Final Report

Title: A model for bone-implant attachment integrity assessment based on vibration techniques.

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Table of contents

1. Introduction.
2. The problem.
4. FEM simulations.
5. Conclusions.
Description of the research training project to be undertaken by the fellow

An early diagnosis of progressing deterioration of the artificial joint implant is of the utmost importance for the timely application of protective measures. In most cases it can not be done by conventional X-ray imaging due to the shielding effects in complicated shapes of implants. Such diagnostics can be based, however, on the systematic monitoring of the state of the replaced joints by means of a modal analysis of the vibrational characteristics of the bone-implant structure.

A simulation study can be conducted for a FE – discretized model of the implant, bone and surroundings in addition. Modeling of the deterioration factors can be based on the available experimental data on the evolution on the mineral content and density of the bones in the result of ageing and osteoporotic degradation, leading to dropping elastic modulae and diminishing energy absorption at damage growth. Sensitivity analysis can be applied in order to establish the ranges of frequency shifts due to the above-mentioned single phenomena of the artificial joint’s deterioration.

A promising tool in that kind investigation seems to be FEM packages augmented with MEMS (Micro-Electric-Mechanic-System), which allow simulating the application of sensors in the implant-stem.
1. Introduction.

Artificial bone replacements used for implants have become widely applied elements of treatments in orthopedic surgery for recent years. Destructive diseases or accidents call for prosthesis, which in many fields have achieved a certain degree of perfection, yielding pain-free functionality and longevity. In the case of joint, e.g. hip or knee replacements can reach 92% probability of good services for more than 10 years, [1]. However, gradual loosening of the implant-bone attachment integrity due to wear, bone regress (related to aging or diseases) and micromechanical damage lead eventually to the failure of the replacement and thus to painful consequences: a repetition of the implanting surgery takes place under unfavorable conditions and total costs of the healing are multiplied.

An early diagnostics of the progressing deterioration of the artificial implants functionality is, therefore, of utmost importance to allow for timely application of protective measures at an early stage of the regressing capabilities of the prosthesis. The existing methods of quality monitoring of the implant are based either on the X-ray imaging or ultrasonic inspection. Therefore they are impaired by shielding effects when complicated shapes of the prosthesis are needed.

MEMS (Micro-Electro-Mechano-Systems) due to their small sizes of a few micrometers create new possibilities of signal measurements and processing. Their application for vibrational signal processing more sophisticated approach to this problem can be archived.

Modal analysis based estimation of the state of fixation between bone and implant is an example of quality based assessment while traditional, widely used so fare X-ray and ultrasonic scans are quantity based estimates.

Due to lack of compatibility between computational environments like Abaqus and Comsol geometrical models created in one of them had to be reduced. Applied geometrical reduction described in subsequent sections made it possible to perform an analysis. From the other hand the MEMS approach was replaced by displacements signal processing either because of model validity checking or the early stage of the project.

2. The problem.

An alternative method of diagnostics is considered, based on the monitoring the integrity of the implant by checking the changes of its vibrational characteristic. Unlike the ultrasonic inspection, which works on the principle of pulse dispersion and reflection for waves of certain frequency chosen for the tested tissue, the proposed method would rely on the changes of the frequency spectrum caused by the changes in the mechanical properties due to the deteriorating state of the implant. A feasibility study for the new method begins with the modeling of structural and mechanical properties of typical cases of artificial bone implants. This constitutes a starting point for the FE-based numerical modal simulation. Great importance is attached here to the analysis of the most critical determinants of the longevity of an implant as known in the orthopedic surgery practice. To these belong the following main deterioration phenomena: bone necrosis of the surrounding tissue, microcracking developing from initial damage left by surgery scars.
and errors, debonding mechanism at the implant / bone interface or adverse bone adaptation in result of redistribution of loads after implanting surgery. The knowledge of the particularly damaged sites and their modes of failure allows to concentrate the attention on those vibrational modal characteristic, which are mainly concerned in result of the deterioration of the implant, leaving aside the remaining bulk of the spectrum.

Mechanical and structural modeling of the replacement in its well-fixed state shortly after the healing of the implant has taken place. In constitutes in the benchmark for the later comparisons for the deteriorating implant. The frequency spectrum analysis is based on actual vibrational signal which is emitted by periodically excited femur and measured by MEMS sensors located around the implant.

Decisions regarding the choice of discretization density, the type of FE elements, boundary conditions as well as material parameters are taken from (sometimes conflicting) available sources. Furthermore, sensitivity analysis is applied to the results for shifting characteristics against the extent of the integrity impairment.

3. The modal analysis based model – an analytical approach.

In this section the procedure of modal superposition will be presented, (see also [2]).

In modal analysis the following second order differential equation of motion is a considered which is valid in static sense for each time step $t$:

$$ M\ddot{U} + C\dot{U} + KU = R(t) $$  \hspace{1cm} (1)

where:

- $M$ - mass matrix,
- $C$ - viscous damping matrix,
- $K$ - stiffness matrix,
- $R(t)$ - external force vector,
- $U$ - displacement vector,
- $\dot{U}$ - velocity vector,
- $\ddot{U}$ - acceleration vector.

The matrices $M, C, K$ have a bandwidth which is determined by number of the nodes of the finite elements in the system. Therefore the topology of the finite element mesh determines the order and bandwidth of the system matrices. Assume that:

$$ U(t) = PX(t) $$  \hspace{1cm} (2)

where:
\( X(t) \) - time dependent vector of order \( n \) with its components referred to as generalize

\[
X(t) = \begin{bmatrix}
x_1(t) \\
x_2(t) \\
\vdots \\
x_n(t)
\end{bmatrix},
\]

\( P \) - a time-independent transformation matrix of size \( n \times n \).

By substituting (2) into (1) and multiplying by \( P^T \) we get:

\[
P^T M P \ddot{X} + P^T C P X + P^T K P X = P^T \mathbf{R}(t)
\]

which can be rewritten:

\[
\ddot{M}X + \ddot{C}X + \ddot{K}X = \ddot{R}(t)
\]

\( P^T M P = \tilde{M} \)
\( P^T C P = \tilde{C} \)
\( P^T K P = \tilde{K} \)
\( P^T R(t) = \tilde{R} \)

In practice one uses \( P \) matrix obtained using the displacement solution of the free-vibration equilibrium equation with the damping neglected:

\[
M \ddot{U} + KU = 0.
\]

Let \( \phi \) be a time independent vector of length \( n \). The solution for one degree of freedom is of the form:

\[
\begin{align*}
U &= \phi \sin \omega(t - t_0) \\
\dot{U} &= \phi \omega \cos \omega(t - t_0) \\
\ddot{U} &= \phi \omega^2 \sin \omega(t - t_0)
\end{align*}
\]

with one and the same oscillating frequency for all nodes:

\[
\begin{align*}
U &= \phi \sin \omega(t - t_0) \\
\dot{U} &= \phi \omega \cos \omega(t - t_0) \\
\ddot{U} &= -\phi \omega^2 \sin \omega(t - t_0)
\end{align*}
\]

After substituting (6)-(8) do (5) we get a generalized eigenvalue problem:

\[
(K - \omega^2 M)\phi = 0.
\]

This transformed to a standard eigenvalue problem in several manners, provided that \( M^{-1} \) exists:

For example:

\[
(K - \omega^2 M)\phi = 0 \Rightarrow (M^{-1} K - \omega^2 I)\phi = 0
\]

where:

\[KM^{-1} = A \]
\[\omega^2 = \lambda \]

Then it can be rewritten to:

\[
(A - \lambda M)\phi = 0
\]
So after all the transformations we get canonic form of eigenproblem with respect to nodal forces:

\[ \mathbf{A} \mathbf{F} = \lambda \mathbf{F} . \]  

(12)

According to the above equation we can rewrite a canonic equation for eigenvalues

\[ \det(\mathbf{H} - \lambda \mathbf{I}) = 0 . \]  

(13)

Eigenproblem yields n eigensolution pairs: \((\omega_1^2, \varphi_1), (\omega_2^2, \varphi_2), \ldots, (\omega_n^2, \varphi_n)\) whose eigenvectors are \(\mathbf{M}\)-orthonormal:

\[ \varphi_i \mathbf{M} \varphi_j \begin{cases} = 1 & i = j \\ = 0 & i \neq j \end{cases} \text{ and } 0 \leq \omega_1^2 \leq \omega_2^2 \leq \ldots \leq \omega_i^2 \ldots \leq \omega_n^2 \]  

(14)

The matrix of eigenvectors is then:

\[ \Phi = [\varphi_1, \varphi_2, \ldots, \varphi_{n-1}, \varphi_n] \]  

(15)

and the matrix of eigenvalues:

\[
\Omega^2 = \begin{bmatrix}
\omega_1^2 \\
& \ddots \\
& & \omega_i^2 \\
& & & \ddots \\
& & & & \omega_n^2
\end{bmatrix}
\]

Now assume that \(\Phi\) is \(\mathbf{M}\)-orthogonal, i.e. assume that (24) holds. Then

\[ \Phi^T \mathbf{M} \Phi = \mathbf{I} \] whence \(\Phi^T \mathbf{K} \Phi = \Omega^2\). Summarized:

\[ \Phi^T \mathbf{K} \Phi = \Phi^T \mathbf{M} \Phi \Omega^2 \]  

(16)

where:

\[ \Phi^T \mathbf{K} \Phi = \Omega^2 \]  

(17)

\[ \Phi^T \mathbf{M} \Phi = \mathbf{I} \]  

(18)

\[ \Phi^T \mathbf{R}(t) = \mathbf{r}(t) - (\mathbf{r} \text{ as in chapter 4, fig. 2}) \]  

(19)

Now \(\mathbf{P}\) becomes \(\Phi\) and we get:

\[ \mathbf{U}(t) = \Phi \mathbf{X}(t) \]  

(20)

\[ \dot{\mathbf{X}}(t) + \Phi^T \mathbf{C} \Phi \dot{\mathbf{X}}(t) + \Phi^T \mathbf{K} \Phi \mathbf{X}(t) = \Phi^T \mathbf{R}(t) \]  

(21)

with initial conditions on \(\mathbf{X}(t)\) using \(\mathbf{M}\)-orthonormality of \(\Phi\):

\[ ^0 \mathbf{X} = \Phi^T \mathbf{M}^0 \mathbf{U} \]  

(22)

\[ ^0 \dot{\mathbf{X}} = \Phi^T \mathbf{M}^0 \dot{\mathbf{U}} \]  

(23)

In the considered system, model properties are: no damping and periodic force applied, modify equation (15) to:

\[ \ddot{\mathbf{X}}(t) + \Omega^2 \mathbf{X}(t) = \mathbf{r} \sin \omega t \]  

(24)

with initial conditions:

\[ x(t)_{t=0} = \Phi_i^T \mathbf{M}^0 \mathbf{U} \]  

(25)

\[ \dot{x}(t)_{t=0} = \Phi_i^T \mathbf{M}^0 \dot{\mathbf{U}} \]  

(26)

Total displacements are defined then:

\[ \mathbf{U}(t) = \sum_{i=1}^{n} \varphi_i x_i(t) \]  

(27)
4. FEM simulations.

In this study an elementary model of implant-bone fixation is considered due to preliminary investigation of problem. It consists of two slender beams. The idea of simplification and main geometrical features are shown in fig. 1, 2. Left edges of beams are clamped so that all possible degrees of freedom for these parts of model are set to be 0. Single cross-section area of the beam is 0,02 x 0,02 m and length of single beam is L=1m.

Prior to any changes in fixation between bone and implant, beams are connected with their bottom top sides. Changes are simulated by disconnection of common nodes. Mesh of the structure consists of square elements with quadratic shape functions, fig. 3. The length of debonding is designated by \( l \). The state of bone-implant attachment integrity is described by dimensional-less coefficient \( f \) called fixation rate and derived from following relation:

\[
\frac{L}{(1-f)100\%} = \frac{l}{L} \quad \text{(28)}
\]

In subsequent study there are a few states of fixation described and taken into consideration. For clearness they are assembled in tab. 1.
Tab. 1. States of fixation considered in this study. Tab. 2. Eigenfrequencies for considered states of debonding.

<table>
<thead>
<tr>
<th>Case nr</th>
<th>l [m]</th>
<th>f [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>0, 25</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>0, 5</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>0, 75</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixation ratio f [%]</th>
<th>Eigenfrequency [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega_1$</td>
</tr>
<tr>
<td>100</td>
<td>15,47</td>
</tr>
<tr>
<td>75</td>
<td>15,45</td>
</tr>
<tr>
<td>50</td>
<td>15,08</td>
</tr>
<tr>
<td>25</td>
<td>13,45</td>
</tr>
<tr>
<td>0</td>
<td>9,06</td>
</tr>
</tbody>
</table>

In this model periodic force is applied to the structure at A point which is a center of the system. Properties of the force are shown in fig. 2. Its magnitude is taken to be 1 N and excitation frequency designated by $\dot{\omega}$ is one of eigenfrequencies calculated for fully attached system. The first three eigenfrequencies for all values of fixation ratios that are considered in this study were depicted in tab. 2. Displacements of the system in vectorial direction are measured at B point. Their corresponding modeshapes were shown in fig. 4, 5, 6.

Fig. 4. Mode shapes $\omega_1$ for different states of fixation:
A) $f=100\%$, B) $f=75\%$, C) $f=50\%$, D) $f=25\%$, E) $f=0\%$.

Fig. 5. Mode shapes $\omega_2$ for different states of fixation:
A) $f=100\%$, B) $f=75\%$, C) $f=50\%$, D) $f=25\%$, E) $f=0\%$.

Fig. 6. Mode shapes $\omega_3$ for different states of fixation:
A) $f=100\%$, B) $f=75\%$, C) $f=50\%$, D) $f=25\%$, E) $f=0\%$. 
For all modifications of bone-implant attachment integrity the first three mode shapes due to eigenfrequencies $\omega_1, \omega_2, \omega_3$ were presented in fig. 4, 5, 6. In this work analysis was applied only to first three eigenfrequencies but their number was equal to the total number of degrees of freedom. From the investigation of chosen frequencies and their mode shapes (i.e. the lowest $\omega_1$) it can be seen that decreasing of fixation rate (increasing of $l$ parameter) results in decrease of eigenfrequency values. The same pattern is observed among higher frequencies $\omega_2$ and $\omega_3$. It can be stated that loosening of integrity results in stiffness decreasing and changes in mass distribution.

In fig. 7, 8, 9, 10 displacements at B point in vertical direction were shown for the structure with applied periodical load. The frequencies used for excitation were $\omega_1, \omega_2, \omega_3$. Also different states of fixation were considered. It can be observed that the features like increase in amplitude and loose of resonance occur due to changing of fixation state.

Fig. 7. Displacements at B point in vertical direction $Y (U_Y)$ for: A) $\dot{\omega}_1 = 15.47$; B) $\dot{\omega}_2 = 96.46$ Hz; C) $\dot{\omega}_3 = 267.81$

Fig. 8. Displacements at B point in vertical direction $Y (U_Y)$ for $\dot{\omega}_1 = 15.47$ Hz and different states of fixation; A) $f = 100\%$, B) $f = 75\%$, C) $f = 50\%$, D) $f = 25\%$. 
Fig. 9. Displacements at B point in vertical direction $Y (U_Y)$ for $\omega_2 = 96.46$ and different states of fixation:
A) $f = 100\%$, B) $f = 75\%$, C) $f = 50\%$, D) $f = 25\%$.

Fig. 10. Displacements at B point in vertical direction $Y (U_Y)$ for $\omega_3 = 267.81$ and different states of fixation:
A) $f = 100\%$, B) $f = 75\%$, C) $f = 50\%$, D) $f = 25\%$.

It can be concluded from the analysis that higher natural frequencies are more sensitive for detection of early loosening of fixation as it can be seen in fig. 8, 9, 10, where the difference in displacements signal for unattached structure within the range of fixation ratio 100\% - 75\% for $\omega_1$ is hardly observed. From this reason the detailed study of influence of fixation rate in range 100\% - 75\% was performed with excitation frequency $\omega_3$. It included displacement signal processing at B point for 25 cases of fixation ratio (100\%-75\%). Moreover detailed comparison study of natural frequencies vs. fixation ratio was performed.

From the analysis of fig. 11 and its detailed views (fig. 12, fig. 13) we can conclude that displacements signal at B point is sensitive for early stage of attachment loosening. It means that reference signal of 100\% fixation rate can be easily distinguished from the one of 85\% rate. In general changes of the displacement signal can be divided into two parts. The first one deals with the range 100\%-82\% fixation ratio and consists of changes in amplitude possessing still some futures of resonance. The second one is concerned with changes of increasing amplitude, which is a characteristic feature of resonance, for beating phenomenon. From the fig. 14 it can be seen that the highest changes in eigenfrequency vs. fixation ratio are observed for third eigenfrequency. Moreover this frequency starts to be sensitive for changes as early as at 85\% of fixation. Base natural frequency is hardly affectionate for any changes.
Fig. 11. Displacements at B point in vertical direction Y ($U_Y$) for $\omega_j = 267.81$ and different states of fixation rate listed in the legend.

Fig. 12. Displacements at B point in vertical direction Y ($U_Y$) for $\omega_j = 267.81$ and different states of fixation rate listed in the legend with detailed view.
Fig. 13. Displacements at B point in vertical direction \( U_y \) for \( \omega_j = 267.81 \) and different states of fixation rate listed in the legend with detailed view.

Fig. 14. Eigenfrequencies \( \omega_1, \omega_2, \omega_3 \) for various states of fixation.
5. Conclusions.

From performed calculations and analysis it can be stated that changes in attachment integrity between two connected systems results in changing stiffness and mass distribution for whole system. All these affect eigenfrequencies and modeshapes calculated for desired fixation rate of system. Also displacements signal processed at desired point of excited system depends on those changes. It was shown in analysis that not all frequencies used for excitation have the same sensitivity for detection in fixation rate. It seems to be that their higher values are more susceptible for diagnosing early loosening of the system.

Described in this work pattern of loosing integrity attachment generates additional nodes which are extra degrees of freedom. It affects the total number of eigenfrequencies and modeshapes in comparison studies between particular states of fixation.

Further work in this area will require model sensitivity validation and applying more sophisticated geometry models.

However it was not possible to conduct any calculations with use of micro-sensors promising outcomes obtained in this work.


