Corrigenda Random Graphs and Complex Networks. Vol. 1

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In this document, we list corrections to Random Graphs and Complex Networks Volume 1 [1]. The line numbers refer to the original Cambridge University Press edition.

1 Page 37, line 13: "and this, too, is not what we want". [Thanks to Christoph Schumacher for noting this typo.]

2 Page 38, [I, (1.6.6)] and the formula below it should be changed to

$$\sum_{j\in[n]} P_{ji}\pi_j = \alpha \sum_{j\in[n]} M_{ij}\pi_j + \frac{1-\alpha}{n} \sum_{j\in[n]} \pi_j = \alpha(M\pi)_i + \frac{1-\alpha}{n}, \tag{1.6.6}$$

where $M = (M_{ij})_{i,j\in[n]}$ is the stochastic matrix

$$M_{ij} = \frac{\mathbb{1}_{\{(j,i)\in E\}}}{d_j^{(\mathrm{out})}} + \frac{\mathbb{1}_{\{j\in\mathcal{D}\}}}{n}. \tag{1.6.7}$$

3 Page 44, [I, (1.7.7)]: The equation for Hill's estimator should read

$$H_{k,m} = \frac{1}{k}\sum_{i=1}^{k} \log(X_{(k+1)}/X_{(i)}), \tag{1.7.7}$$

i.e., [I, (1.7.7)] misses a minus sign. [Thanks to Christoph Schumacher for noting this typo.]

4 Page 60: The proof how [I, Theorem 2.4] follows from [I, (2.1.13)] is somewhat confusing. Indeed, rewrite [I, (2.1.14)] as

$$\sum_{r=k}^{N} (-1)^{k+r} \frac{\mathbb{E}[(X)_r]}{(r-k)!k!}, \tag{2.1.14}$$

which is alternatingly larger than $\mathbb{P}(X = k)$ (for $N + k$ even) and smaller than $\mathbb{P}(X = k)$ (for $N + k$ odd). Then, one can apply this to $\mathbb{P}(X_n = k)$, and obtain finite-sum upper and lower bounds to $\mathbb{P}(X_n = k)$. Since for the Poisson random variables, these upper and lower bounds will become identical for $N \to \infty$, this completes the proof. the fact that [I, (2.1.14)] was using the same n as the n in X_n in [I, Theorem 2.4] was highly confusing. [Thanks to Christoph Schumacher for noting this confusion.]

5 Page 73, line 2: 'For $\lambda + t \leq n$....' [Thanks to Christoph Schumacher for noting this typo.]

6 Page 84, [I, Exercise 2.7]: (2.8.3) should read

$$\sum_{r\geq k} \frac{\mathbb{E}[(X)_r]}{(r-k)!} < \infty, \tag{2.8.3}$$

[Thanks to Christoph Schumacher for noting this typo.]

7 Pages 84-85, [I, Exercises 2.18 and 2.19]: It would be more appropriate to say that 'X and Y have a normal distribution', or 'X and Y are normal random variables [Thanks to Christoph Schumacher.]

8 Page 94, line -10: 'It follows that T' in (3.3.2)....' [Thanks to Christoph Schumacher for noting this typo.]

9 Page 94, line 3 of the proof of [I, Lemma 3.6]: It should read that $S_0 = S_0' = 1$.

10 Page 126, formula [I, (4.3.12)]: Here k_n should be an integer, so that the text should read "By the union bound, for $k_n = \lceil \log n \rceil$,

$$\mathbb{P}_\lambda(|\mathscr{C}_{\max}| > a \log n) \leq n\mathbb{P}_\lambda(|\mathscr{C}(1)| > k_n) \leq \mathrm{e}^{I_\lambda} n^{1-aI_\lambda} = \mathrm{e}^{I_\lambda} n^{-\delta}, \tag{4.3.12}$$

with $\delta = aI_\lambda - 1 > 0$ whenever $a > 1/I_\lambda$."
[Thanks to Christoph Schumacher for noting this typo.]

11 Page 130, [I, (4.3.40)] should read

$$\chi_{\geq k_n}(\lambda) = k_n P_{\geq k_n}(\lambda) + \sum_{t=k_n+1}^{n} P_{\geq t}(\lambda) \leq k_n \mathrm{e}^{-(k_n-1)I_\lambda} + \sum_{t=k_n+1}^{n} \mathrm{e}^{-I_\lambda(t-1)} \tag{4.3.40}$$
$$\leq k_n \mathrm{e}^{-(k_n-1)I_\lambda} + \frac{\mathrm{e}^{-k_n I_\lambda}}{1-\mathrm{e}^{-I_\lambda}} = O(k_n n^{-aI_\lambda}).$$

[Thanks to Christoph Schumacher for noting this typo.]

12 Page 133, line 7: Since $\lambda_n = \lambda(1 - k_n/n)$, it should read that $\lambda_n - \lambda = -\lambda k_n/n$. This makes no difference in what follows. [Thanks to Christoph Schumacher for noting this typo.]

13 Page 134, [I, (4.4.27)] should read

$$\mathbb{P}_\lambda(\mathscr{C}(j) < k \mid i \longleftrightarrow j, \mathscr{C}(i) = l) - \mathbb{P}_\lambda \mathscr{C}(j) < k) \leq lk\lambda/n. \tag{4.4.27}$$

[Thanks to Christoph Schumacher for noting this typo.]

14 Page 136, line 11: It should read that $g(\alpha; \lambda) > 1$ (as it does on Page 137, line 3).

15 Page 136, [I, (4.4.38)]: To be consistent with [I, (4.4.36)], all $\mathbb{P}_\lambda$'s except for the first should be $\mathbb{P}$'s. [Thanks to Christoph Schumacher for noting this typo.]

16 Page 136, [I, (4.4.39)]: The last line of [I, (4.4.39)] should read

$$= \exp\left(st - n(1 - \mathrm{e}^{-\lambda t/n})(1 - \mathrm{e}^{-s})\right), \tag{4.4.39}$$

[Thanks to Christoph Schumacher for noting this typo.]

17 Page 136, line -2: This should read "by the fact that $\mathrm{e}^x - 1 > x$ for every $x \in \mathbb{R} \setminus \{0\}$. As a result, $s^* \geq 0$ precisely when $t \leq \lfloor \alpha n \rfloor$ with $\alpha \leq \zeta_\lambda$." [Thanks to Christoph Schumacher for noting this typo.]

18 Page 137: [I, (4.4.42)] should read

$$\mathbb{P}_\lambda(S_t = 0) \leq \mathrm{e}^{-t\left(-\log g(t/n;\lambda) - 1 + g(t/n;\lambda)\right)} = \mathrm{e}^{-tI_{g(t/n;\lambda)}}. \tag{4.4.42}$$

[Thanks to Christoph Schumacher for noting this typo.]

19 Page 171: The last line of [I, (5.4.7)] should read "$\leq \frac{2\lambda+2\lambda^2}{n}$". On the next line, this means that the inequality should become "Since $\frac{2\lambda+2\lambda^2}{n} \leq \frac{\varepsilon_n}{2}$,..." [Thanks to Christoph Schumacher for noting this typo.]

20 Pages 171-172: The probability measure $\mathbb{P}_\lambda$ should be replaced by $\mathbb{P}$ in [I, (5.4.12)–(5.4.17)] for all events involving X_1 and X_2. [Thanks to Christoph Schumacher for noting this typo.]

21 Page 172, line 10: "See Theorem 6.10 below." [Thanks to Christoph Schumacher for noting this typo.]

22 Page 174, [I, Exercise 5.8]: The exercise should read $M_n \xrightarrow{\mathbb{P}} \infty$ when $a < \frac{1}{2}$, while $M_n \xrightarrow{\mathbb{P}} 0$ when $a > \frac{1}{2}$. [Thanks to Bas Kleijn for noting this typo.]

23 Page 174, [I, Exercise 5.9]: The norming is not quite correct. It should read $\lambda = \frac{1}{2}\log n + \frac{1}{2}\log\log n + t$. [Thanks to Bas Kleijn for noting this typo.]

24 Page 175, [I, Exercises 5.11 and 5.12]: It should be $\lambda = \log n + 2\log\log n + t$, rather than $\lambda = 2\log n + t$.

25 Page 175, [I, Exercises 5.16 and 5.15]: [I, (5.6.6)] should read

$$D_{\max}/\log n \xrightarrow{\mathbb{P}_\lambda} 0. \qquad (5.6.6)$$

Similarly, [I, (5.6.7)] should read

$$D_{\max}/a_n \xrightarrow{\mathbb{P}_\lambda} 0. \qquad (5.6.7)$$

[Thanks to Christoph Schumacher for noting this typo.]

26 Page 180, after (I.1): 'for each integer $k \leq n$'. The counterexample is $n = 1$, $d_1 = d_2 = 2$. [Thanks to Christoph Schumacher for noting this typo.]

27 Page 196: The first inequality in [I, (6.4.13)] is in fact an equality. [Thanks to Christoph Schumacher for noting this typo.]

28 Page 211, between [I, (6.4.17)] and [I, (6.4.18)]: This should read "If $X_{ij} = 1$, then $\hat{X}_{ij} = 0$ and $Y_{j;i}+\hat{X}_{ij} = k-1$, while, if $X_{ij} = 0$, then $\hat{X}_{ij} = 1$ and $Y_{i;j} + X_{ij} = k$. Therefore, by the independence of the events $\{\hat{X}_{ij} = 1\}$ and $\{Y_{i;j} + X_{ij} = k-1\}$, as well as the independence of the events $\{X_{ij} = 1\}$ and $\{Y_{j;i} + \hat{X}_{ij} = k\}$,..." As a result, [I, (6.4.18)] should be replaced by

$$\mathbb{P}(D_i = D_j = k) - \mathbb{P}(D_i = k)\mathbb{P}(D_j = k) \leq p_{ij}[\mathbb{P}(D_i = k) + \mathbb{P}(D_j = k-1)] \qquad (6.4.18)$$

and [I, (6.4.19)] by

$$\sum_{k\geq 0} \mathrm{Var}(P_k^{(n)}) \leq \frac{1}{n} + \frac{2}{n^2} \sum_{i,j\in[n]} p_{ij} \to 0, \qquad (6.4.19)$$

This causes no necessary additional changes. [Thanks to Marta Milewska and Christoph Schumacher for noting this typo.]

29 Page 199, below [I, (6.6.7)], it should read "in which case we return to (6.2.1) since, for $i \neq j$,..." [Thanks to Christoph Schumacher for noting this omission.]

30 Page 206, above [I, (6.7.21)]: "then we can find a sequence $\varepsilon_n \searrow$ such that..." [Thanks to Christoph Schumacher for this addition.]

31 Page 206, line -3: The statement that $c = c(\varepsilon) \geq 0$ is not very helpful. It should read "$c > 0$". A more precise version is "$c = \frac{1}{16}$", as shown in the following argument, where we assume without loss of generality that $p \leq q$:

$$\begin{aligned}
\rho(p,q) &= \big(\sqrt{p} - \sqrt{q}\big)^2 + \big(\sqrt{1-p} - \sqrt{1-q}\big)^2 \\
&= \Big(\frac{p-q}{\sqrt{p}+\sqrt{q}}\Big)^2 + \Big(\frac{(1-p)-(1-q)}{\sqrt{1-p}+\sqrt{1-q}}\Big)^2 \\
&= (p-q)^2 \frac{(\sqrt{1-p}+\sqrt{1-q})^2 + (\sqrt{p}+\sqrt{q})^2}{(\sqrt{p}+\sqrt{q})^2(\sqrt{1-p}+\sqrt{1-q})^2} \\
&\overset{0\leq q\leq p\leq 1}{\geq} (p-q)^2 \frac{(\sqrt{1-p}+\sqrt{0})^2 + (\sqrt{p}+\sqrt{0})^2}{(\sqrt{p}+\sqrt{p})^2(\sqrt{1}+\sqrt{1})^2} \\
&= \frac{(p-q)^2}{16p}.
\end{aligned} \qquad (1.1.1)$$

[Thanks to Christoph Schumacher for this comment, and the nice argument in (1.1.1).]

32 Page 203, [I, Theorem 6.18]: The above has some repercussions in [I, Theorem 6.18]. In [I, Theorem 6.18], we additionally assume

$$\limsup_{n\to\infty} \max_{1\leq i<j\leq n} q_{ij} \leq 1 - \varepsilon. \qquad (1.1.2)$$

Then we conclude the asymptotic equivalence of $\mathrm{IRG}_n(\boldsymbol{p})$ and $\mathrm{IRG}_n(\boldsymbol{q})$ if and only if [I, (6.7.2)] holds.

$$\lim_{n\to\infty} \sum_{1\leq i<j\leq n} \frac{(p_{ij}-q_{ij})^2}{p_{ij}\vee q_{ij}} = 0. \tag{6.7.2}$$

The modified condition (6.7.2) is indeed equivalent to the asymptotic equivalence of the graphs $\mathrm{IRG}_n(\boldsymbol{p})$ and $\mathrm{IRG}_n(\boldsymbol{q})$, since

$$\begin{aligned} \sum_{1\leq i<j\leq n} \frac{(p_{ij}-q_{ij})^2}{p_{ij}\vee q_{ij}} &\overset{(1.1.1)}{\leq} \frac{1}{c}\sum_{1\leq i<j\leq n} \rho(p_{ij},q_{ij}) \\ &\overset{[\text{I, }(6.7.19)]}{\leq} -\frac{2}{c}\log H(\mathbb{P}_n,\mathbb{Q}_n) \\ &\overset{[\text{I, }(6.7.16)]}{=} -\frac{2}{c}\log\big(1-(d_{\mathrm{H}}(\mathbb{P}_n,\mathbb{Q}_n))^2\big) \\ &\overset{[\text{I, }(6.7.12)]}{\leq} -\frac{2}{c}\log\big(1-d_{\mathrm{TV}}(\mathbb{P}_n,\mathbb{Q}_n)\big) \to 0, \end{aligned} \tag{1.1.3}$$

when $d_{\mathrm{TV}}(\mathbb{P}_n,\mathbb{Q}_n)\to 0$. [Thanks to Christoph Schumacher for this comment, and the nice argument in (1.1.3).]

33 Page 203, [I, Theorem 6.19]: In [I, Theorem 6.19], replace [I, (6.8.4)] by

$$\sum_{i\in[n]} w_i^3 = o(\ell_n^{3/2}). \tag{6.8.4}$$

Indeed, [I, Theorem 6.19] was written with [I, Condition 6.4] in mind. [I, Condition 6.4] implies that $\ell_n = \Theta(n)$. However, since [I, Theorem 6.19] does not assume [I, Condition 6.4], there are counterexamples to the current statement. For example, taking $w_1 = w_2 = 1$ and $w_i = 1/(n-2)$ for $i \in [n]\setminus\{1,2\}$ produces an obvious counterexample, since $p_{ij}^{(\mathrm{CL})} = \frac{1}{3}$ and $p_{ij} = p_{ij}^{(\mathrm{GRG})} = \frac{1}{4}$. [Thanks to Christoph Schumacher for this correction, and the counterexample.]

34 Page 203, [I, (6.8.8)]: Replace this chain of inequalities by

$$\ell_n^{-3/2}\sum_{i\in[n]} w_i^3 \leq \ell_n^{-3/2}\max_{i\in[n]} w_i \sum_{i\in[n]} w_i^2 = \frac{\max_{i\in[n]} w_i}{\sqrt{n}} \frac{\mathbb{E}[W_n^2]}{(\mathbb{E}[W_n])^{3/2}} \to 0. \tag{6.8.8}$$

[Thanks to Christoph Schumacher for this correction.]

35 Page 208, above [I, (6.8.10)]: " that there is at least one edge between vertices i and j is, conditionally on the weights,..." [Thanks to Christoph Schumacher for this correction.]

36 [I, Exercise 6.18] can be considered to be a special case of [I, Exercise 6.3] with random weights. It is arguably preferable to restrict [I, Exercise 6.3] to deterministic weights. [Thanks to Christoph Schumacher for this observation.]

37 Page 217, two lines below [I, Example 7.4]: "(recall (I.1) on page 180)" [Thanks to Christoph Schumacher for this correction.]

38 Page 222, [I, Definition 7.5]: [I, (7.2.3)] in [I, Definition 7.5] should hold for all $m \in [\ell_n/2]$.

39 Page 230, two lines below [I, (7.3.21)]: "Note that when $\ell_n \leq 3$, there cannot be any multiple edges, so from now on, we assume that $\ell_n \geq 4$." [Thanks to Christoph Schumacher for this correction.]

40 Page 230, [I, Proposition 7.13]: "Then $(S_n, \widetilde{M}_n)$ converges in distribution to (S, M)." This is in fact what we prove. [Thanks to Christoph Schumacher for this correction.]

41 Page 230, [I, below Proposition 7.13]: "Indeed, Theorem 7.12 is a simple consequence of Proposition 7.13, since $\mathrm{CM}_n(\boldsymbol{d})$ is simple precisely when $S_n = \widetilde{M}_n = 0$. By the weak convergence result stated in Proposition 7.13 and the independence of S and M, the probability that $S_n = \widetilde{M}_n = 0$ converges to $\mathrm{e}^{-\mu_S-\mu_M}$, where μ_S and μ_M are the means of the limiting Poisson random variables S and M." [Thanks to Christoph Schumacher for this correction.]

42 Page 233, proof of [I, Theorem 7.12 and Proposition 7.13]: I am grateful to Clara Stegehuis for pointing out a problem in the proof of [I, Proposition 7.13]. On page 233, line -3, it is written that 'the upper bound in [I, (7.4.9)] always holds.' This is incorrect. Indeed, fix a pair of vertices i and j for which two edges must be present, say s_1t_1 and s_2t_2 are paired. That actually makes it *more* likely that also s_1t_1 and s_3t_3 are being paired. These issues arise exactly when there are several $m_i^{(2)}$ that involve the *same* pair of vertices, as then there are less half-edges that need to be paired. Here we complete the proof given, by an induction on the number of pairs of vertices that have several edges between them.

We refer to the notation in the [I, proof of Proposition 7.13]. Recall that

$$\mathcal{I}_1 = \mathcal{I}_1(\boldsymbol{d}) = \{(st, i)\colon i \in [n], 1 \leq s < t \leq d_i\}, \tag{1.1.4}$$

$$\mathcal{I}_2 = \mathcal{I}_2(\boldsymbol{d}) = \{(s_1t_1, s_2t_2, i, j)\colon 1 \leq i < j \leq n, 1 \leq s_1 < s_2 \leq d_i, 1 \leq t_1 \neq t_2 \leq d_j\}, \tag{1.1.5}$$

where we make the dependence on the degree sequence $\boldsymbol{d}$ explicit. Recall the notation $\sum^*$ for the sum over distinct indices introduced in [I, Theorem 2.7]. By [I, Theorem 2.7],

$$\mathbb{E}[(S_n)_s(\widetilde{M}_n)_r] = \sum^*_{\substack{m_1^{(1)},\ldots,m_s^{(1)}\in\mathcal{I}_1\\ m_1^{(2)},\ldots,m_r^{(2)}\in\mathcal{I}_2}} \mathbb{P}\big(I_{m_1^{(1)}}^{(1)} = \cdots = I_{m_s^{(1)}}^{(1)} = I_{m_1^{(2)}}^{(2)} = \cdots = I_{m_r^{(2)}}^{(2)} = 1\big). \tag{1.1.6}$$

Denote $\vec{m}^{(2)} = (m_1^{(2)}, \ldots, m_r^{(2)})$. For $i, j \in [n]$ with $i \neq j$ and $m^{(2)} \in \mathcal{I}_2$, we write that $(i, j) \in \mathcal{I}_2$ when there exist s_1t_1 and s_2t_2 such that $m^{(2)} = (s_1t_1, s_2t_2, i, j)$. We let

$$p(\vec{m}^{(2)}) = \#\{(i, j)\colon \exists r_1, r_2 \in [r] \text{ such that } (i, j) \in m_{r_1}^{(2)}, (i, j) \in m_{r_2}^{(2)}\}. \tag{1.1.7}$$

We perform induction with respect to $p(\vec{m}^{(2)})$. The induction hypothesis is that there exists $\varepsilon_n(\boldsymbol{d}) = o(1)$ such that

$$\begin{aligned} Q_n^{(s,r)}(l, \boldsymbol{d}) &\equiv \sum^*_{\substack{m_1^{(1)},\ldots,m_s^{(1)}\in\mathcal{I}_1\\ m_1^{(2)},\ldots,m_r^{(2)}\in\mathcal{I}_2}} \mathbb{P}\big(I_{m_1^{(1)}}^{(1)} = \cdots = I_{m_s^{(1)}}^{(1)} = I_{m_1^{(2)}}^{(2)} = \cdots = I_{m_r^{(2)}}^{(2)} = 1\big) \\ &\leq \varepsilon_n(\boldsymbol{d})^l \Big(\frac{|\mathcal{I}_1(\boldsymbol{d})|}{\ell_n - 2s - 4r + 1}\Big)^s \Big(\frac{|\mathcal{I}_2(\boldsymbol{d})|}{(\ell_n - 2s - 4r + 1)^2}\Big)^r, \end{aligned} \tag{1.1.8}$$

when the sum is restricted to those $\vec{m}^{(2)}$ for which $p(\vec{m}^{(2)}) = l$.

The claim in (1.1.8) completes the argument in the proof of the upper bound in [I, proof of Theorem 2.7], where an upper bound on the case where $l = 0$ was proved in [I, (7.4.11)]. The claim in (1.1.8) shows that the contributions for $l \geq 1$ vanish, which, in turn, completes the proof.

The induction hypothesis is initialised for $l = 0$ by [I, (7.4.11)]. To advance the induction hypothesis, we assume that $l \geq 1$, and pick the smallest pair (i, j) in the lexicographic order for which there exist $r_1, r_2 \in [r]$ such that $(i, j) \in m_{r_1}^{(2)}, (i, j) \in m_{r_2}^{(2)}$. Let $e \geq 3$ be the number of edges involved in the pairing needed for $I_{m_{r_1}^{(2)}}^{(2)} = 1$ to occur for all r_1 for which $(i, j) \in m_{r_1}^{(2)}$ and let $q = \#\{r_1 \in [r]\colon (i, j) \in m_{r_1}^{(2)}\}$. Then

$$Q_n(l, \boldsymbol{d}) \leq \sum_{e\geq 3}\sum_{q=2}^{r}\sum_{i,j\in[n]} \frac{(d_i)_e(d_j)_e}{(\ell_n - 2s - 4r + 1)^{2e}} Q_n^{(s,r-q)}(l - 1, \boldsymbol{d}'), \tag{1.1.9}$$

where $d'_v = d_v$ for all $v \notin \{i, j\}$ and $d'_v = d_v - e$ for $v \in \{i, j\}$. Using that $|\mathcal{I}_1(\boldsymbol{d}')| \leq |\mathcal{I}_1(\boldsymbol{d})|, |\mathcal{I}_2(\boldsymbol{d}')| \leq |\mathcal{I}_2(\boldsymbol{d})|$, and by the induction hypothesis,

$$Q_n^{(s,r-q)}(l - 1, \boldsymbol{d}') \leq \varepsilon_n(\boldsymbol{d})^l \Big(\frac{|\mathcal{I}_1(\boldsymbol{d})|}{\ell_n - 2s - 4r + 1}\Big)^s \Big(\frac{|\mathcal{I}_2(\boldsymbol{d})|}{(\ell_n - 2s - 4r + 1)^2}\Big)^{r-q}. \tag{1.1.10}$$

Let

$$b_n = \min\Big\{1, \frac{|\mathcal{I}_2(\boldsymbol{d})|}{(\ell_n - 2s - 4r + 1)^2}\Big\}^{-1}, \tag{1.1.11}$$

and

$$\varepsilon_n(\boldsymbol{d}) = \sum_{e\geq 3}\sum_{q=2}^{r} b_n^q \sum_{e\geq 3}\Big(\sum_{i\in[n]} \frac{(d_i)_e}{(\ell_n-2s-4r+1)^e}\Big)^2, \tag{1.1.12}$$

then indeed

$$Q_n(l,\boldsymbol{d}) \leq \varepsilon_n(\boldsymbol{d})^l\Big(\frac{|\mathcal{I}_1(\boldsymbol{d})|}{\ell_n-2s-4r+1}\Big)^s\Big(\frac{|\mathcal{I}_2(\boldsymbol{d})|}{(\ell_n-2s-4r+1)^2}\Big)^r. \tag{1.1.13}$$

This advances the induction hypothesis, and we are left to prove that $\varepsilon_n(\boldsymbol{d}) = o(1)$ under [I, Condition 7.8(a)-(c)]. This follows since, under these conditions, $d_{\max} = o(\sqrt{n})$ and $b_n = O(1)$ since we assumed that $\nu > 0$, so that

$$\varepsilon_n(\boldsymbol{d}) \leq \sum_{q=2}^{r} b_n^q \sum_{e\geq 3}\Big(\sum_{i\in[n]} \frac{(d_i)_2}{(\ell_n-2s-4r+1)^2}\Big)^2\Big(\frac{d_{\max}^2}{\ell_n-2s-4r+1}\Big)^{e-2} \leq C_r\frac{d_{\max}^2}{n}, \tag{1.1.14}$$

for some $C_r > 0$. □

43 Page 238, [I, Theorem 7.18]: "Let $\mathcal{E}_n$ be a subset of multi-graphs for which $\mathbb{P}(\mathrm{CM}_n(\boldsymbol{d}) \in \mathcal{E}_n) \xrightarrow{\mathbb{P}} 1$ when $\boldsymbol{d}$ satisfies Conditions 7.8(a)-(c) in probability." [Thanks to Christoph Schumacher and Louigi Addario-Berry for this correction.]

44 Page 238, before [I, (7.5.7)]: "By (7.5.6), for every set $\mathcal{E}_n$,..." [Thanks to Christoph Schumacher for this correction.]

45 Page 239, line -4: "Further, since Condition 6.4(c) holds, Exercise 6.3 implies that..." [Thanks to Christoph Schumacher for this correction.]

46 Page 239, line -4: "so that $p_{ij}^{(\mathrm{CL})} = (w_iw_j/\ell_n) \wedge 1 = w_iw_j/\ell_n$..." [Thanks to Christoph Schumacher for this correction.]

47 Page 239, [I, (7.5.12)]: While strictly speaking not being wrong, it is more illuminating to rewrite [I, (7.5.12)]: as

$$\sum_{i\in[n]} d_i(d_i-1) = \sum_{i,j,k\in[n]:\, j\neq k} I_{ij}I_{ik} = 2\sum_{i,j,k\in[n]:\, j<k} I_{ij}I_{ik}, \tag{7.5.12}$$

[Thanks to Christoph Schumacher for this correction.]

48 Page 239, [I, (7.5.13)]: Replace the first line of this equation as

$$\mathbb{E}\Big[\frac{1}{n}\sum_{i\in[n]} d_i(d_i-1)\Big] = \frac{2}{n}\sum_{i,j,k\in[n]:\, j<k,j\neq i\neq k} \frac{w_iw_j}{\ell_n}\frac{w_iw_k}{\ell_n}. \tag{7.5.13}$$

The third equality is stated without argument. It can be obtained by inclusion-exclusion, by first adding and subtracting the $j = i$ contribution, followed by the $k \in \{i,j\}$ contributions. The subtracted contributions give rise to the subtracted sums. [Thanks to Christoph Schumacher for this correction.]

49 Page 240, [I, (7.5.15)]: The last line should be replaced by

$$+\frac{8}{n^2}\sum_{i,j,k,l\in[n]} p_{ij}p_{ik}p_{il} + \frac{8}{n^2}\sum_{i,j,k,l\in[n]} p_{ij}p_{jk}p_{kl}. \tag{7.5.15}$$

[Thanks to Christoph Schumacher for this correction.]

50 Page 241, below [I, (7.5.16)]: "By Condition 6.4(c), $\mathbb{E}[W_n^2]$ converges, and also $\max_{i\in[n]} w_i = o(\sqrt{n})$." [Thanks to Christoph Schumacher for this correction.]

51 Page 247, line 5: "The general set-up assuming Conditions 6.4(a)-(c), together with the degree asympotics stated in Theorem 7.19, allow us to easily extend Theorem 7.25 to $\mathrm{GRG}_n(\boldsymbol{w})$:" [Thanks to Christoph Schumacher for this correction.]

52 Page 247, [I, Corollary 7.26]: In [I, Corollary 7.26], it should be assumed that the weights $\boldsymbol{w}$ satisfy [I, Conditions 6.4(a)-(c)], since otherwise [I, Theorem 7.19] cannot be invoked. [Thanks to Christoph Schumacher for this correction.]

53 Page 247, (7.7.11) should be replaced by

$$\mathbb{P}(D^\star = k) = \mathbb{P}\big(\mathsf{Poi}(W^\star) = k-1\big). \tag{7.7.11}$$

[Thanks to Christoph Schumacher for this correction.]

54 Page 247, proof of [I, Corollary 7.26]: "By Theorem 7.19, the degrees $\boldsymbol{d} = (d_i)_{i\in[n]}$ satisfy Conditions 7.8(a)-(c) with $D \sim \mathsf{Poi}(W)$." Further, it is useful to observe that $D^\star \sim \mathsf{Poi}(W^\star) + 1$ when $D \sim \mathsf{Poi}(W)$. [Thanks to Christoph Schumacher for this correction.]

55 Page 252, [I, Exercise 7.7:] It is instructive to add here "and that Condition 7.8(c) holds whenever $\mathbb{E}[D^2] < \infty$." [Thanks to Christoph Schumacher for this suggestion.]

56 Page 255, [I, Exercise 7.25:] Here, one should assume that $\tau \in (2, \frac{5}{2})$. Indeed, when $\tau > 2$, the number of multi-edges between vertices i and j with $i, j \in [3]$ is of the order $n^{2/(\tau-1)-1}$, so that the number of triangles equals $n^{6/(\tau-1)-3}$, where

$$6/(\tau - 1) - 3 > 1$$

precisely when $\tau < \frac{5}{2}$.

57 Page 260, line -2: "We start with $\mathrm{PA}_1^{(m,\delta)}$ consisting of a single vertex with m self-loops." [Thanks to Christoph Schumacher for this correction.]

58 Page 262, below [I, (8.2.2)]: The text here is a little confusing, since there will almost surely be infinitel;y many self-loops, even though the probability of adding one *at any given time* is small. It is better to write "... since the probability to add a self-loop in $\mathrm{PA}_t^{(m,\delta)}$ is quite small when t is large." [Thanks to Christoph Schumacher for this suggestion.]

59 Page 263, [I, (8.3.10)] should be replaced by

$$\begin{aligned}\frac{D_i(t)+\delta}{t^{1/(2+\delta)}} &= M_i(t)\frac{(1+\delta)\Gamma(i-1/(2+\delta))}{\Gamma(i)}(1+O(1/t)) \qquad (8.3.10)\\ &\xrightarrow{a.s.} M_i\frac{(1+\delta)\Gamma(i-1/(2+\delta))}{\Gamma(i)} \equiv \xi_i.\end{aligned}$$

[Thanks to Christoph Schumacher for this correction.]

60 Page 265, [I, (8.4.9)]. This formula should read

$$\mathbb{P}(X = k) = \frac{\Gamma(k+r)}{k!\Gamma(r)} p^r (1-p)^k. \tag{8.4.9}$$

61 Page 265, [I, (8.4.10)]. This formula should read

$$p_k = \mathbb{E}[\mathbb{P}(X = k - m)]. \tag{8.4.10}$$

62 Page 298-299, [I, Exercise 8.16]: The above two changes also have consequences for [I, Exercise 8.16], where also $p_k = \mathbb{E}[\mathbb{P}(X = k)]$ should be replaced with $p_k = \mathbb{E}[\mathbb{P}(X = k - m)]$.

63 Page 273, [I, (8.6.18)]. This formula should read

$$\sup_k \sup_t |\varepsilon_k(t)| \le C. \tag{8.6.18}$$

[Thanks to Christoph Schumacher for this correction.]

64 Page 274, below [I, (8.6.22)]. “When $t = 1$, we have that $\mathrm{PA}_1^{(1,\delta)}$ consists of a vertex with a single self-loop.” [Thanks to Christoph Schumacher for this correction.]

65 Page 276, two lines above [I, (8.6.40)]. “Note that then $k \geq \lceil t(2+\delta) \rceil + 2 \geq t + 2$, since $\delta > -1$. Since the maximal degree of $\mathrm{PA}_t^{(1,\delta)}$ is $t + 2$ (which happens precisely when all edges are connected to the initial vertex), we have that $\bar{N}_k(t+1) = 0$ for $k \geq t + 4$.” As a result, the argument for $k = t + 2$ and $k = t + 3$ is missing. For these cases, we can use that $\bar{N}_k(t+1)$ is exponentially small, since at least $t - 2$ of the edges should be connected to vertex 1, and the probability to connect to vertex 1 is at most $\frac{3}{4}$. Aside from this, the argument can be followed. This also has some minor consequences in the $m \geq 2$ setting around [I, (8.6.91)]. [Thanks to Christoph Schumacher for this correction.]

66 Page 276, [I, (8.6.44)] should be replaced by

$$(T_{t+1}Q)_k = \Big(1 - \frac{k+\delta}{t(2+\delta') + (1+\delta')}\Big)Q_k + \frac{k-1+\delta}{t(2+\delta') + (1+\delta')}Q_{k-1}. \tag{8.6.44}$$

67 Page 279, below [I, (8.6.64)] it is claimed that T_u is a contraction. However, that is not quite true (and also not what is used in the proof). Indeed, for $\delta = 0$ and for $m = 1$,

$$\begin{aligned}(T_{t+1}p)_k &\overset{\text{[I, (8.6.44)]}}{=} \Big(1 - \frac{k}{2t+1}\Big)p_k + \frac{k-1}{2t+1}p_{k-1} \\ &\overset{\text{[I, (8.6.56)]}}{=} \Big(1 - \frac{k}{2t+1} + \frac{k-1}{2t+1}\frac{k+2}{k-1}\Big)p_k \\ &= \Big(1 + \frac{2}{2t+1}\Big)p_k \\ &> p_k.\end{aligned} \tag{1.1.15}$$

The precise statements that are needed in the proof are [I, (8.6.66)], as well as the precise contraction property in [I, Lemma 8.13]. [I, (8.6.66)] follows from a double application of [I, (8.6.63)], for which we need that $\sup_k k^2|Q_k| \leq K$, which is true for . The proof for $(p_k)_{k\geq 1}$. The proof of [I, Lemma 8.13] is self-contained. [Thanks to Christoph Schumacher for this observation.]

68 Page 281, [I, (8.6.87)] should read

$$\begin{aligned}\|T_{t+1}^{(m)}Q\|_\infty \leq \|T_{mt+1}Q\|_\infty &\leq \Big(1 - \frac{1}{mt(2+\delta') + (1+\delta')}\Big)\|Q\|_\infty \\ &\leq \Big(1 - \frac{1}{t(2m+\delta) + (m+\delta)}\Big)\|Q\|_\infty.\end{aligned} \tag{8.6.87}$$

[Thanks to Christoph Schumacher for this correction.]

69 Page 292, [I, (8.9.9)] should be replaced by

$$p_k = (2+\delta)\frac{\Gamma(k+\delta)}{\Gamma(1+\delta)}\frac{\Gamma(2+2\delta)}{\Gamma(k+3+2\delta)}, \tag{8.9.9}$$

see also [2, Section 4.2].

70 Page 298, [I, Exercise 8.14]: [I, (8.11.3)] should read

$$\mathbb{E}[D_i(t) + \delta] = (1 + \mathbb{1}_{\{i\in\{1,2\}\}} + \delta)\frac{\Gamma(t + 1/(2+\delta))\Gamma(i)}{\Gamma(t)\Gamma(i + \mathbb{1}_{\{i=1\}} + 1/(2+\delta))}. \tag{8.11.3}$$

[Thanks to Christoph Schumacher for this correction.]

71 Page 302, [I, Definition A.3]: [I, (A.1.3)] should read

$$\limsup_{K\to\infty}\limsup_{n\to\infty}\mathbb{E}[|X_n|\mathbb{1}_{\{|X_n|>K\}}] = 0. \tag{A.1.3}$$

[Thanks to Christoph Schumacher for this correction.]

72 Page 302, [I, Definition A.3]: In order to apply [I, Theorem A.1] after [I, (A.1.5)], one has to invoke Skorohod's representation theorem first to get a coupling $((\hat{X}_n)_n, \hat{X})$ with $\hat{X}_n \xrightarrow{a.s.} \hat{X}$. This representation can also be obtained using the coupling in the proof of [I, Theorem A.4]. [Thanks to Christoph Schumacher for this correction.]

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[1] R. van der Hofstad. *Random graphs and complex networks. Volume 1*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, 2017.

[2] A. Rudas, B. Tóth, and B. Valkó. Random trees and general branching processes. *Random Structures Algorithms*, **31**(2):186–202, 2007.