Optimal Design of Composite Laminates

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October 29, 2008

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1 Introduction

A composite laminate is a material which consists of two or more distinct layers with specific structure assembled to achieve certain physical properties. The layers can be of different thicknesses, their inner fiber orientations may differ, and they may consist of different materials.

In the design of composite laminates, the goal is to obtain a resistant material with high loading capacity and low weight, for example, for the production of wings for aircrafts, cross-country-skis and many more applications.

In classical elasticity theory the composite is modeled using 2-dimensional plane approximation of the layers, and the forces are considered in the midsurface.

We apply normal forces in the x and y directions which are denoted by N_x and N_y, respectively, and shear forces N_{xy}, whose units are given by N/m. Likewise, we apply moments in the x and y direction which are denoted by M_x and M_y, respectively, and shear moments M_{xy}. A shear stress or force is defined as a stress which is applied parallel or tangential to the face of a material, as opposed to a normal stress which is applied perpendicularly. The forces and moments are illustrated in Figure 1.

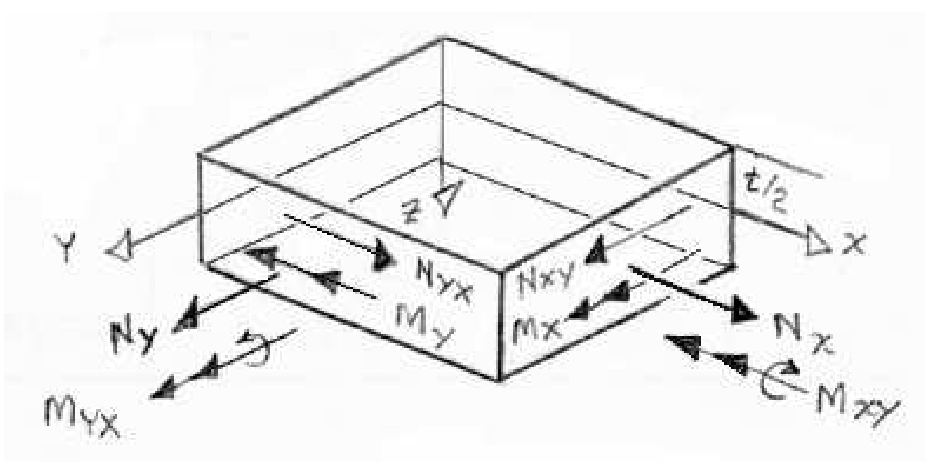

Figure 1: Forces and moments acting on a laminate

It is interesting to consider composite laminates because of the intrinsic structure of some materials. Graphite-epoxy for example is called an *orthotropic* material, which means that it has different properties and strenghts in its different orthogonal directions. Other materials, such as aluminium, behaving uniformly in all directions are called *isotropic*.

2 Problem statement

There are several properties that are required to satisfy certain conditions when considering composite laminates in industrial applications. Thus, the design of composite laminates may be formulated as a multiobjective optimization problem.

The objectives might be, e.g. the mass of the structure ρ_A or the total elastic strain energy U, i.e. the work done in deforming a solid within its elastic limit. Other objectives, such as, the total thickness of the structure t_{tot}, could be also used. In this work, we concentrate on the mass and the total elastic strain energy, and assume that t_{tot} is fixed.

According to the classical laminated plate theory the total elastic strain energy U is computed as:

$$\begin{aligned} U &= \int\int\int_V \frac{1}{2}\sigma_{ij}\epsilon_{ij} dzdydx \\ &= \int\int\int \frac{1}{2}\boldsymbol{\sigma}^T(\boldsymbol{\epsilon}^0 + z\boldsymbol{\kappa})dzdydx \\ &= \int\int_A \frac{1}{2}\begin{bmatrix} \boldsymbol{N}^T & \boldsymbol{M}^T \end{bmatrix}\begin{bmatrix} \boldsymbol{\epsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix} dydx, \end{aligned} \tag{1}$$

where $N^T = \begin{bmatrix} N_x & N_y & N_{xy} \end{bmatrix}$ and $M^T = \begin{bmatrix} M_x & M_y & M_{xy} \end{bmatrix}$ are the forces and moments, respectively, $\boldsymbol{\epsilon}^0$ is the vector of middle-surface strains and $\boldsymbol{\kappa}$ the vector of middle-surface curvatures. The middle-surface strains and curvatures depend on the thickness, orientation of the materials and the properties of the individual materials. For more details on the mechanics of composite laminates the interested reader is referred to [4].

We consider the simple case where the strain tensors are constant over the middle surface. This is the case when the external forces and moments act in the laminate plane. So in this case, we are facing the following objective function:

$$U = \begin{bmatrix} \boldsymbol{N}^T & \boldsymbol{M}^T \end{bmatrix}\begin{bmatrix} \boldsymbol{\epsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix}. \tag{2}$$

And for the mass m_A the objective function becomes:

$$m_A = \sum_{i=1}^{n} t_i m_i, \tag{3}$$

where t_i and m_i are the thickness and the mass of the ith layer, respectively.

Due to the properties of certain materials, the orientation of the fibers in the material θ_i $(i = 1, ..., n)$ might have an effect on the total elastic strain energy. Thus, we consider the angle of the fibers as one of our design variables. Other design variables we use are the thicknesses of the individual layers in the material t_i, the number of layers n and the order of materials in the structure.

The objective function is, in general, a non-linear function of the design variables, and we also impose some constraints on our design variables. So, we are dealing with a constrained non-linear optimization problem, which, in general, cannot be solved analytically.

In this paper, we explore two numerical algorithms for solving the problem, namely, a gradient based method FMINCON and a genetic algorithms NSGA.

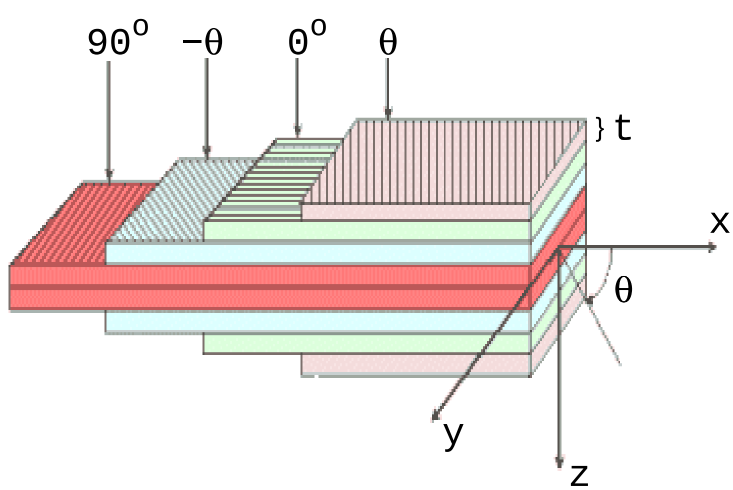

Figure 2: Layers and fiber orientations

3 Algorithms

In this section we present the algorithms we used to solve the optimization problem.

Gradient-based methods

We explore different kinds of subproblems of the multiobjective optimization problem described in Section 2 using two different algorithms. To investigate the effect of different fibre angles in a two-layer composite consisting of graphite-epoxy, we used a gradient-based algorithm FMINCON. This algorithm is a gradient-based minimization algorithm for constrained non-linear minimization problems. The objective function has to be a scalar valued function, and thus, we choose the total elastic strain energy as our objective.

Genetic Algorithms

Genetic Algorithms (GA) are heuristic optimization methods that try to mimic the evolution process in nature by simulating inheritance, mutation, selection and crossover (recombination). Genetic Algorithms work very well on mixed (continuous and discrete, combinatorial) problems. Genetic algorithms as proven to work well in solving complex optimization models which sometimes, beacause of their complexity, are not treatable with gradient based methods. One of the advantages of GA, is that it is less prone to get stuck on local minima. The pseudo-code of the basic GA is presented below

Initialize: Generate random population of n chromosomes

Fitness: Evaluate the fitness $f(x)$ of each chromosome x in the population

New population: A new population is created by repeating following steps until the new population is complete

- **Selection:** Select two parent chromosomes from a population according to their fitness (the better fitness, the greater probability to be selected)
- **Crossover:** With a crossover probability cross over the parents to form a new offspring. Offspring is a copy of parents without any crossover.
- **Mutation:** Mutate new offspring at each position in chromosome (locus).
- **Accepting:** Accept the new offspring in a new population

Replace: Use the new generated population for a further run of algorithm

Test: If the end condition is satisfied, stop, and return to the best solution in current population

Loop: Go to step 2

The Non-dominated Sorting Genetic Algorithm (NSGA)[1] is an improved version of the basic GA. It differs from the GA only in the way the selection operation functions, that is, in what way the parent chromosomes are chosen. The improved selection is based on the ranking selection method that emphasizes good chromosomes. The population is ranked prior to the selection on the basis of choromosomes' non-domination. The non-dominant ones are then assigned with higher fittness value. More detailed describtion can be found from [3].

4 Tests

Recall the objective function for total elastic strain energy.

$$U = \begin{bmatrix} \boldsymbol{N}^T & \boldsymbol{M}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}^0 \\ \boldsymbol{\kappa} \end{bmatrix}, \tag{4}$$

It is used in all of the following test cases.

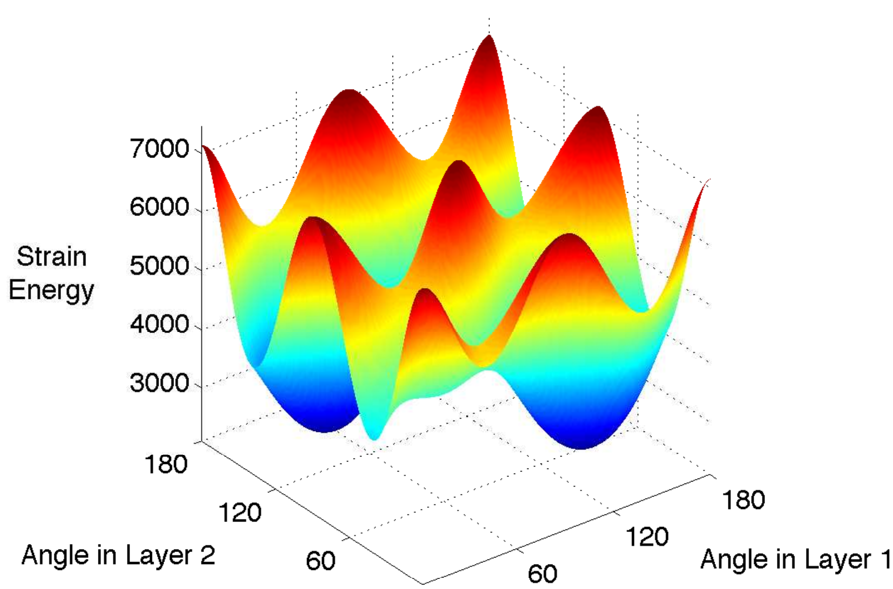

Figure 3: B

FMINCON

For this series of testing we used the MATLAB function FMINCON, which is useful in finding minima of a constrained non-linear multivariable function. In our case the objective function was given by the strain-energy-function W. As design variables we used the angles θ_i and the thicknesses t_i, with the following constraints:

$0° \leq \theta_i \leq 180°$,
$10^{-3} \leq t_i \leq 10^{-2}$
$\sum t_i = t_{\text{tot}}$

As materials we used graphite-epoxy and aluminium.

Case 1: 1-Layer Graphite Epoxy

This basic case was chosen to demonstrate the behavior of the material when we apply a single force in the direction x or y to only one layer. With the boundary conditions for strain forces and moments we get the following results:

Load conditions:
$N = [\ 100 \quad 0 \quad 0\ ]^T$
$M = [\ 0 \quad 0 \quad 0\ ]^T$
Optimal solution:
$\hat{\theta} = 0°, 180°$
$\hat{t} = t_{\text{tot}}$

Load conditions:
$N = [\ 100 \quad 100 \quad 0\ ]^T$
$M = [\ 0 \quad 0 \quad 0\ ]^T$
Optimal solution:
$\hat{t} = t_{\text{tot}}$

Load conditions:
$N = [\ 100 \quad 100 \quad 100\ ]^T$
$M = [\ 0 \quad 0 \quad 0\ ]^T$
Optimal solution:
$\hat{\theta} = 45°$

$\hat{t} = t_{\text{tot}}$

We find that the fibers are aligned in the direction of the force, which shows that the algorithm returns expected results. As we only consider a single layer, the optimal thickness of the layer is the total thickness.

Case 2: 2-Layers Graphite Epoxy

We optimized material composed of two layers of the same material, graphite epoxy and applied equal normal forces N_x and N_y to the material but omitting shear forces. In this case, the optimal angles lie on two lines: $\theta_2 = \theta_1 + 90°$ and $\theta_2 = \theta_1 - 90°$ which means that the fibers of the two layers show an orthogonal structure. We also see, that the layers are of equal thickness.

Load conditions:
$N = [\ 100 \quad 100 \quad 0\ ]^T$
$M = [\ 0 \quad 0 \quad 0\ ]^T$
Optimal solution:
$\hat{\theta}_1 = 45°$
$\hat{\theta}_2 = 45°$
$\hat{t}_1 = 0.5 \cdot 10^{-2}$
$\hat{t}_2 = 0.5 \cdot 10^{-2}$

Case 3: 2-Layers Aluminium and Graphite-Epoxy

In this case, we optimized material composed of two layers, using aluminium in the first layer and graphite-epoxy in the second layer. Since aluminium is an isotropic material the fiber orientation is irrelevant and hence the design variables are the fiber orientation θ_2 of Graphite-Epoxy and the thicknesses t_1 and t_2 of the layers.
(i)
Load conditions:
$N = [\ 100 \quad 0 \quad 0\ ]^T$
$M = [\ 0 \quad 0 \quad 0\ ]^T$
Optimal solution:
$\hat{\theta}_2 = 45°$
$\hat{t}_1 = 0.1 \cdot 10^{-2}$

$\hat{t}_2 = 0.9 \cdot 10^{-2}$

(ii)

Load conditions:

$N = [\ 0 \quad 100 \quad 0\ ]^T$

$M = [\ 0 \quad 0 \quad 0\ ]^T$

Optimal solution:

$\hat{\theta}_2 = 90°$

$\hat{t}_1 = 0.1 \cdot 10^{-2}$

$\hat{t}_2 = 0.9 \cdot 10^{-2}$

(iii)

Load conditions:

$N = [\ 0 \quad 0 \quad 100\ ]^T$

$M = [\ 0 \quad 0 \quad 0\ ]^T$

Optimal solution:

$\hat{\theta}_2 = 45°$

$\hat{t}_1 = 0.9 \cdot 10^{-2}$

$\hat{t}_2 = 0.1 \cdot 10^{-2}$

These results show, that applying just forces in the x and y direction respectively favor orthotropic material, aluminium, with the fibres aligned in the direction of the force. When we apply only shear forces, we get that isotropic material, graphite-epoxy, performs better. In all these test results we should keep in mind, that we did not take the overall mass of the structure into account.

(iv)

Load conditions:

$N = [\ 100 \quad 0 \quad 100\ ]^T$

$M = [\ 0 \quad 0 \quad 0\ ]^T$

Optimal solutions:

$\hat{t}_1 = 0.75 \cdot 10^{-2}$

$\hat{t}_2 = 0.25 \cdot 10^{-2}$

In this case, we apply forces in the x direction and shear forces in the x and y direction and the results show that the structure is optimal with 75% of aluminium and 25% of graphite epoxy. With respect to the angle no outstanding optimum was found due to the high participation of aluminium, which is an isotropic material and responds equally to the forces in x and y direction.

Because of the specific structure (local and global minima) of U the result of the optimization process is highly dependent on the starting point - this is one of the disadvantages of using this algorithm.

In the figure 4 (below), you can see that the objective function for this example is composed of two valleys separated by a hill. As a consequence, when using a gradient based algorithm, starting from the left side of the hill, implies to end up in one of the minima of the left side valley. The analogous occurs when starting from the right side of the hill. There exist even a region in the top of the hill (where the maxima of the function are) which can not be taken as a starting point for being stationary points.

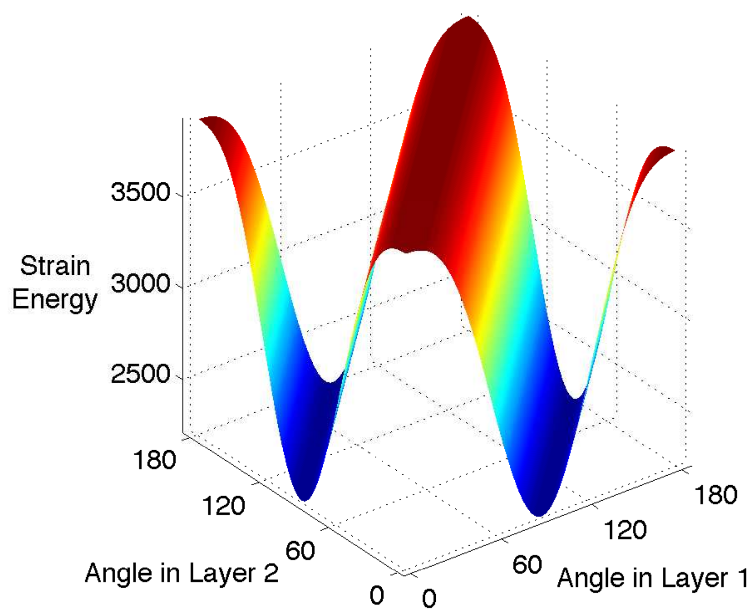

Figure 4: 2-Layers Graphite Epoxy

NSGA

Case 1

Problem description: The task is to minimize

- the total elastic strain energy

- the overall mass of the composite

with respect to

- the thicknesses of the layers,
- the fiber direction of each layer,
- the material of each layer and
- the total number of layers

while

- the total thickness of the composite is fixed
- the possible materials are
  - aluminium
  - steel
  - copper
  - graphite-epoxy

1. shear forces and moments are applied with varying values of loading forces from 1000 to 5000 units, that is,

$$\begin{cases} N^{(1)} & = \begin{bmatrix} 0 & 0 & 1000 \end{bmatrix}^T \\ M^{(1)} & = \begin{bmatrix} 0 & 0 & 1000 \end{bmatrix}^T \end{cases}, \quad \ldots, \quad \begin{cases} N^{(5)} & = \begin{bmatrix} 0 & 0 & 5000 \end{bmatrix}^T \\ M^{(5)} & = \begin{bmatrix} 0 & 0 & 5000 \end{bmatrix}^T \end{cases}.$$

2. normal forces in x-direction are applied with varying values from 1000 to 5000 units, that is,

$$N^{(1)} = \begin{bmatrix} 1000 & 0 & 0 \end{bmatrix}^T, N^{(2)} = \begin{bmatrix} 2000 & 0 & 0 \end{bmatrix}^T, \ldots, N^{(5)} = \begin{bmatrix} 5000 & 0 & 0 \end{bmatrix}^T.$$

Results:
We used NSGA to obtain the as pareto fronts for the total mass and total elastic strain energy. For fixed shear forces and moments, the pareto fronts for different number of layers are plotted in the same figure. It is seen that the optimal number of layers depend on the shear forces applied.

Consider a situation where a composite of a certain mass, say $2.25kg$, is optimized with respect to shear forces and moments of 5000 units. The optimal composite laminate in such case consists of four layers. The graphite epoxy covers the most part of the composite with 90% of the whole thickness. Since graphite epoxy is orthotropic material the fiber directions are approximately 135 and 45 degrees. This phenomena was seen in earlier cases where gradient based methods were applied. For the isotropic materials, aluminum and steel, the response to the forces applied does not depend on the angle.

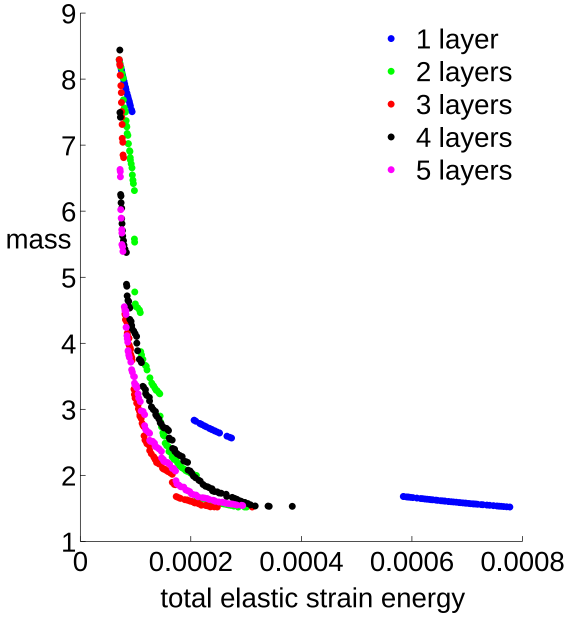

Figure 5: 1000 units

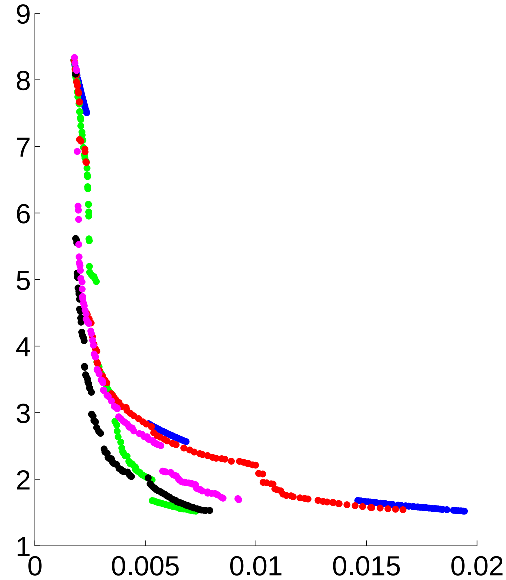

Figure 6: 5000 units

For a mass of 2.25 and total thickness of 1, using 4 layers gives the best result, and the design variables are as follows:

# Layer	1	2	3	4
Material	Gr-Ep	Gr-Ep	Al	steel
Thickness	75%	15%	1%	9%
Fiber angle	133°	46°	-	-

5 Conclusions

The design of a proper composite laminate is a delicate process that leads to a complicated mixed variable optimization problem with multiple objectives. Some simple spesific tasks were solved successfully with a gradient based method using a Matlab's built-in function *fmincon*. The total strain energy was minimized with respect to the fiber orientations and the layer thicknesses while the materials and number of layers were kept fixed. It was shown that, in case of shear forces and moments, isotropic materials perform better, while for single normal forces, orthotropic materials perform better.

In order to be able to tackle more complicated tasks the non-dominant sorting genetic algorithm (NSGA) was implemented because of its suitability to solve mixed variable multiobjective problems. The total elastic strain energy and the overall mass of the laminate was minimized with respect to the number of layers and the material properties (material, fiber orientations) therein. The total thickness of the laminate was fixed.

It was seen that introducing only normal forces the optimal laminate favoured simple structures; one layer of anisotropic material. In case of both shear forces and moments the number of layers in the optimal structure depend on the applied amount of force and moment.

For shear forces, isotropic material performs better than orthotropic, while for normal forces, orthotropic material is more convenient in the design of the structure. In the design of an optimal structure, the load condition is determinant in deciding the amount of orthotropic and isotropic material that will be used.

A Material properties

Table 1: The properties of the materials used.

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	ρ (kg/m^3)
Aluminum	73.1	73.1	27.5	0.33	2700
Steel	207.0	207.0	77.5	0.29	7900
Copper	124.1	124.1	44.1	0.33	8920
Gr.-Ep(AS4)	126.0	11.0	6.6	0.28	1600

References

[1] `http://www.iitk.ac.in/kangal/codes.shtml`,
cited at 22.8.2008.

[2] Melanie Mitchell, *An Introduction to Genetic Algorithms*, MIT Press, 1996

[3] N. Srinivas, K. Deb, *Multiobjective Function Optimization Using Nondominated Sorting Genetic Algorithms*, Evolutionary Computation, MIT Press, 1995

[4] Robert M. Jones, *Mechanics of Composite Materials*, Taylor & Francis, 1998