A first result concerning βη-Church-Rosser-property
(to apply to AUT-QE)

1.1 The reduction considered is generated by β-reduction and a severely restricted form of η-reduction. It is proposed in order to just answer — up till now at least only — the practical need of η-reduction when actually checking mathematics written in AUT-QE. The objection one might have however is that efficiency of the checking can probably be increased considerably by adding a less restricted form of η-reduction.

1.2 The η-reduction intended allows us to replace expressions $[x,\alpha]\{x\}A$ by A only if following conditions are fulfilled:
(i) $x \notin FV(A)$ — as usual —
(ii) A must not reduce to form $[y,A_1]A_2$
(iii) A is a 2-expression (i.e. it has degree 2, so is a type or a type-valued function)

Motivation for (ii) is in thus avoiding the problematic case of an expression $[x,\alpha]\{x\}[y,\beta]C$ which both β-reduces to $[y,\alpha]C$ and η-reduces to $[y,\beta]C$. After imposing restriction (ii), condition (iii) is motivated by our desire to keep the substitution lemma valid: $B \geq B' \Rightarrow (S^x_A B \geq S^x_A B')$.

Condition (ii) is not as bad as it looks. Firstly it can clearly be weakened to (ii'): A must not β-reduce to form $[y,A_1]A_2$ and secondly — as A is a 2-expression by (iii) — verification of condition (ii') seems to require a small amount of administration and a simple computation only.

2.1 Let $CR(A)$ abbreviate the Church-Rosser property for A:
$A \geq B,\ A \geq C \Rightarrow$ C and D have common reduct.
We only want some weak conditions to hold for A (in order to prove $CR(A)$), namely
(i) all variables and constants — so all expressions — have a degree
(ii) application expressions — e.g. A in $\{A\}B$ — have degree 3
We shall even restrict to correct (AUT-QE) expressions A only so (i) and (ii) are certainly satisfied. We need some property then, which [illegible] we prove it for full βη-reduction.
The restriction to correct A allows us to use strong normalization which is not necessary but simplifies the proof slightly. At first we assume that no abbreviations and no definitional reductions are present.

2.2. Definition of reduction and equality.

(1) $\{A\}[x,B]C > \mathcal{S}^x_A C$

(2) $[x,B]\{y\}C > C$ if (i) $x \notin FV(C)$

(ii) $C \not> [y,G]C_2$

(iii) C is a 2expression

(3) If $A > A'$, $B > B'$, $A_i > A_i'$ for $i = 1,\dots,l$ then

$\{A\}B > \{A'\}B$, $(x,A)B > [x,A']B'$, $p(A_1,\dots,A_l) > p(A_1',\dots,A_l')$

(4) $\geq$ is reflexive and transitive closure of $>$

(5) $\vdash A$, $\vdash B$, $A \geq C \leq B \Rightarrow A = B$

(6) $=$ is transitive

3. Proof of SR (A).

3.1. Two substitution lemmas:

1. $A \geq A' \Rightarrow \mathcal{S}^x_A B \geq \mathcal{S}^x_{A'} B$

Proof: First assume $A > A'$ then $\mathcal{S}^x_A B \geq \mathcal{S}^x_{A'} B$ by ind. on length of B. Then the lemma holds by ind on $A \geq A'$.

2. If $\vdash B$, $\vdash A$, A a 3expression, x has degree 3, $B > B'$ or $B \geq B'$. $\mathcal{S}^x_A B > \mathcal{S}^x_A B'$ (suppose no confusion of variables arises)

Proof: Assume $B > B'$. Ind on $B > B'$. Let E^* abbreviate $\mathcal{S}^x_A E$

(1) $B \equiv \{B_1\}[y,B_2]B_3$, $B' \equiv \mathcal{S}^y_{B_1} B_3$. $B^* \equiv \{B_1^*\}[y,B_2^*]B_3^*$, $(B')^* \equiv (\mathcal{S}^y_{B_1} B_3)^*$. Now $B^* > \mathcal{S}^y_{B_1^*} B_3^*$ which indeed equals $(B')^*$ because $y \notin FV(A)$.

(2) $B \equiv [y,B_1]\{z\}B_2$, $B' \equiv B_2$. We know by the conditions on reduction that B is a 2expression and $B_2 \not> [z,B_3]B_4$. Now $B^* \equiv [y,B_1^*]\{z\}B_2^*$, $(B')^* \equiv B_2^*$. Well, $y \notin FV(B_2^*)$, namely $y \notin FV(A)$, B_2^* is a 2-expression, and $B_2^* \not\geq [y,G]G_2$ by following argument: If $B_2^* \geq [y,G]G_2$ then $B_2^* \geq [y,G']G_2'$. Now by main lemma of "A cheap proof of strong normalization" we have that $B_2 \geq [y,G^0]G_2^0$ or $B_2 \geq \{D_1\}\dots\{D_k\}x$. Last case does not apply so first one applies which contradicts the assumption on B_2.

(3) E.g. $B \equiv \{B_1\}B_2$, $B_1 > B_1'$, $B_2 > B_2'$, $B' \equiv \{B_1'\}B_2'$. Now $B^* \equiv \{B_1^*\}B_2^* > (B')^*$ by ind. hyp. Other cases similar.

Now by ind. on $\geq$ we are done (use closure property for B)

3.2 Main lemma. $\vdash A$, $A \geq B$, $A \geq C \Rightarrow B \geq D \leq C$

Proof: Ind. on length of A. Distinguish following cases

(1) $A \equiv \{A_1\} A_2$, $B \equiv \{B_1\} B_2$, $C \equiv \{C_1\} C_2$, $B_1 \leq A_1 \geq C_1$, $B_2 \leq A_2 \geq C_2$. By ind. hyp. $B_1 \geq D_1 \leq C_1$, $B_2 \geq D_2 \leq C_2$. So $B \geq \{D_1\} D_2 \leq C$.

(2) $A \equiv \{A_1\} [y, A_2] A_3$, $B \equiv \{B_1\} [y, B_2] B_3$, $C \equiv \dot{S}^{y}_{A_1} A_3$, $A_1 > B_1$, $A_2 > B_2$, $A_3 > B_3$. Now take D to be $\dot{S}^{y}_{B_1} B_3$, so $B > D$ and $C \geq D$ by lemma 3.1.2. and 3.1.1

(3) $A \equiv \{A_1\} [y, A_2] \{y\} A_3$, $B \equiv \{A_1\} A_3$ by η-red. and $C \equiv \{A_1\} A_3$ by β-reduction. Take $B \equiv D \equiv C$

(4) $A \equiv p(A_1, \ldots, A_k)$. Both sides must be applications of monotone rules, so proof as in case (1)

(5) $A \equiv [y, A_1] A_2$, both sides monotony rules applied then as in case (1)

(6) $A \equiv [y, A_1] \{y\} A_2$, $B \equiv [y, B_1] \{y\} B_2$, $A > C \equiv A_2$ by η-reduction, so we know (i) $y \notin FV(A_2)$ (so $y \notin FV(B_2)$), (ii) $A_2 \geq [z, A_3] A_4$ (so $B_2 \geq [z, B_3] B_4$), (iii) A_2 a λ-expression (so B_2 is). Take $D \equiv B_2$.

(7) $A \equiv [y, A_1] \{y\} [z, B] C$; This is the problem for general η-reduction. Now however the η-reduction $A > [z, B] C$ is forbidden so the problem does not arise.

3.3.1 Theorem: If $\vdash A$ then $CR(A)$

Proof: Induction on reduction tree of A. Let $A \geq B$, $A \geq C$. In case either side is application of reflexivity of $\geq$, then it is easy. In the other case $A > B_1 \geq B$, $A > C_1 \geq C$. By main lemma $B_1 \geq D_1 \leq C_1$. By closure property $\vdash B_1, \vdash C_1, \vdash D_1$ and B_1, C_1, D_1 have smaller reduction tree than A. So ind. hyp gives $B \geq D_2 \leq D_1$, $C \geq D_3 \leq D_1$, $D_2 \geq D_4 < D_3$. Take D to be D_4.

3.3.2. Corollary1: $A = B \Rightarrow A \geq C \leq B$

Corollary2: Uniqueness of normal form.

4. Some comments

4.1 It seems that the proposed weak form of η-reduction is of some use in writing mathematics in AUT-QE actually. Namely what one doing is the definitional equality of

$$[x, \alpha] \{x\} z \quad \text{and} \quad z$$

where z is e.g. a predicate variable

4.2. However the substitution lemma:

$$B \geq B' \Rightarrow J^x_A B \geq J^x_A B'$$

gets lost [illegible] for [illegible] x – although [illegible] if [illegible] (x and A)

The best we get here is:

$$B \geq B' \Rightarrow J^x_A B \geq C \leq J^x_A B', \qquad (*)$$

which gives:

$$A = A', B = B' \Rightarrow J^x_A B = J^x_{A'} B'$$

4.3. In proving (*) one has to simulate [illegible] q-reduction by a sequence of β- and principal definitional (δ-) reductions. This possibly yields a large amount of extra work for the verification to be done.

4.4. One might wonder if answering the question whether shortcuts via q-reduction are allowed, requires about the same amount of work that was to be avoided by the shortcut. Luckily this seems not to be the case. Because we deal with expressions A in this respect only to decide whether A reduces to abstraction form amounts to counting the number of "starting abstractors" of A. If one adds abbreviation constants and definitional reductions, this counting must be slightly more sophisticated only.

v. Daalen, 11-1-'74 - 1

Yet another equivalence concerning equality of proofs

1. Terminology. We adopt Jeff's proposal to call "remarkable" things "generalized" from now on, so we speak about generalized logic, generalized function, generalized if then else etc.

2. In our previous remarks on this subject we conjectured that the scheme IRPR (irrelevance of proofs) or equivalently, GITE (generalized if-then-else) is really stronger than the scheme CAN (canonical map) ~~or~~ or, equivalently EQST (equality on subtypes).
This conjecture is <u>wrong</u>. Both schemes are provably equivalent in a system of generalized logic — which is already necessary for the formulation only of schemes IRPR and GITE —.

3. The proof of IRPR $\Rightarrow$ EQST, e.g, we gave already. Now we go the other direction. Assume EQST, ~~...~~ i.e.

$\langle a, p \rangle = \langle a, g \rangle$ where $a : \alpha$, $P : \alpha \to \pi$, $p : \{a\}P$, $g : \{a$

Assume further

$\beta :=$	——	π
$\gamma :=$	——	τ
$\gamma^* :=$	$ST(\gamma, [x,\gamma]\beta)$	τ
$f :=$	——	$\beta \to \gamma$ (gen. function)
$x :=$	——	γ^*
$x1 :=$	$\{k_2\, x\} f$	γ
$f * g :=$	① $x1$ (which means $[x,\gamma^*]\, x1(x)$	$\gamma^* \to \gamma$
$c :=$	——	β
$d :=$	——	β

Now we like to prove

$IRPR :=$	$\{c\}f = \{d\}f$	
$c1 :=$	$EQST(\{c\}f, c, d)$	$\langle \{c\}f, c \rangle = \langle \{c\}f, d \rangle$
$c2 :=$	equality axiom	$\{\langle \{c\}f, c \rangle\}g = \{\langle \{c\}f, d \rangle\}g$
	or by suitable reduction	$IRPR$

dit is een eenvoudige parafrase van [illegible] proof — vóór AUT 38

v. Daalen, 20-2-79-1

A proof of CR-property for full $\beta\eta$-reduction
(to apply to AUT-QE)

1.1. Let a usual definition of AUT-QE be given. We list some basic notions: (i) $\sigma \vdash \;:\Leftrightarrow\; \sigma$ is a correct indicatorstring / context

(ii) $\sigma \vdash A \;:\Leftrightarrow\; \sigma \vdash$ and A correct expression relative to σ

(iii) $\sigma \vdash A \in B \;:\Leftrightarrow\; \sigma \vdash A$, $\sigma \vdash B$ and $A \in B$ a correct $\in$-formula with respect to σ

(iv) $\geq_0$ denotes the usual reduction relation generated by β and η-reduction

(v) $\equiv$ denotes syntactic identity modulo α-equivalence (renaming of variables)

(vi) $=_0$ denotes definitional equality relation generated by $\geq_0$

1.2. We assume following properties:

(i) Strong normalization for $\geq_0$: any proper $\geq_0$-reduction sequence terminates

(ii) Closure property: $\sigma \vdash A$, $A \geq_0 B \Rightarrow \sigma \vdash B$

(iii) Various soundness conditions of $=_0$, e.g.

$\xi \vdash A \in B$, $\xi \vdash A =_0 C$, $\xi \vdash B =_0 D \Rightarrow \xi \vdash C \in D$ and

$\xi \vdash A =_0 A'$, $\xi, x \in A \vdash B =_0 B' \Rightarrow \xi \vdash [x, A]B =_0 [x, A']B'$

(iv) Uniqueness of domains, i.e.

$\xi \vdash [x, A]B =_0 [x, C]D \Rightarrow \xi \vdash A =_0 C$, $\xi \vdash B =_0 D$ and

$\xi \vdash [x, A]\{x\}[y, B]C \Rightarrow \xi, x \in A \vdash A =_0 B$

2.1. We introduce by simultaneous definition an extended α-equivalence $\approx$, a more liberal reduction relation $\geq$ and a new definitional equivalence $=$ by (for correct expressions only):

1. (i) $\{A\}[x,B]C > S^x_A C$ (elementary β-reduction)
   (ii) $[x,B]\{x\}C > C$ if $x \notin FV(C)$ (elementary η-reduction)
   (iii) $A \approx B \Rightarrow A > B$ (elementary $\approx$-reduction)
   (iv) $A_1 > B_1, A_2 > B_2 \Rightarrow \{A_1\}A_2 > \{B_1\}B_2$
   (v) $B_1 > B_2 \Rightarrow [x,A]B_1 > [x,A]B_2$
   (vi) $A_1 > A_1', \ldots, A_k > A_k' \Rightarrow p(A_1, \ldots, A_k) > p(A_1', \ldots, A_k')$

   } monotony rules

2. (i) $\geq$ is transitive closure of $>$
   (ii) $=$ is equivalence relation generated by $\geq$
3. (i) $A \equiv B \Rightarrow A \approx B$
   (ii) $A_1 = A_2,\ B_1 \approx B_2 \Rightarrow [x,A_1]B_1 \approx [x,A_2]B_2$
   (iii) $A_1 \approx A_2,\ B_1 \approx B_2 \Rightarrow \{A_1\}B_1 \approx \{A_2\}B_2$
   (iv) $A_1 \approx A_1', \ldots, A_k \approx A_k' \Rightarrow p(A_1, \ldots, A_k) \approx p(A_1', \ldots, A_k')$

2.2. Remarks:

(i) It is not necessary to define the above notions simultaneously. By result below we might as well have taken instead of 3.(ii)
$A_1 =_0 A_2,\ B_1 \approx B_2 \Rightarrow [x,A_1]B_1 \approx [x,A_2]B_2$ (among 1.i)–1.(vi))

(ii) Note that only 1.(iii) and 1.(v) are unusual. One can conceive of $\approx$ as α-equivalence but for possible domains which have to be definitionally equal however.

(iii) It is possibly more honest to put in contexts in definition 2.1. However we feel that there is no danger of confusion.

2.3. 1. Property: $A = B \Leftrightarrow A =_0 B$
Proof: $\Rightarrow$. By simultaneous induction on the definition 2.1.
$\Leftarrow$ By induction on definition of $=_0$ (and $\geq_0$)

2. Proof: (1) Closure property for $\geq$
(2) Monotony of $=$

3. Property: $A \approx B \Rightarrow A = B$
Proof: Induction on definition of $\approx$

4. Property: $\approx$ is an equivalence relation

5. Prop: Monotony of $\geq$

3.1. In λ-calculus, when speaking about proper reduction sequences, one means reduction sequences in which, say, only a finite number of α-reductions occurs (Here I don't want to exclude the case that, by specific choice of substitution operator, sometimes α-reductions are necessary before applying some β-reduction)
So strong normalization means that in any reduction sequence $\Sigma_1 > \Sigma_2 > \Sigma_3 \cdots$ from a certain expression Σ_k on, only α-reductions are applied.
For Σ to be normal, is defined as $\Sigma \geqslant \Delta \Rightarrow \Sigma =_\alpha \Delta$.

3.2. Analogously, we define one-step reduction by 2.11.(iii) as single proper reduction. Proper reduction sequences are red. sequences in which only a finite number of these, say, $\tilde{\approx}$-reductions are applied.
An expression Σ is called normal iff $\Sigma \geqslant \Delta \Rightarrow \Sigma \tilde{\approx} \Delta$
Remark that no (more) new possibilities for β-reduction are caused by $\approx$-reduction (><!) than by α-reduction: $A \approx B \geqslant_\beta C \Rightarrow A =_\alpha B' \geqslant_\beta C'$, and that η-reductions also don't increase the number of β-reductions.

3.3. Now to get strong normalization for $\geqslant$, we argue as follows: Let a reduction sequence of some correct Σ be given. By strong normalization for $\geqslant_0$, we know that the number of applications of β-reduction in the sequence is bounded. Call $\theta(\Sigma)$ the maximal number of β-reductions in the red. sequences of Σ. Now the number of proper reductions can be proved to be finite by induction: (i) on $\theta(\Sigma)$ strictly (ii) on structure of Σ.

3.4. We don't need strong normalization for $\geqslant$ in the sequel, but having it makes one particular proof (Theorem 4.2) simpler.

4.1. Substitution Lemma 1: If $\begin{cases} A > A' \\ A \geqslant A' \\ A = A' \\ A \approx A' \end{cases}$ then $J_A^B \begin{cases} > \\ \geqslant \\ = \\ \approx \end{cases} J_{A'}^B$ (provided both are correct)

Proof: Ind. on length of B.

4.2. Substitution Lemma 2: If $\begin{cases} B > B' \\ B \geqslant B' \\ B = B' \\ B \approx B' \end{cases}$ then $J_A^B \begin{cases} \geqslant \\ = \\ \approx \end{cases} J_A^{B'}$ (again ↷)

Proof: Ind. on $>, \geqslant, =, \approx$

4.1. Main lemma (CR-1): If $\vdash A$, $B \leftarrow A \rightarrow C$ then $B \geq D \leq C$

Proof: By induction on length of A (as is the case $A \equiv x$)

The case $A \approx B$ or $A \approx C$ is straightforward. So we distinguish following cases: (1) $A \equiv \{A_1\}A_2$, $B \equiv \{B_1\}B_2$, $C \equiv \{C_1\}C_2$, $C_1 < A_1 > B_1$, $C_2 < A_2 > B_2$. By ind.hyp. $C_1 \geq D_1 \leq B_1$, $C_2 \geq D_2 \leq C_2$ and by monotony of $\geq$, $C \geq \{D_1\}D_2 \leq B$

(2) $A \equiv \{A_1\}[x, A_2]A_3$, $B \equiv \{B_1\}[x, B_2]B_3$, $C \equiv \int_{A_1}^{x} A_3$, $A_1 > B_1$ and $A_2 = B_2$, $A_3 \rightarrow B_3$. Now by substitution Lemmas above $C \equiv \int_{A_1}^{x} A_3 \geq \int_{B_1}^{x} B_3 < B$

(3) $A \equiv \{A_1\}(x, A_2)\{x\}A_3$, $B \equiv \{B_1\}A_3$ (by η-reduction), $C \equiv \{A_1\}A_3$ by β-reduction, $x \notin FV(A_3)$, $A_1 > B_1$ well, $C \rightarrow B$ so ready

(4) $A \equiv (x, A_1)A_2$, $B \equiv [x, B_1]B_2$, $C \equiv (x, C_1)C_2$, $A_1 = B_1$, $A_1 = C_1$, $B_2 < A_2 > C_2$. By ind.hyp. $B_2 \geq D_2 \leq C_2$ so, e.g. $B \geq [x, A_1]D_2 \leq C$

(5) $A \equiv (x, A_1)\{x\}A_2$, $B \equiv [x, B_1]\{x\}B_2$, $C \equiv A_2$ (by η-reduction), $A_1 = B_1$, $A_2 > B_2$, $x \notin FV(A_2)$. We know that we got B_2 from A_2 by applying some (either proper or improper) elementary reduction on disjoint subexpression of A_2, we can do the proper reductions first and then get $A_2 \rightarrow E_2 \approx B_2$. So $B \geq [x, B_1]\{x\}E_2 \geq E_2 \leq A_2 \equiv C$ ($x \notin FV(E_2)$)

(6) $A \equiv (x, A_1)\{x\}[y, A_2]A_3$, $B \equiv [y, A_1]A_3$ (by β-red), $C \equiv (y, A_2)A_3$, by η-reduction. By language theory $A_1 = A_2$ so $A_1 = A_2$. So $B \rightarrow C$ ready

(7) $A \equiv p(A_1, \dots, A_k)$, $B \equiv p(B_1, \dots, B_k)$, $C \equiv p(C_1, \dots, C_k)$, $C_i < B_i > A_i$ for $i = 1, \dots, k$. As in case (1)

4.2. Theorem (CR) (properly) If $\vdash A$, $B \leftarrow A \rightarrow C$ then $B \geq D \leq C$

Proof: The case that $A > B$, $A > C$ is produced by the lemma. Further we do induction on red. tree of A. In the other case namely $B \leq B_1 \leq A \geq C_1 \geq C$ where B_1, C_1 have strictly smaller reduction trees of A. So $B_1 \geq D_1 \leq C_1$. Again by ind.hyp. $B \geq E \leq D$ and $D \geq F \leq C$. Thus by ind.hyp once more $B \geq E \geq D \leq F \leq C$.

4.3. Corll 1: $\vdash A$, $\vdash B$, $A = B \Rightarrow A \rightarrow C \leftarrow B$ (Church-Rosser theorem)

Corll 2: Uniqueness of normal form: $\vdash A$, $\vdash B$, both normal, $A = B \Rightarrow A \approx B$

20-2-24-5

5.1. Definition: A is σ-normal if $A \geq_\sigma B \Rightarrow A \equiv B$ (i.e., $A =_\sigma B$)

2. Property: If A σ-normal and $A \geq B$ then in this reduction no elementary β-reductions occur.

Proof: As in 3.2. If after some non-β-reductions a β-reduction is possible then it [illegible] of the possibly one η-reduction only. So there occur no β-reductions! (Even $A \geq A' \geq_\eta B$)

3. Property: If A σ-normal, A does not have abstraction form (so $A \not\equiv (x, A_1)A_2$), $A \geq B$ then A keeps its outer structure in the following cases: (i) $A \equiv x \Leftrightarrow B \equiv x$

(ii) $A \equiv \{A_1\}A_2 \Leftrightarrow B \equiv \{B_1\}B_2$, $A_1 \geq B_1$, $A_2 \geq B_2$

(iii) $A \equiv p(A_1, \ldots, A_k) \Leftrightarrow B \equiv p(B_1, \ldots, B_k)$ and $A_i \geq B_i$ for $i = 1, \ldots, k$

Proof: Both α-reductions and η-reductions leave the outer structure intact.

Theorem:

5.2. Uniqueness of σ-normal form: If $\xi \vdash A$, $\xi \vdash B$, both σ-normal, $A =_\sigma B$, then $A \equiv B$

Proof: By induction on sum of lengths of A and B. Firstly, $A = B \Rightarrow A \geq C \leq B$. Remark that not necessarily σ-normal expressions are normal again. As for $A \geq C$, $B \geq C$ the situation from 5.1.3. also applies. Distinguish:

(i) Both A and B have abstraction form then $A \equiv (x, A_1)A_2$, $B \equiv (x, B_1)B_2$. By language theory $A_1 =_\sigma B_1$, $A_2 =_\sigma B_2$, all expressions are σ-normal $\Rightarrow A_1 \equiv B_1$, $A_2 \equiv B_2$, $A \equiv B$.

(ii) Neither A nor B has abstraction form. Use 5.1.3. Example: $A \equiv \{A_1\}A_2 \Rightarrow C \equiv \{C_1\}C_2 \Rightarrow B \equiv \{B_1\}B_2$, $A_1 =_\sigma B_1$, $A_2 =_\sigma B_2$, $A_1 \equiv B_1$, $A_2 \equiv B_2 \Rightarrow A \equiv B$

(iii) A has abstr. form, $A \equiv [x, A_1]A_2$ and B has not. Then C has not by 5.1.3. So $A \geq [x, D_1]\{x\}D_2 \geq_\eta D_2 \geq C \leq B$, here $A_1 = D_1$, $A_2 \geq \{x\}D_2$. By closure property $\xi, x \in D_1 \vdash \{x\}D_2$. So $\xi, x \in A_1 \vdash \{x\}B$. Certainly $x \notin FV(B)$. We see $A_2 = \{x\}B$ both σ-normal so $A_2 \equiv \{x\}B$ and apparently A not [illegible]. Contradiction, this case does not really apply.

notice that indeed in that step $\checkmark$ applies

5.3. Coroll: Church Rosser theorem for $\geq_\sigma$