Bachelor's or Master's graduation project: Self-interacting random walks

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Modelling the motion of particles has been and remains a source of inspiration in mathematics. In probability, often random walks are used to describe the irregular motions of particles, like dust particles in a glass of water. In many applications, the decision of where to step to depends on the details of the path up to that point, i.e., the transition rules depend on the history of the path. Think of a person exploring a new city. This person is likely to prefer to traverse a street which he/she has already traversed, simply because it is familiar. Another example is a certain type of bacteria that produces a slime, of which it prefers to slide. In the past decades, such problems have attracted enormous attention, the basic question being how the details of the dependency on the history influence the characteristics of the motion, e.g., does the walker get stuck on a single edge, does every site gets visited, or does the walker visit the starting point infinitely often?

In this project, we shall focus on a particular version of a self-interacting random walk model, in which, at each time, the probability of stepping along a particular edge incident to the current position is the normalized weight of that edge, and the weight of the edge is either 1 (when the edge has not yet been traversed), or $1+\beta$ (when the edge has been traversed in the past). Here $\beta > 0$ is the parameter of the model. Thus, the weights depend on the history of the path through the edges that have been traversed. This is called *once-edge reinforced random walk* (OERRW). It is expected that OERRW behaves very similarly to random walk, and it is the main goal in this project to investigate to what extent this expectation is valid.

The key questions are:

(1) Simulate OEERW to investigate what the displacement of the process is, in different dimension. Is this displacement consistent with central limit theorem behavior?

(2) Suppose we fix an $n \in \mathbb{N}$ and $x \in \mathbb{Z}^d$, and suppose we could give each edge either initial weight 1 or $1+\beta$ (this would fix a particular history of the path). Which edges would we give weight 1 and which edges weight $1+\beta$ in order to maximize the probability for the process to end up in x at time n? This question sheds light on the influence of the history upon the stepping rules of the process.

For more background on OERRW, see e.g., [1, 2, 3].

References

[1] R. Durrett, H. Kesten, and V. Limic. Once edge-reinforced random walk on a tree. *Probab. Theory Related Fields*, **122**(4):567–592, (2002).

[2] R. van der Hofstad and M. Holmes. An expansion for self-interacting random walks. Submitted, Preprint (2006).

[3] S. Rolles. *Random Walks in Stochastic Surroundings.* PhD thesis, University of Amsterdam, (2002).