Bachelor's or Master's graduation project: Preferential Attachment Random Graphs and Real Networks

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Empirical studies on real networks, such as the Internet, the World-Wide Web, social and sexual networks, and networks describing protein interactions, show fascinating similarities. Most of these networks are *small worlds*, meaning that typical distances in the network are small, and have *power-law degree sequences*, which means that the number of vertices with degree k falls off as an inverse power of k. This phenomenon is illustrated by the fact that many real networks have a huge variability in the interconnectedness of their vertices. Most vertices have small degrees, but there exist vertices with gigantic degrees.

Incited by these empirical findings, random graph models have been proposed to *model* and/or *explain* these two paradigms. This project is about *preferential attachment* random graphs, which are graphs evolving in time in such a way that new vertices are more likely to attach to vertices that already have high degree, i.e., the 'rich get richer'. See [1] for a popular account of preferential attachment and its consequences.

In the non-rigorous literature, it is suggested that many graph properties, such as the typical graph distance, the amount of clustering in the graph, the diameter of the graph and its connectivity properties, should be similar in preferential attachment models and in other models having similar degrees. The goal of this project is to investigate this, both by simulations and by analysis. The key questions are:

(1) Simulate distances in preferential attachment graphs, and compare these simulations to known results about distances simpler graphs with a similar degree structure (such as the so-called configuration model).

(2) Mathematically investigate the degrees of one specific preferential attachment graph.

(3) Investigate the literature on network properties of one example of a real network. Examples of real networks are protein-protein interaction networks [2], the Internet [3], the WWW [4, 5] or an example of your own choice.

References

[1] A.-L. Barabási. *Linked: The New Science of Networks.* Perseus Publishing.

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[3] C. Faloutsos, P. Faloutsos, and M. Faloutsos. On power-law relationships of the internet topology. *Computer Communications Rev.*, **29**:251–262, (1999).

[4] R. Kumar, P. Raghavan, S. Rajagopalan, D. Sivakumar, A. Tomkins, and E. Upfal. Stochastic models for the web graph. In *42st Annual IEEE Symposium on Foundations of Computer Science*, pages 57–65, (2000).

[5] R. Kumar, P. Raghavan, S Rajagopalan, and A. Tomkins. Trawling the web for emerging cyber communities. *Computer Networks*, **31**:1481–1493, (1999).