MATH ENCOUNTERS

SAVING *face*:
information tricks for
LIFE *and* **LOVE**

Tom Verhoeff
Eindhoven University of Technology
The Netherlands
http://www.win.tue.nl/~wstomv/talks/momath
mailto:T.Verhoeff@tue.nl

Contents

1 ACAN Trick: Any Card, Any Number

1.1 Ingredients

- 27 distinct cards (e.g. 26 regular playing cards and 1 joker)

- 1 magician

- an audience, with 1 or 2 volunteers

1.2 Procedure

1. The magician asks the/a volunteer to select 1 card, keeping it secret for the magician.

2. The magician asks the/another volunteer to pick a number N from 1 to 27 (inclusive), and announce it. It is recommended to write this number down.

3. The magician now announces that the selected (secret) card will be moved to the selected position in the deck.

4. The magician writes $N-1$ in *ternary* notation ($xyz = 9x + 3y + z$, using only digits 0, 1, 2), *reverses* the digit order, and interprets the digits as 0=Top, 1=Middle, 2=Bottom as *recipe*.

N	$N-1$	Recipe		N	$N-1$	Recipe		N	$N-1$	Recipe
1	000	T T T		10	100	T T M		19	200	T T B
2	001	M T T		11	101	M T M		20	201	M T B
3	002	B T T		12	102	B T M		21	202	B T B
4	010	T M T		13	110	T M M		22	210	T M B
5	011	M M T		14	111	M M M		23	211	M M B
6	012	B M T		15	112	B M M		24	212	B M B
7	020	T B T		16	120	T B M		25	220	T B B
8	021	M B T		17	121	M B M		26	221	M B B
9	022	B B T		18	122	B B M		27	222	B B B

5. The magician repeats the following 3 times:

   (a) Lay out the cards, row-by-row, starting top left, in 9 rows of 3 cards. Each row partly overlaps the preceding row, clearly making 3 columns.

   (b) Ask the (first) volunteer, in which column the selected card appears.

   (c) Collect each column into a piles; while turning the piles over, put the selected pile into the position indicated by the recipe.

6. From the pile of face-down cards, the magician counts off $N-1$ cards (face down), and shows the N-th card face up. This is the selected card.

2 A Five-Card Trick

2.1 Ingredients

- 1 complete deck of 52 regular playing cards (4 suits with 13 values)
- 1 (trained) assistant, 1 magician
- an audience

2.2 Procedure

1. The magician asks the assistant to let (5 volunteers from) the audience pick 5 cards from the deck.
2. The assistant lays these cards in a row, with the leftmost card face down, and the others face up (further details below).
3. The magician then looks at the face-up cards, and announces the face-down card (further details below).

Card suits are ordered: Spades ($\spadesuit$), Hearts ($\heartsuit$), Clubs ($\clubsuit$), Diamonds ($\diamondsuit$)


Card values are ordered: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J (11), Q (12), K (13 = 0)
Cards are ordered first by suit, then by value.

Assistant

1. Picks a *same-suit* pair from the 5 cards. Their values differ $d \leq 6$. [Pigeon-hole principle]
2. Places the (cyclicly) higher card face down on the left.
3. Places the (cyclicly) lower card rightmost.
4. Assuming `A B C` is the sorted order of the remaining 3 cards, permutes them for rank d (see table), and places them between the other two cards.
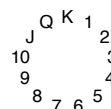
d	Permutation		d	Permutation		d	Permutation
1	A B C		3	B A C		5	C A B
2	A C B		4	B C A		6	C B A

Magician

1. Deduces the suit of the face-down card from the rightmost card.
2. Deduces the distance d from the permutation of middle 3 cards.
3. Deduces the value of the face-down card by adding d to the value of the rightmost card.

Practice on-line at `http://www.win.tue.nl/~wstomv/misc/cardtrick`

M. Kleber, R. Vakil. "The Best Card Trick", Math. Intelligencer, **24**(1):9–11

3 Ulam's Game: Number Guessing with Lies

3.1 Ingredients

- 1 magician

- an audience, with 1 or 2 volunteers

3.2 Procedure

1. The magician asks the/a volunteer to pick a number N from 0 to 15 (inclusive), and announce it. It is recommended to write this number down.

2. The magician asks the/another volunteer 7 Yes-No questions, where at most 1 lie is permitted. Is you number one of these?

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q_1		1		3	4		6		8		10			13		15
Q_2		1	2			5	6		8			11	12			15
Q_3									8	9	10	11	12	13	14	15
Q_4		1	2		4			7		9	10		12			15
Q_5					4	5	6	7					12	13	14	15
Q_6			2	3			6	7			10	11			14	15
Q_7		1		3		5		7		9		11		13		15

3. The magician writes down the answers A_i, does a little calculation, and announces the number N, and which answer was a lie (if any).

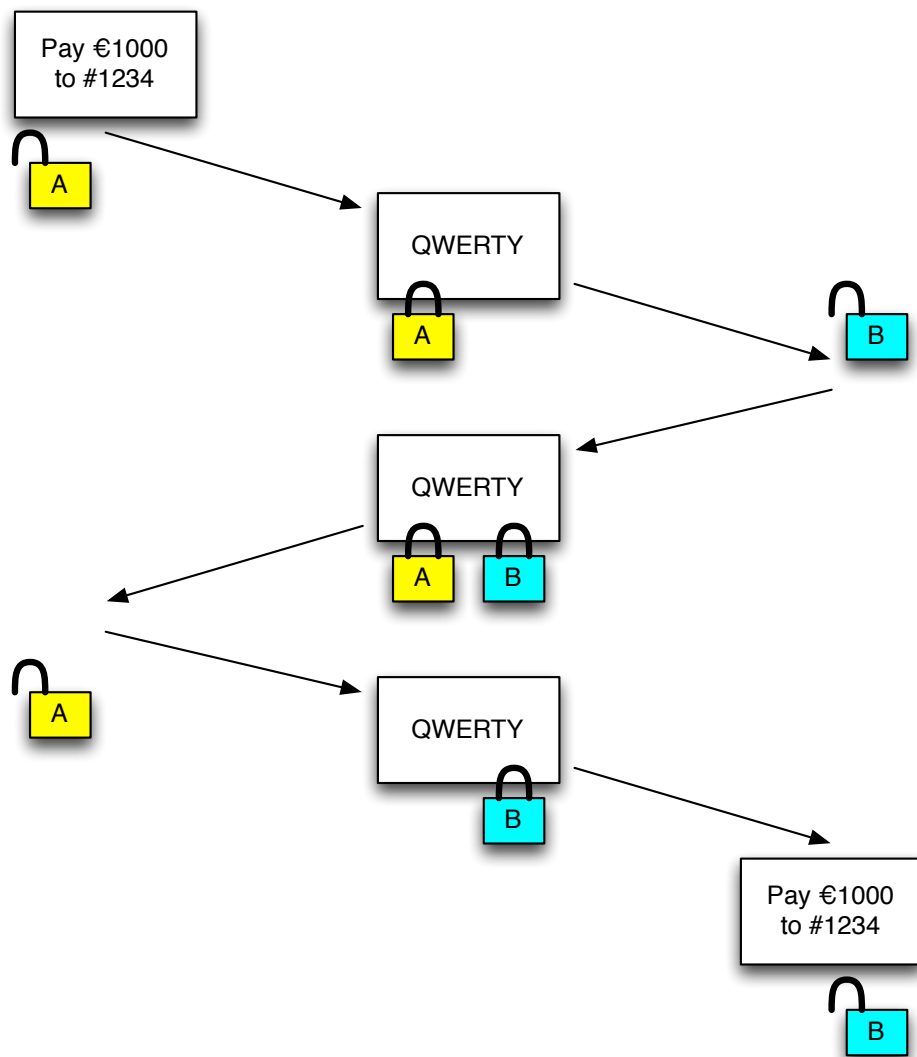**Calculation** The magician places the answers in the 3 circles of the diagram:
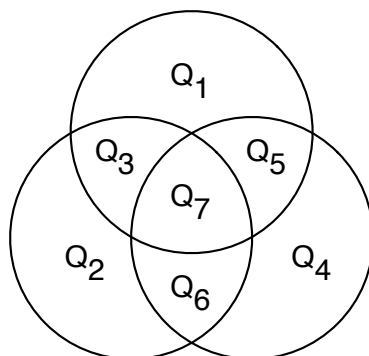
The magician makes each circle with an *odd* number of No's *red*. A lie is inside all *red* circles, and outside all non-red circles. A lie can now be *corrected*. The number $N = 8A_3 + 4A_5 + 2A_6 + A_7$.

4 Shamir's Three-Pass Protocol

4.1 Ingredients
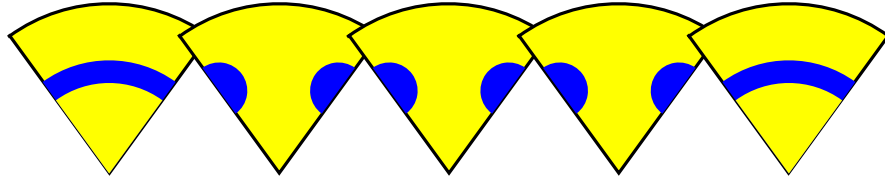
- 1 strongbox
- 2 padlocks with different keys

4.2 Procedure

5 Zero-Knowledge Match Making

5.1 Ingredients

- 5 sectors, 2 of one type ("head"), and 3 of another type ("line")

- 1 spinner

- 1 magician

- an audience with 2 volunteers

5.2 Procedure

1. The magician asks for two volunteers who want to decide on whether to go to MoMath together, in a *face-saving way*.

2. The magician gives each volunteer two distinct sectors.

3. Each volunteer places the two sectors side by side, making an "arrow" that points *toward* the other for "Yes", and *away* from the other for "No".

4. The magician places the remaining sector upside-down on the spinner (at 6 o'clock), and lets the volunteers add in their sectors, also upside-down.

5. The magician spins the spinner and turns it upside down.

6. The outcome is determined as follows:

No   Yes

See: http://demonstrations.wolfram.com/ZeroKnowledgeMatchmaker/
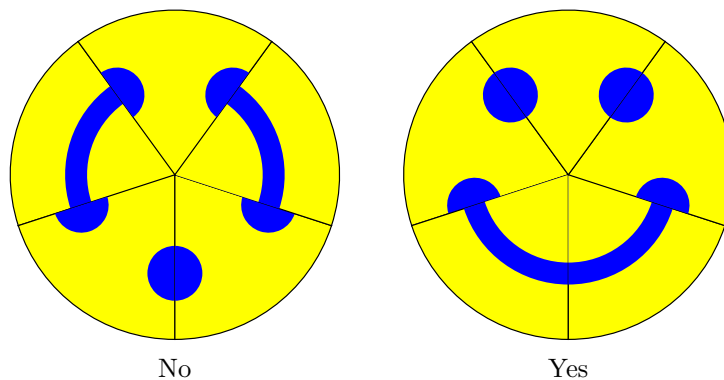
Five-card Trick: Exercises for Assistant

Given two cards of the same suit, determine the lower and higher value, and their distance (on the clock). That distance is always between 1 and 6.

Cards	Lower value	Higher value	Distance d
7♠ 2♠	2	7	5
3♠ 6♠			
10♡ J♡			
Q♡ 8♡			
10♣ 4♣			
K♣ J♣			
8♢ K♢			
J♢ 5♢			
2♠ Q♠	Q	2	3 (not 10)
K♠ 1♠			
3♡ J♡			
Q♡ 1♡			
9♣ 2♣			
4♣ K♣			
1♢ J♢			
5♢ Q♢			

Given three cards, put them in sorted order.

Cards			Sorted order		
2♡	1♣	3♠	3♠	2♡	1♣
6♡	4♢	5♣			
H♡	B♢	1♡			
V♠	10♣	2♢			
3♢	7♡	3♠			
B♣	3♣	6♣			
9♠	8♠	V♡			

Given three cards (in sorted order), permute them to encode the given distance d.

Sorted order			Distance d	Encoded order		
3♠	2♡	1♢	4	2♡	1♢	3♠
V♠	10♣	2♢	1			
7♡	5♣	4♢	2			
9♠	10♠	V♡	3			
1♡	V♡	B♣	4			
3♢	7♢	B♢	5			
4♠	7♡	3♢	6			

Given five cards, choose which card to put face down, and in what order to put the face-up cards? NOTE: There can be multiple solutions!

Cards					d	Order				
3♠	8♢	1♣	4♢	2♡	4	8♢	2♡	1♣	3♠	4♢
4♢	5♣	2♠	6♡	K♠						
J♢	10♣	1♡	K♡	1♣						
10♣	10♠	2♢	J♠	Q♠						
5♡	7♡	1♠	3♢	Q♡						
K♢	6♣	3♣	9♢	J♣						
8♠	9♠	Q♡	Q♠	9♠						

8

Five-Card Trick: Exercises for Magician

Given the lower value and the desired distance, determine the higher value (on the clock).

Lower value	Distance d	Higher value
2	5	7
8	3	
3	6	
6	4	
J	2	
7	5	
Q	1	
7	4	
Q	3	2
B	4	
K	1	
J	3	
10	5	
K	2	
10	6	
9	5	

Given three face-up cards, which distance d do they encode?

Face-up cards			Encoded distance d
2♡	1♣	3♠	4
6♡	4♢	5♣	
H♡	B♢	1♡	
V♠	10♣	2♢	
3♢	7♡	1♠	
B♣	3♣	6♣	
9♠	8♠	V♡	

Given four face-up cards, which distance d do they encode, and what is the face-down card?

Face-up cards					Encoded distance d	Face-down card
?	2♡	1♣	3♠	4♢	4	8♢
?	6♡	4♢	5♣	K♠		
?	K♡	B♢	1♡	10♣		
?	Q♠	10♣	2♢	10♠		
?	3♢	7♡	1♠	Q♡		
?	J♣	3♣	6♣	9♢		
?	9♠	8♠	Q♡	9♠		