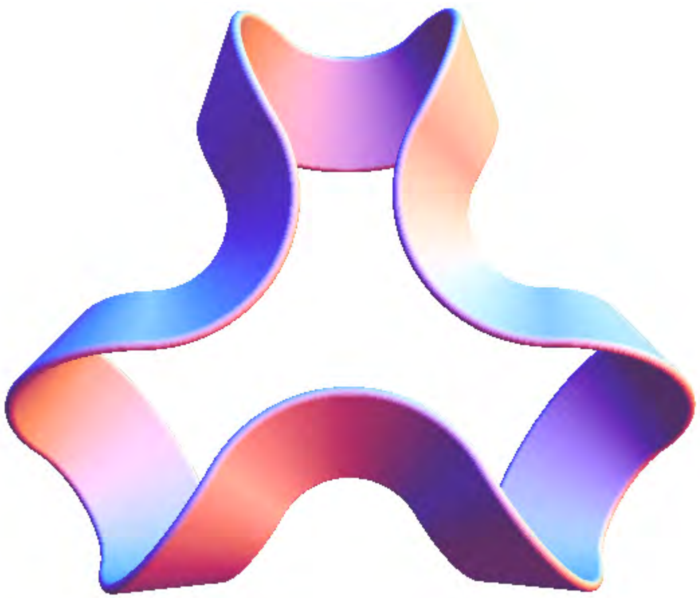
Discrete Approximations of Conical Segments

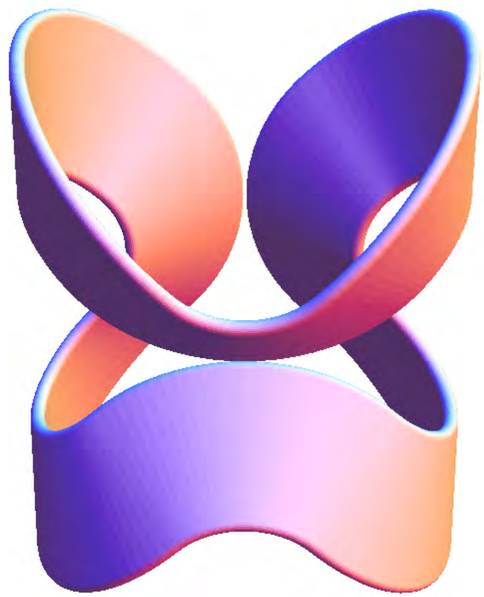
Same Shapes with Straight Trapezoidal Segments



More Examples of Closed Strips of Conical Segments







Related Work

- *Seat of Wisdom* and *Circle Squared* by Vic Pickett
- *Bronze Spheric Theme* and *Model for 'Spheric Theme'* by Naum Gabo
- *Snake*, *Berlin Junction*, and other sculptures by Richard Serra
- *Borsalino* and other sculptures by Henk van Putten, using cylindrical segments with a square cross section

  Also see "LEGO®" Knots by Séquin and Galemmo (Bridges 2014)
- *Arabesque XXIX* by Robert Langhurst resembles *Lobke*, but it has no hole and it is not a developable surface.

Conclusion

- Explore constructions with congruent conical segments

  Two parameters: cone aperture, cone fraction

- Challenge: find properly closed strips

- Describe with Turtle Geometry

- Relationship with mitered constructions

- Relationship with constant torsion paths

- Rotate segments about center line

- Square cross section

Rotate segment about the center line; square cross section