Bachelor's or Master's graduation projects: Mathematical models for heat conduction.

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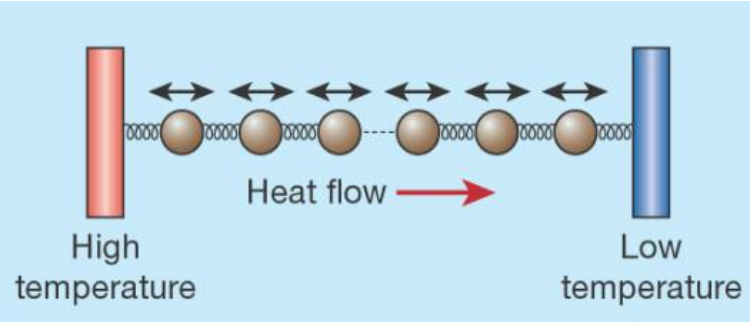
When a system is in contact with two heat (or particles) reservoirs, a non-equilibrium steady state sets in after sufficiently long time, in which there is a flux of energy (or particles) through the system (see figure) from one boundary to the other.



When dealing with transport of particles a class of convenient mathematical models is given by *exclusion processes.* These are basically Markov chains defined for particles on the lattice which moves subject to an exclusion rule, that is there can not be more than one particle on each site of the lattice. For this class of models there are many mathematical results, which derive from an explicit solution of the steady states weights of the particle configurations [1]. This rigorous results allowed to derive a number of results for non-equilibrium quantities, like the density or current large deviations.

We recently introduced a model for heat conduction which is very close to exclusion processes [2]. It can actually be seen as the counterpart of exclusion process, in which particles (carrying quanta of energy) feel an attractive interaction, contrary to the hard core interaction of exclusion processes. The similarity has been recognized as a consequence of a duality property. The aim of the project will be to extend to this bosonic model the results which are know for the fermionic exclusion process.

Some of the key questions are:

1. Is there an explicit solution for the probability density function in the steady states? Here the matric ansatz of exclusion process [1] needs to be generalized to an operator ansatz.
2. What are the large deviations properties of heat flux and temperature profiles?
3. Find efficient algorithms for simulations of large deviation events [3].

References

1. B. derrida, *Non-equilibrium steady states: fluctuations and large deviations of the density and of current*, http://arxiv.org/abs/cond-mat/0703762
2. C. Giardinà, J. Kurchan. F. Redig, *Duality and exact correlations for a model of heat conduction* http://arxiv.org/abs/cond-mat/0612198
3. C. Giardinà, J. Kurchan, L. Peliti, *Direct evaluation of large-deviation functions*, http://arxiv.org/abs/cond-mat/0511248