MASTER PROJECT: SCALE-EQUIVARIANT PARAMETER CHOICE FOR WEDGELET-RECONSTRUCTIONS

Olaf Wittich HG 10.21

DESCRIPTION: Wedgelets are roughly speaking a hierarchically organized class of step functions serving as domain for suitable functionals in image analysis such as for example the complexity penalized least square

$$F_d(w,\gamma) := \gamma\,|w| + \|w-d\|^2$$

where d denotes the data, a function $d \in L^2([0,1]^2)$ called *image*, $\gamma \geq 0$ is a parameter and w is a step function that belongs to $W \subset L^2([0,1]^2)$, the set of wedgelets under consideration. The expression $|w|$ denotes the number of connected subsets of $[0,1]^2$ where the wedgelet w is constant. Since $|w|$ will generically determine the number of parameters that are necessary to determine w, we think of this term as describing the *complexity* of w. A minimizer of the functional F_d for some data d can thus be thought of as a *least complex wedgelet* which is sufficiently close to the data in the sense of L^2-distance.

The minimizer $w^*(\gamma)(d)$ of the functional and hence the wedgelet-reconstruction of the image depends on the choice of the parameter $\gamma > 0$. γ can be interpreted as a scale parameter since due to

$$\text{argmin}_w F_{sd}(w,\gamma) = \text{argmin}_{\tilde{w}=w/s} F_{sd}(s\tilde{w},\gamma) = s\,\text{argmin}_{\tilde{w}} F_d(\tilde{w},\gamma/s^2),$$

the change of scale in signal strength by some $s > 0$ leads to a reconstruction with a different parameter $\tilde{\gamma} = \gamma/s^2$. In particular, for the reconstruction to a particular choice of $\gamma > 0$ we do not have

$$w^*(\gamma)(sd) = s\,w^*(\gamma)(d)$$

for all $s > 0$, i.e. the reconstruction is not scale-equivariant. However, the full space of minimizers $\mathcal{M}(d) := \{w^*(\gamma)(d)\,:\,\gamma \geq 0\}$ can be efficiently computed. Everybody is invited to download the corresponding software from

http://ibb.gsf.de/homepage/laurent.demaret/wedgelet/

and to consider the reconstructions of the parrot. $\mathcal{M}(d)$ is is the wedgelet-analogue to the *scale - spaces* in diffusion imaging. The map $\gamma \mapsto w^*(\gamma)(d)$ is locally constant with changes only at a finite set $\mathcal{T}(d) := \{\gamma_1 < \ldots < \gamma_{n(d)}\}$ of γ-values. $\mathcal{T}(d)$ is thought of as the wedgelet-analogue of *top points* in diffusion imaging.

The topic of the masters thesis is to use the full information of the scale space $\mathcal{M}(d)$ to construct scale-invariant reconstructions of d.

REFERENCES: See the literature quoted at

http://ibb.gsf.de/homepage/laurent.demaret/wedgelet/