Factory Fires

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1 Introduction

In a factory where chipboards are manufactured fires can start due to the spontaneous ignition of sawdust piling on top of the hot press. If heat emitted from the press cannot diffuse fast enough through the sawdust, then dangerously high temperatures may result. The thicker the pile of sawdust, the more likely ignition will occur. In this report, we are going to determine the critical thickness of the sawdust layer for spontaneous ignition. First we formulate a mathematical model for this problem, which we simplify assuming that the temperature in the sawdust is in steady state. We try both analytical and numerical methods to obtain the solution, then we return to our original problem to find the critical thickness. In a bifurcation analysis we further simplify the steady model and using some well known properties we can determine the critical value. In the last part of this report, we give the summary of our results and the limitations of our approach.

2 Mathematical model

Now we are going to formulate the mathematical model for our problem. The first law of thermodynamics states that the change (per unit time) of internal energy (potential and kinetic energy) equals the sum of supplied heat and power (work done), hence we have

$$\frac{dK}{dt} + \frac{dW}{dt} = Q + P$$

where K is the kinetic energy, W the potential energy, Q the supplied heat and P the work done (power). From this law we can get the following equation, see e.g. [1],

$$\rho\frac{d\varepsilon}{dt} = \mathrm{div}(TD) + \rho r - \mathrm{div}\mathbf{q}.$$

Since in our case the velocity $\mathbf{v} = \mathbf{0}$, we have the rate of deformation $D = 0$, hence we have

$$\rho\frac{d\varepsilon}{dt} = \rho r - \mathrm{div}\mathbf{q} \tag{1}$$

Here, $\rho\varepsilon = \rho\varepsilon(\mathbf{x}, t)$ is the internal energy density (will be specified later on), $\rho r = \rho r(\mathbf{x}, t)$ a given heat source and $\mathbf{q} = \mathbf{q}(\mathbf{x}, t)$ the heat flux vector; a constitutive equation for this flux will be given later.

In our problem, the sawdust is a thermally active medium which can be characterized by

(i) the assumption that the internal energy only depends on temperature, i.e.,

$$\varepsilon = \varepsilon(\theta)(= \varepsilon(\theta(\mathbf{x}, t))); \tag{2}$$

here, θ is the absolute temperature in Kelvin(K).

(ii) a constitutive equation for the heat flux; for this we take here Fourier's law, i.e.,

$$\mathbf{q} = -\lambda(\theta)\mathrm{grad}\theta, (q_i = \lambda\theta_i). \tag{3}$$

Hence, the heat flux is proportional to the gradient of the temperature; the coefficient of proportionality λ (in $N/Ksec$) is called heat conduction coefficient, and can still depend on temperature. In (3), $\lambda > 0$ is a scalar, we assume it is a constant; in the most general case λ is a 2-tensor, i.e., $q_i = \lambda_{ij}\theta_{,i}$. The restriction to (3) is valid only in isotropic media (to which we shall restrict ourselves here). A medium is isotropic if in any point of that medium the material properties are equal in all directions. Hence, an isotropic media exhibits no preferred directions.

According to (2) we have

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon}{d\theta}\frac{\partial\theta}{\partial t} = c(\theta)\frac{\partial\theta(\mathbf{x}, t)}{\partial t}, \tag{4}$$

where we have introduced

$$c(\theta) := \frac{d\varepsilon}{d\theta}; \tag{5}$$

where c is called the specific heat (c in J/Kkg; $J : Joule = Nm$). Substituting (3) and (4) into the energy equation (1), we obtain

$$\rho c(\theta)\frac{\partial\theta}{\partial t} = \rho r + \mathrm{div}(\lambda(\theta)\mathrm{grad}\theta). \tag{6}$$

Next let us obtain the heat source ρr, in our case the heat source should come from the reaction between the sawdust and the hot press. From equation (6), the dimension of $\rho c(\theta)\partial\theta/\partial t$ is $J/m^3 s$, so the dimension of the heat source ρr should also be $J/m^3 s$. Furthermore, heat source ρr is related to the concentration of sawdust $C = C(\mathsf{x}, t)$ (in kg/m^3) ($\rho r \sim C$). We will use the Arrhenius formula which gives the dependence of the rate constant k of the chemical reaction on the temperature T (in Kelvin) and activation energy E_a, i.e., $k = Ae^{-E_a/R\theta}$, so we can write our heat source ρr in the following form

$$\rho r = Q_0 C k = Q_0 C A e^{-E_a/R\theta} \tag{7}$$

where Q_0 is the heat of reaction and its dimension is J/kg. Substituting (7) into (6), we get

$$\rho c(\theta)\frac{\partial\theta}{\partial t} = Q_0 C k + \lambda\nabla^2\theta. \tag{8}$$

Hence (8) is the general equation for our problem.

To solve this nonlinear equation, we need to add boundary conditions at $x = 0$ and $x = \delta$ where δ is the thickness of the sawdust. We consider a sawdust layer of uniform thickness.

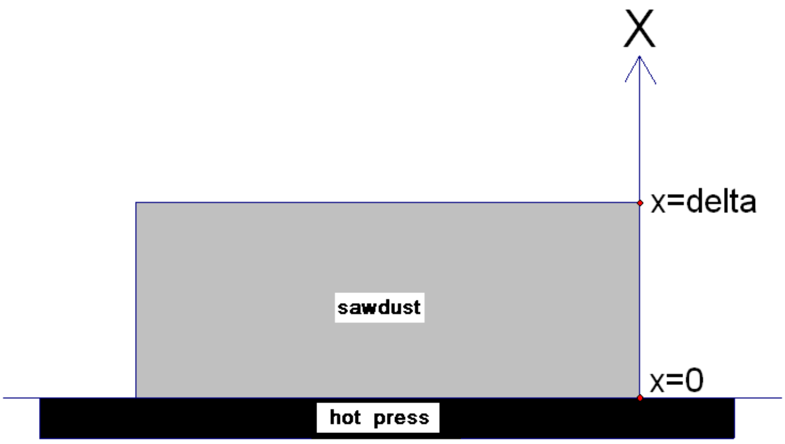

Figure 1: Model

At $x = 0$, the temperature of the sawdust equals the temperature of the hot press which is noted by θ_p, so we have

$$\theta(0) = \theta_p.$$

At $x = \delta$, the normal component of the heat flux vector at this boundary is proportional to the difference in temperature of the sawdust at the boundary and the ambient temperature θ_a, which is a constant. So we have the boundary condition

$$-\lambda\theta_x(\delta) = H(\theta(\delta) - \theta_a)$$

where H is positive scalar constant.

From the above discussion, we get our mathematical model for factory fire problem:

$$\rho c(\theta)\frac{\partial\theta}{\partial t} = Q_0CAe^{-E_a/R\theta} + \lambda\frac{\partial^2\theta}{\partial x^2}, \tag{9a}$$

$$\theta(0) = \theta_p, \tag{9b}$$

$$-\lambda\theta_x(\delta) = H(\theta(\delta) - \theta_a) \tag{9c}$$

In our report we take the parameter values in the mathematical model as follows

- $\lambda := 0.103\frac{J}{smK}$
- $E_a/R = Ta := 16220K$
- $\theta_p := 473.15K$
- $\theta_a := 298.15K$
- $Q_0CA = 7.1 \times 10^{17}\frac{J}{m^3}$
- $\frac{H}{\lambda} = 80m^{-1}$

3 Analytical solution

Now we have constructed a mathematical model in (9) which describes the variation of temperature $\theta(x,t)$ at time t and at a height x above the hot press. This is a time-dependent differential equation. However, before the spontaneous ignition of the sawdust, we can assume that equilibrium is reached and the temperature within the sawdust is steady, i.e. $\theta(x,t) = \theta(x)$. As a result, equation (9a) becomes

$$Q_0 C A e^{-E_a/R\theta} + \lambda \frac{d^2\theta}{dx^2} = 0. \tag{10}$$

Our target is to solve equation (10) for $\theta(x)$ where $x \in [0, \delta]$, together with the boundary conditions (9b) and (9c). The height of the sawdust δ will serve as a parameter and will be regarded as a fixed constant. When δ increases from zero and reaches the critical value δ_0, spontaneous ignition occurs. The temperature of the sawdust rises quickly and no equilibrium can be maintained. This means that we cannot find a steady-state solution of $\theta(x)$ satisfying equation (10). In other words, when $\delta = \delta_0$, bifurcation occurs and there is no solution for equation (10).

So now we start to solve equation (10). Rearranging terms, we have

$$\frac{d^2\theta}{dx^2} + K e^{-T_a/\theta} = 0, \tag{11}$$

where $K = Q_0 C A/\lambda$ and $T_a = E_a/R$ is the activation temperature for the chemical reaction, which is much higher than θ. Thus the exponential term is quite small, and we would like to approximate it in the form of e^θ since it would then allow us to obtain an analytical solution. Because we know that the temperature of the sawdust will be comparable to that of the hot press, we expand $1/\theta$ around $\theta = \theta_p$ by its Taylor series up to the first order and obtain

$$\frac{1}{\theta} \approx \frac{1}{\theta_p} + (\theta - \theta_p)\left(\frac{-1}{\theta_p^2}\right) = \frac{2}{\theta_p} - \frac{\theta}{\theta_p^2}$$

Thus equation (11) becomes

$$\frac{d^2\theta}{dx^2} + K e^{T_a(\theta - 2\theta_p)/\theta_p^2} = 0. \tag{12}$$

To simplify the expression, we change the variable from θ to ϕ, according to

$$\phi = \frac{T_a}{\theta_p^2}(\theta - 2\theta_p) \tag{13}$$

and we finally obtain

$$\phi_{xx} + K_1 e^{\phi} = 0 \tag{14}$$

where $K_1 = T_a K/\theta_p^2$. Also, we write $\frac{d\phi}{dx} = \phi_x$ and $\frac{d^2\phi}{dx^2} = \phi_{xx}$. The two boundary conditions become

$$\phi(0) = \phi_p \tag{15a}$$

$$\lambda \phi_x(\delta) = -H(\phi(\delta) - \phi_a) \tag{15b}$$

where ϕ_p and ϕ_a are obtained by substituting θ_p and θ_a respectively into (13). Equation (14) only depends implicitly on x, so we may directly integrate it with respect to ϕ and obtain

$$\begin{aligned}
\int \phi_{xx} d\phi + \int K_1 e^{\phi} d\phi &= \text{Constant} \\
\int \frac{d\phi_x}{dx} \phi_x dx + K_1 e^{\phi} &= \text{Constant} \\
\int \phi_x d\phi_x + K_1 e^{\phi} &= \text{Constant} \\
\frac{1}{2}\phi_x^2 + K_1 e^{\phi} &= \text{Constant} = C_1 \qquad (16)
\end{aligned}$$

where C_1 is an integration constant. We notice that at the point x_0 where the temperature θ_0 is maximum, i.e., at $\theta(x_0) = \theta_0$, we have $\theta_x(x_0) = 0$. It follows that $\phi_x(x_0) = 0$, so we may express C_1 as

$$C_1 = K_1 e^{\phi_0}. \qquad (17)$$

Substituting (17) into (16) gives

$$\phi_x = \pm\sqrt{2K_1\left(e^{\phi_0} - e^{\phi}\right)}$$

This equation can be integrated easily by separating the variables ϕ and x. We have

$$-2\text{Artanh}\sqrt{1 - e^{\phi-\phi_0}} = \pm\sqrt{2C_1}x + C_2, \qquad (18)$$

where C_2 is another integration constant, and Artanh(z) is the inverse of tanh(z) [1]. At $x = x_0$, we have

$$C_2 = \mp\sqrt{2C_1}x_0. \qquad (19)$$

Since Artanh$(x) \geq 0$ for $x \geq 0$, by substituting (19) into (18), we have

$$\text{Artanh}\sqrt{1 - e^{\phi-\phi_0}} = \sqrt{\frac{K_1 e^{\phi_0}}{2}}|x - x_0|. \qquad (20)$$

So the solution to (14) is given by

$$\phi(x) = \phi_0 - 2\ln\left[\cosh\left(\sqrt{\frac{K_1 e^{\phi_0}}{2}}|x - x_0|\right)\right]. \qquad (21)$$

To determine the unique solution, we have to solve ϕ_0 and x_0 which satisfy the two boundary conditions (15a) and (15b). Substituting $x = 0$ into (21) and compare with the first boundary condition (15a), we have the first equation

$$\phi_0 - 2\ln\left[\cosh\left(\sqrt{\frac{K_1 e^{\phi_0}}{2}}x_0\right)\right] = \phi_p. \qquad (22)$$

[1] Artanh(z) is sometimes written as Arctanh(z)

Next we differentiate the solution (21) with respect to x. Evaluate the derivative at $x = \delta$ and compare with the second boundary condition (15b), we have the second equation

$$\tanh[\sqrt{\frac{K_1 e^{\phi_0}}{2}}(\delta - x_0)] = \frac{H}{\lambda\sqrt{2K_1 e^{\phi_0}}}\left[\phi_0 - \phi_a - 2\ln\left(\cosh\left[\sqrt{\frac{K_1 e^{\phi_0}}{2}}(\delta - x_0)\right]\right)\right] \quad (23)$$

So we have two simultaneous equations (22) and (23) for the two unknowns ϕ_0 and x_0. By tuning the parameter δ, we obtain different solutions of ϕ_0 and x_0. We solve the system by a graphical method, i.e., we plot the graphs of equations (22) and (23) on the $x_0 - \theta_0$ plane and look for the point of intersection, which is the solution to the system. Note that we choose to show θ_0 instead of ϕ_0 since it gives a better physical picture.

To illustrate the solutions, we choose a particular instance where the temperature of the surrounding air is $\theta_a = 25^0C$ and the temperature of the hot press is $\theta_p = 200^0C$. For $\delta = 37cm$, we obtain two solutions for ϕ_0 and x_0, corresponding to two possible equilibrium conditions. However, one of these corresponds to a negative value of x_0, which is physically impossible. When we increase δ such that $\delta = 38.33cm$, the two graphs touche each other and we only have one solution for ϕ_0 and x_0. This corresponds to the point of spontaneous ignition where $\delta = \delta_0$, since beyond that we will have no solution at all. For instance, when $\delta = 39cm$, the two graphs never intersect and no equilibrium can be attained. So in this case the critical value of the thickness of sawdust for spontaneous ignition is $\delta_0 = 38.33cm$.

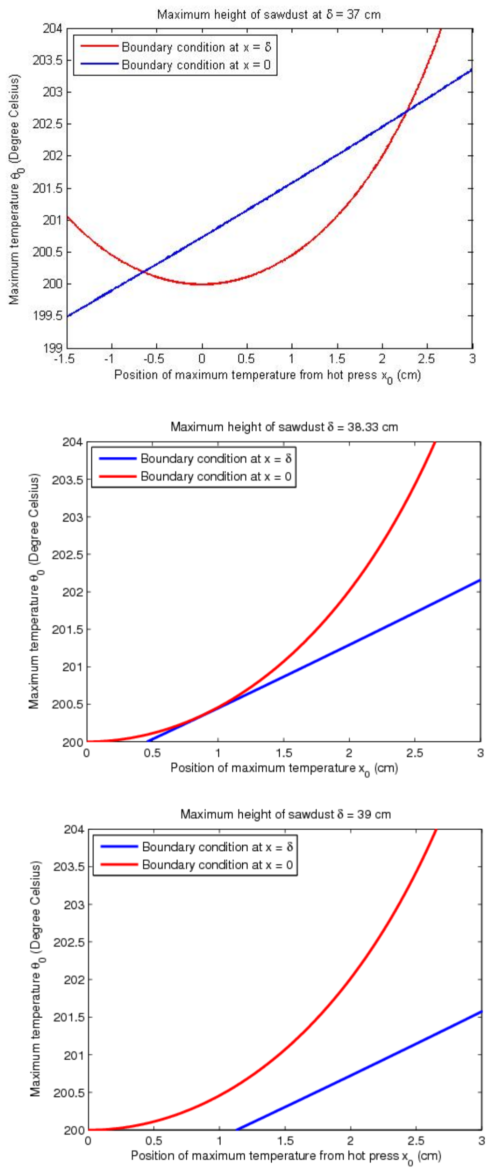

Figure 2: Plots of the boundary conditions when δ=$37cm, 38.33cm$ and $39cm$.

4 Numerical solution

To get a numerical solution, we simplify our mathematical model as follows:

$$\nabla^2\theta + \alpha e^{\beta\theta} = 0, \tag{24a}$$

$$\theta(0) = \theta_p, \tag{24b}$$

$$\lambda\theta'(\delta) = -H(\theta(\delta) - \theta_a) \tag{24c}$$

where $\alpha = Q_0 CA/(\lambda e^{\frac{2T_a}{\theta_p}})$ and $\beta = T_a/\theta_P^2$. Scaling x to x^* ($x^* \in [0,1]$), $x^* := \frac{x}{\delta}$, then our model becomes:

$$\frac{d^2\theta}{dx^{*2}} + \delta^2\alpha e^{\beta\theta} = 0, \tag{25a}$$

$$\theta(0) = \theta_p, \tag{25b}$$

$$\frac{\lambda}{\delta}\theta'(1) = -H(\theta(1) - \theta_a) \tag{25c}$$

In order to compute a numerical solution of our model (25), we introduce the grid

$$x_j^* = (j-1)\Delta x^*, j = 1, 2, \ldots, M,$$

with $\Delta x^* := 1/(M-1)$ the grid size. We denote the numerical approximation of $\theta(x_j^*)$ by θ_j and use the central difference scheme.

From boundary condition (11b) we get

$$\theta_1 = \theta_p \tag{26}$$

and from boundary condition (11c), we obtain

$$\frac{\lambda}{\delta}\frac{\theta_{M+1} - \theta_{M-1}}{2\Delta x^*} = -H(\theta_M - \theta_a)$$

so, we have

$$\theta_{M+1} = \frac{-2H\Delta x^*\delta(\theta_M - \theta_a)}{\lambda} + \theta_{M-1}. \tag{27}$$

Next, we apply the central difference scheme to (25a), with (26) and (27), and obtain:
for $j = 2$,

$$\frac{\theta_p - 2\theta_2 + \theta_3}{(\Delta x^*)^2} + \delta^2\alpha e^{\beta\theta_2} = 0,$$

for $j = 3, \ldots, M-1$,

$$\frac{\theta_{j-1} - 2\theta_j + \theta_{j+1}}{(\Delta x^*)^2} + \delta^2\alpha e^{\beta\theta_j} = 0,$$

for $j = M$,

$$\frac{\theta_{M-1} - 2\theta_M + \theta_{M+1}}{(\Delta x^*)^2} + \delta^2 \alpha e^{\beta\theta_M} = 0.$$

Then we get the following system of equations

$$A\Theta + f(\Theta) = \mathbf{0} \tag{28}$$

where $\Theta := (\theta_2, \theta_3, \ldots, \theta_M)^T$ is the vector of unknowns and where the matrix A and the vector function $f(\Theta)$ are defined by

$$A = \frac{1}{(\Delta x^*)^2} \begin{pmatrix} -2 & 1 & \cdots & \cdots & \cdots & \cdots \\ 1 & -2 & 1 & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & 1 & -2 & 1 \\ \cdots & \cdots & \cdots & \cdots & 2 & -\frac{2H\Delta x^* \delta}{\lambda} - 2 \end{pmatrix}.$$

and

$$f(\Theta) = \begin{pmatrix} \delta^2 \alpha e^{\beta\theta_2} + \frac{\theta_p}{(\Delta x^*)^2} \\ \delta^2 \alpha e^{\beta\theta_3} \\ \vdots \\ \delta^2 \alpha e^{\beta\theta_{M-1}} \\ \delta^2 \alpha e^{\beta\theta_M} + \frac{2\theta_a H\delta}{\lambda\Delta x^*} \end{pmatrix},$$

respectively.

Next, we use Newton iteration to compute a numerical solution of our mathematical model. In the corresponding MATLAB-script, we first compute an initial solution (matrix A cannot be singular), then we use Newton iteration and finally we use bisection to get the best value of thickness. see Appendix (6.2). After running the Matlab script, we find for the critical thickness $38.3236cm$. To conclude, we add the picture below to show the variation of temperature in the sawdust.

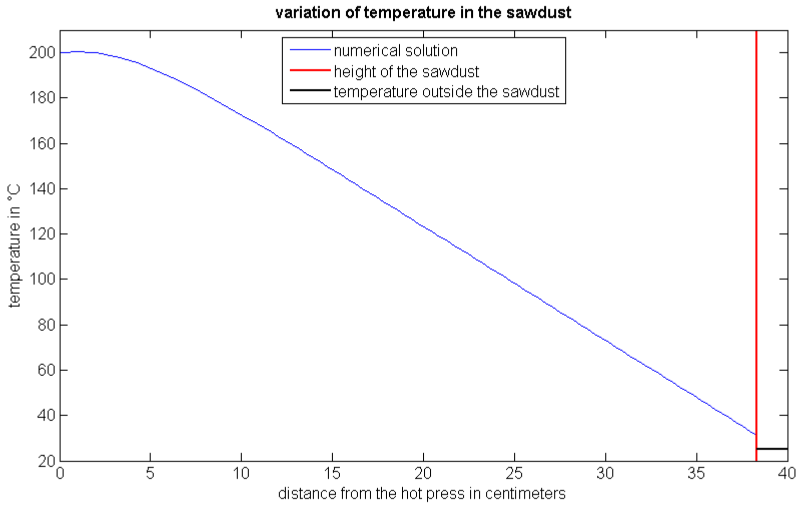

Figure 3: Variation of the temperature in the sawdust

5 Bifurcation analysis

To demonstrate the phenomenon of bifurcation, we consider the following lumped model for heat conduction in the sawdust layer, i.e.,

$$\frac{d\theta}{dt} = -\theta + \Lambda e^{\theta},$$

where Λ depends on the thickness of the sawdust and e^{θ} stands for the reaction term.

Let us consider equilibrium solutions, i.e., $\frac{d\theta}{dt} = 0$. Therefore

$$\frac{\theta}{\Lambda} = e^{\Lambda}.$$

Let us discuss the behaviour of the two curves involved in the equation above , i.e., $y = \frac{\theta}{\Lambda}$ and $y = e^{\Lambda}$.

- For sufficiently small Λ:

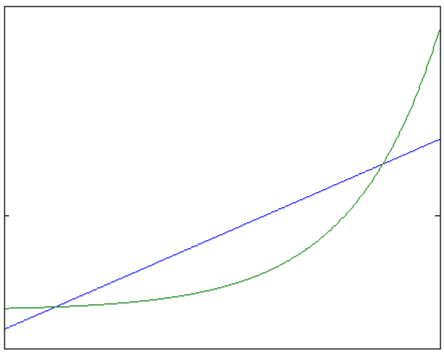

Figure 4: Two intersection points

- for sufficiently large Λ

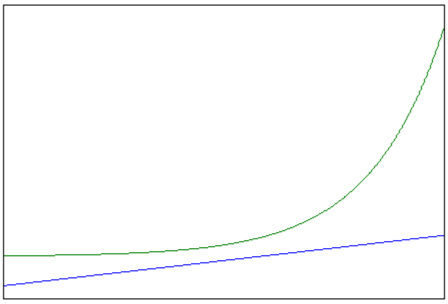

Figure 5: No points in common

- for the critical value of $\Lambda = \Lambda^c$

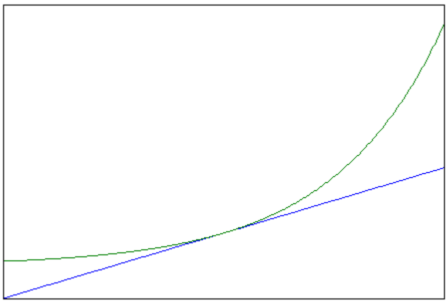

Figure 6: One point in common

We can find the critical Λ^c and the critical equilibrium temperature θ^c. From $\frac{\theta^c}{\Lambda^c} = e^{\theta^c}$ and the property that both curves are tangent, i.e., $\frac{1}{\Lambda^c} = e^{\theta^c}$ we have

$$\theta^c = 1 \qquad \text{and} \qquad \Lambda^c = e^{-1}.$$

Let $\theta_1 < \theta_2$ denote the intersection points of the curves (pic4) and let θ_0 denote the initial temperature.

- if $\theta_0 \in [0, \theta_1]$ then $\frac{d\theta}{dt} > 0$ and $\theta(t) \longrightarrow \theta_1$ as $t \longrightarrow \infty$
- if $\theta_0 \in [\theta_1, \theta_2]$ then $\frac{d\theta}{dt} < 0$ and $\theta(t) \longrightarrow \theta_1$ as $t \longrightarrow \infty$
- if $\theta_0 \in [\theta_2, \infty]$ then $\frac{d\theta}{dt} > 0$ and the temperature $\theta(t) \longrightarrow \infty$ as $t \longrightarrow \infty$

The solution $\theta(t) \equiv \theta_1$ is the stable equilibrium temperature while $\theta(t) \equiv \theta_2$ is the unstable equilibrium temperature. If Λ is large ($\Lambda > \Lambda^c$), then there are no equilibrium points. It means that $\frac{d\theta}{dt} = -\theta + \Lambda e^u > 0$, and the temperature $\theta(t)$ tends to infinity as $t \to \infty$. To avoid ignition the temperature may not exceed θ_2

In the bifurcation analysis we simplify the model and using some well known properties we can get the critical value. We can observe that if θ_0 exceeds θ_2 then the temperature goes up and ignition occurs. If $\Lambda < \Lambda^c$ then the initial temperature θ_0 should be smaller than θ_2 to avoid ignition. If $\Lambda > \Lambda^c$ then for any initial temperature θ_0 we are not able to avoid ignition.

6 Conclusion and discussion

In this section we conclude our investigation in this project. First, we describe our approach to the problem and show the main results. Then, we discuss the major assumptions and limits in our model. Together with this discussion, we also suggest some possible extensions.

6.1 Summary of results

To sum up, we have solved the problem of spontaneous ignition of sawdust (which can lead to fires in chip factories) by first describing the situation in terms of a partial differential equation (PDE) and then investigating the behavior of the solutions of the PDE. We derived a criterion for the occurrence of spontaneous ignition, which is the central idea of the solution: spontaneous ignition occurs when heat equilibrium cannot be maintained, in other words, we can no longer obtain a stationary (or time-independent) solution for the PDE.

This criterion reduces the problem to a simpler one in which we look for the solution of a time-independent differential equation. We study two approaches, viz. the analytical and the numerical solution method. In the analytical approach, we need to approximate the equation by a Taylor expansion. This assumes that the temperature within the sawdust is close to the temperature of the hotpress, which we will justify later. In the numerical approach, we discretize the domain and use Newton's method to solve the discretized differential equation. Note that in this case, we do not need to apply any approximation.

In both cases, we find that when we take the temperature of hot press to be 200^oC and that of the surrounding air to be 25^oC, the critical height of the sawdust is about $38cm$. This is the height of the sawdust where spontaneous ignition occurs. Physically it means that the chemical reaction of the sawdust produces heat at rate faster than that of the heat loss to the surrounding air, and thus the temperature within the sawdust blows up.

Finally, as mentioned above, the occurrence of spontaneous ignition corresponds to a structural change of the solution of the differential equation while the parameter δ changes continuously. This means that bifurcation of solution happens, and a technique called bifurcation analysis is useful here. We discussed this approach in section (5).

6.2 Limitations of our approach

There are, however, some limitations to our solution. We can distinguish two aspects, viz. the mathematical model and the approximations that we have made in our calculations. First, there are a number of simplifications that we have imposed in our mathematical model in order to describe the situation in a parsimonious yet practical way. They are listed as follows:

- We only consider a one-dimensional problem where the interest is in the temperature variation along the height of the sawdust. In real situations, heat diffusion occurs in three dimensions.

- We have assumed that the height of sawdust is uniform throughout the surface. This is reasonable if we are considering a one-dimensional problem, but it will be more general to assume that it varies with location. In our result, the critical height of the sawdust is about $38cm$, and it may not be likely that the distribution of sawdust is even. A practical approach is to assume that the sawdust is more concentrated in some locations and decreases smoothly around that point.
- Similar to the point above, we have assumed that the concentration of sawdust is constant.

Second, there are some limitations to our solutions since we have made approximations in obtaining them. The limitation is on our analytical solution, and theoretically the numerical solution could be as accurate as possible. For the analytical solution, we have assumed that the expansion of the Taylor series for the function $1/\theta$ is a good approximation. We mentioned that it would be correct if the temperature of the sawdust is close to that of the hot press, but clearly this is not the case for points that are far away from it. However, since we are mainly interested in the entire variation of the temperature within the sawdust, this will not affect much on our final solution for the critical height δ. Instead, this will affect the location of the maximum temperature and its value. For this, it is reasonable to assume that the point with maximum temperature is very close to the hot press. As a result, the Taylor expansion is good at the region close to the hot press, which is the region that plays the most significant role.

Appendix

Matlab script for numerical solution

```
function theta=fireApp(M,delta,stepnum)
%Create the Matrix A with M steps for high delta
%Parameter



lamda=0.103;
r=80;
W=1;
Q0=7.1e017;
Ta=16220;
thetaPress=473.15;
b=Ta/thetaPress^2;
K=Q0*W*exp(-2*Ta/thetaPress)/lamda;

%step size
h=1/(M-1);

%right side
xi=-473.15/h^2;
eta=-596.3*r*delta/h;
rH=[xi;zeros(M-3,1);eta];

%create matrix A
e=ones(M-1,1);
A=spdiags([e -2*e e], -1:1, M-1, M-1);
%right boundary conditions
A(M-1,M-2)=2;
A(M-1,M-1)=-2-2*r*h*delta;

A=A/h^2;

%newton operation
%theta=ones(M-1,1);
%thetaStart=ones(M-1,1);
%v=ones(M-1,1);
%initial guess
v=A\(-(K*delta^2*ones(M-1,1)-rH));
thetaStart=A\(-(K*delta^2*exp(b*v)-rH));


epsilon=1.0e-009;
theta=thetaStart;
tol=1;

%newton-step
k=0;
while tol>epsilon && (k<stepnum)
    %function A*theta+K*delta^2*exp(b*theta)-rH
    p=A*theta+(K*delta^2*exp(b*theta)-rH);
    %derivative of function
    a=A+diag(K*delta^2*b*exp(b*theta));
    %newton-step
    J=a\p;
    theta=theta-J;
    %norm-check
    tol=norm(J,2);
    k=k+1;
end

%include the left boundary condition
theta=[473.15;theta];

%find max
%[maxt,I]=max(theta);

%generate grid points
figure(2);
x=0:delta/(M-1):delta;
%plot solution scaled in kelvin and centimeters
plot(x*100,theta-273.15,[delta*100,delta*100],[20,250],'r',[delta*100,45],[25,25])...
    ,xlabel('distance from the hot press in centimeters'),ylabel('temperature in kelvin'),title('variation of temperature in the sawdust');
h=legend('numerical solution','height of the sawdust','temperature outside the sawdust');
legend(h,'Location','North');
```

References

[1] A.A.F. van de Ven, *FIELD THEORY FOR CONTINUOUS MEDIA*. Chapter 4 (2004)