Bachelor's project:
A Performance Study for Simulated Annealing

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Description: The minimization of functions with a huge number of local minima is a difficult problem. This problem arises for instance if one follows the variational ansatz in image analysis where the reconstruction of a given noisy image is given by the global minimizer of such a functional. In those cases, one frequently uses some stochastic algorithm to determine the minimizer. This iterative Makov chain Monte Carlo algorithm is called *simulated annealing*. By a Theorem of Dobrushin from the 1950's, the output of the algorithm converges to the minimizer of the functional with probability one. However, for principal reasons, there is no general about the accuracy of the output of the algorithm after n iterations.

The topic of the thesis is to study the performance of simulated annealing for the minimization of the functional

$$F_{x,\gamma}(z) := \gamma\, j(z) + \|z - x\|^2$$

where $\gamma > 0, x \in \mathbb{R}^N$ are fixed, $z \in \mathbb{R}^N$ and $\| - \|$ denotes euclidian distance in $\mathbb{R}^N$. For $z = (z_1, ..., z_N)$, we have

$$j(z) := \{i = 1, ..., N-1 \,:\, z_i \neq z_{i+1}\}.$$

This functional arises frequently in signal analysis. The number of its local minima increases with N. On the other hand, there is a dynamical optimization algorithm which effectively computes the exact minimizer. F is thus an ideal testbed for the performance of other minimization procedures. The topic of the thesis is thus to implement simulated annealing for the functional F and to compare the performance of the algorithm for different cooling schedules with the known exact solution.

References.

[1] N. Madras (2002) *Lectures on Monte Carlo Methods*, American Mathematical Society, Providence, RI

[2] G. Winkler (2003) *Image Analysis, Random Fields and Markov Chain Monte Carlo Methods*, Springer Verlag, Berlin