Proposed content course on
"Local convergence, giants and small-world properties in inhomogeneous random graphs"

(based on Random Graphs and Complex Networks Volume 2)

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In this document, we propose a course content for a course on connectivity properties in inhomogeneous random graphs, based on Random Graphs and Complex Networks Volume 2. We focus on the local limit, the giant component and small- and ultra-small-world properties in inhomogeneous random graphs.

(1) Start with a recap of the generalised random graphs, by discussing some of their properties in [II, Chapter 1]. Particularly [II, Section 1.3.2] is then convenient.

(2) Discuss the theory of local convergence in [II, Chapter 2]. Cover the basic notation of rooted graphs and their metric properties in [II, Section 2.2], the notion of local convergence of deterministic graphs in [II, Section 2.3], and that of random graphs in [II, Section 2.4].

(3) Continue with consequences of local convergence in [II, Section 2.5]. Here, you can pick those consequences that you like best.

(4) Discuss the relation between the size of the giant and the local limit in [II, Section 2.6]. This is useful as intuition, and the proof is relatively straightforward.

(5) Discuss general inhomogeneous random graphs in [II, Section 3.2], and their degree structure in [II, Section 3.3].

(6) Discuss multi-type branching processes in [II, Section 3.4], as these will be essential in describing the local limit of inhomogeneous random graphs.

(7) Discuss local convergence of inhomogeneous random graphs in [II, Section 3.5].

(8) Treat the size of the giant in inhomogeneous random graphs in [II, Section 4.3]. The main result, [II, Theorem 3.19], is not fully proved here. A part of the proof is deferred to [II, Chapter 6], see [II, Section 6.5.3].

(9) Close the course with the small-world properties in inhomogeneous random graphs in [II, Chapter 6]. [II, Section 6.2] gives an outline of the results, which are then proved in [II, Sections 6.3–6.5]. The lower bounds on distances are the simplest, and can be found in [II, Section 6.3]. The doubly logarithmic upper bounds on distances appear in [II, Section 6.4], and are obtained by investigating how quickly a vertex connects to a giant-weight vertex. The logarithmic upper bounds are performed in [II, Section 6.5], and rely on a second-moment bound for the number of paths connecting vertices in [II, Section 6.5.1]. This argument is technically somewhat challenging, as it involves an intricate combinatorial analysis of the probability that two paths exist between a pair of vertices.

(10) Time permitting, you could discuss extensions to the diameter of inhomogeneous random graphs and other distance results in [II, Section 6.6], directed inhomogeneous random graphs in [II, Section 9.2.1] and/or spatial inhomogeneous random graphs in [II, Section 9.5.3].