Mathematical Modeling of the Pumping Kite Wind Generator:
Optimization of the Power Output

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Abstract

This problem has arisen as a logical consequence of the Kite Power Project (Tampere University of Technology, Project Leader Risto Silvennoinen) devoted to analytical modeling of the so-called pumping kite generator whose operating principle is to mechanically drive a ground based electric generator using tethered kites. The main purpose of the study was to estimate the mechanical energy output of the pumping kite wind generator. The analytical mathematical model based on the refined crosswind motion law in the case of equilibrium motion of the kite was constructed. A simple approximate formula for the mean mechanical power generated by the deploying kite was obtained. We propose to apply the obtained analytical mathematical model for the constrained structural optimization of the pumping kite generator maximizing its power output by adjusting the key parameters of the kite generator.
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1 Introduction

About 80% of the world electric energy is produced from fossil source. Among the various problems associated with using fossil fuel for the power generation, environmental pollution (which consequently results in drastic climate changes) is of utmost concerns \[10\]. Thus, there is the need for exploring renewable source of energy. One of the latest idea in this direction is the Kite Wind Generator (see, e.g., patents \[11, 13, 14\] and papers \[2, 4, 8, 6\]).

The kite wind generator concept’s operating principle is to mechanically drive a ground-based electric generator using a tethered kite, instead of attempting to locate a wind turbine system at high altitudes. On the ground-station the lower portion of the tether is wound around a drum connected to the generator. Energy is extracted from high altitude by letting the kite fly at a lying-eight orbit with high crosswind speed. During the fast crosswind motion the kite develops a large pulling force, and thus the generator generates electricity while the kite pulls the tether out of the ground-station. Then the kite is controlled so, that the pulling force is reduced, and the lower part of the tether is wound back onto the drum using the generator as a motor. This cycle is repeated, and thus the system is called a pumping kite generator. It should be emphasized that the requirement for energy generation with this system is that the kite dynamics must be controlled to get large and small pulling force alternately \[9, 11, 13, 14\].

![Figure 1: Principle of operation of a kite wind generator.](image)

2 Problem Statement

The main aim of this project is not to evolve a new mathematical optimization model of a kite wind generator but to use the given mathematical model \[1\] for structural optimization of the pumping kite wind generator.

The aerodynamic forces controlling the motion of the kite are essentially the drag and lift forces. The lift force is the component of the net force acting on the kite perpendicular to the flow direction, and the component along the flow direction is referred to as the drag force. It is known that the lift and drag forces are given by the formulas

\[
L = \frac{1}{2} \rho_a A C_L |W_e|^2, \quad D = \frac{1}{2} \rho_a A C_D |W_e|^2.
\]

Here, \(W_e = W - v\) is the effective wind vector, \(W = Ve_1\) is the wind speed vector, \(V\) is the wind speed, \(v\) is the kite velocity, \(\rho_a\) is the density of air, \(A\) is the characteristic area of the
kite, $C_L$ and $C_D$ are the lift and drag coefficients.

In this work, we will adapt the formula for the mean mechanical power output of a kite wind generator obtained in [1]. The mean mechanical power is given by the following formula:

$$P_M = \frac{1}{2} \rho a A C_L V^3 \cos^3 \vartheta k_0 k_*.$$  \hspace{1cm} (1)

Here, $\vartheta$ is the mean angle of inclination of the tether, the coefficients $k_0$ and $k_*$ are given by

$$k_0 = \frac{4}{27} G_e \sqrt{1 + G_e^2},$$

$$k_* = 1 - \frac{2}{3} \frac{\langle F_{gra}^r \rangle + \langle w^r \rangle}{\rho_a A C_L k_0 V \cos \vartheta}.$$

The coefficient $k_*$ gives the correction on weight of the kite and the tether that may be found to be

$$\langle F_{gra}^r \rangle = mg \sin \vartheta, \quad \langle w^r \rangle = \mu rg \sin \vartheta,$$

where $m$ is the mass of the kite and $\mu$ is the linear density of the tether.

The linear density of the tether is given by the formula

$$\mu = \rho_t \frac{\pi d^2}{4},$$

where $\rho_t$ is the density of the tether’s material, $d$ is the diameter of the tether.

The parameter $G_e$, describing the aerodynamic efficiency of the system kite — tether, is given by the formula [7]

$$G_e = \frac{C_L}{C_D + \frac{C_{\perp} r d}{4 A}},$$

where $C_{\perp}$ is the normal reaction coefficient, $r$ is the length of the tether.

For preliminary analytical considerations, we shall use the simplified formula

$$P_M = \frac{1}{2} \rho a A C_L V^3 \cos^3 \vartheta \frac{4}{27} G_e^2.$$  \hspace{1cm} (2)

We can estimate the ratio of electrical power output versus mechanical power output by using Williams assumptions (2006). If we suppose that the velocity of the kite is periodic

$$V_L = \Delta V_L \sin \frac{2\pi t}{T_p},$$  \hspace{1cm} (3)

where $T_p$ is the period of oscillations of the kite; and assume that the line tension is periodic as well (with the same period)

$$T = T_0 + \Delta T \sin \frac{2\pi t}{T_p},$$  \hspace{1cm} (4)

the mean mechanical power output over a period is given by

$$P_m = \frac{1}{T_p} \int_0^{T_p} T V_L dt = \frac{\Delta V_L \Delta T}{2}. $$  \hspace{1cm} (5)
The maximum mechanical power output is given by the maximum of $T \times V_L$ that is $(T_0 + \Delta T)\Delta V_L$.

It shows that the ratio of effective mechanical power output versus maximum mechanical power output is

$$\chi_M = \frac{1}{2} \cdot \frac{1}{1 + \frac{T_0}{\Delta T}},$$

Additionally we have to take into account the loss of power in the gearbox (with ratio $\chi_{GB}$) and in the generator unit (with ratio $\chi_{G}$).

Hence the overall power ratio is

$$\chi = \chi_M \cdot \chi_{GB} \cdot \chi_{G}$$

$$P_{el} = \chi \cdot P_M$$

and a typical value of $\chi_M$ is about 0.2. If we assume that the gearbox output has $\chi_{GB} = 0.9$ and the generator output has $\chi_{G} = 0.95$, the kite ratio of maximum mechanical output versus electrical output is about $\chi = 0.171$.

We consider a mobile version of a kite wind generator (which will hence forth be referred to as Mobile Kite Generator). The Mobile Kite Generator is made up of a tether attached and wound around a drum. The drum is mounted on a tractor, with which the kite could be transported from one place to another.

Based on the analytical mathematical model [1], we propose justification to three of Prof. M. Diehl’s conjectures which were formulated at the First International Workshop on Modelling and Optimization of Power Generating Kites [5].

3 Optimization

In order to maximize the power output we want to choose all parameters (in the feasible region) in an optimal way. To solve this task we applied various local search optimization codes and implemented them in matlab.

Due to the nice behavior of the power output function we did not face any major problems with the optimization task but rather had to focus on finding the adequate limiting constraints.

4 Design Variables

In this section, we will choose appropriate design variables among all the parameters on which the power output depends. Recall that the design variables describe the configuration of a structure, element quantities like cross-sections, wall-thicknesses, shapes, etc., and physical properties of the material. In this project, we consider the following variables as the design parameters:

- $\vartheta$ is the angle of inclination of the tether in radian (rad),
- $r$ is the length of the tether rope in meters (m),
- $A$ is the area of the kite in square meters ($m^2$),
- $d$ is the diameter of the tether in meters (m).

In addition to the main design variables, the following are preassigned (constant) parameters that influence the power output of the Kite Generator Design.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lift Coefficient</td>
<td>$C_L$</td>
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</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_D$</td>
<td>0.04</td>
</tr>
<tr>
<td>Normal reaction coefficient</td>
<td>$C_\perp$</td>
<td>1.0</td>
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<tr>
<td>Friction coefficient</td>
<td>$C_\parallel$</td>
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<td>Line density of the tether</td>
<td>$\mu$</td>
<td>-</td>
</tr>
<tr>
<td>Area density of the kite</td>
<td>$\mu_k$</td>
<td>970 kg/m$^3$</td>
</tr>
<tr>
<td>Reference height/altitude</td>
<td>$z_0$</td>
<td>10 m</td>
</tr>
<tr>
<td>Reference wind velocity at $z_0$</td>
<td>$V_0$</td>
<td>8 m/s</td>
</tr>
<tr>
<td>Aerodynamical pressure at height 0</td>
<td>$p_0$</td>
<td>1013 Pa</td>
</tr>
<tr>
<td>Density of air</td>
<td>$\rho_a$</td>
<td>1.225 kg/m$^3$</td>
</tr>
</tbody>
</table>

Table 1: Fixed parameter values of the kite model

5 Design Constraints

In this section, we discuss the conditions which limit construction of a Kite Generator.

5.1 Safety Factor Constraint

We derive from Hooke’s law for elastic materials, the safety constraint on the kite’s rope

$$\frac{T}{\pi d^2} \leq \frac{1}{k} \sigma_0;$$  \hspace{1cm} (7)

where $k$ is the safety factor, $T$ is the pulling force, $\sigma_0$ is the tensile strength, $d$ is the diameter of the tether.

We assume that the tether is manufactured from the modern polymeric material Dyneema with the following mechanical properties: density $\rho_t = 970$ kg/m$^3$, elastic modulus $E = 89$ GPa, tensile strength $\sigma_0 = 2700$ MPa.

This constraint ensures that the tether does not break as a result of large pulling force.

5.2 Height Constraint

Currently in the United States, due to the Federal Aviation regulations (FAR), kiting and all high altitude activities are regulated to the height restriction of 500 feet or 153 meters. At the same time, in many developing countries, in particular, in many places in Africa, this strict height limitation can be altered. However, the height limitation of several hundreds of meters is an important limitation to be considered.

The height constraint can be formulated as follows:

$$r \sin \theta \leq h_*,$$

where $h_* = 300$ m. We have chosen the constraint of 300 meters height in view of the average thickness of the boundary layer.
5.3 Weight Constraint

A very strong wind results in a high pulling force, so the kite generator must be designed in such a way that the ground base system will not be pulled off the ground. Therefore, the weight of the ground station on which the generator is mounted will also be considered as a constraint.

Introducing the coefficient of safety \( k \), we can formulate the weight constraint as follows:

\[
T \leq \frac{1}{k} W_s,
\]

where \( W_s \) is the weight of the ground station.

This constraint is essential in the case of a mobile kite generator.

6 Diehl’s Conjectures

On the 1st International Workshop on Modeling and Optimization of Power Generating Kites "KITE-OPT 07", Prof. M. Diehl formulated the following so-called provocative claims:

- Kite lines will be far from vertical, kites fly at low angles
- Lift control will play a crucial role
- Kites will be ‘pumping’ rather than turning a carousel
- Plants will be built rather on sea than on land
- Connection to ground by only one line, not two or more

6.1 Low Angle of Inclination of the Tether

To analyze the first Diehl’s conjecture "Kite lines will be far from vertical, kites fly at low angles" we examine the optimal angle \( \vartheta \) that maximizes the power output for different surface friction coefficients \( \alpha \).

The first approach is based on the simplified analytic power model:

\[
P_M = \frac{1}{2} \rho_a AC_L V^3 \cos^3 \vartheta \frac{4}{27} G^2_c
\]

The wind speed \( V \) is given by the power law

\[
V(z) = v_0 \left( \frac{z}{z_0} \right)^\alpha,
\]

where

\[
z = r \sin \vartheta
\]

with the assumption that \( \rho_a \approx \rho(0) \) and all parameters but \( \vartheta \) are fixed, we obtain: \( P_M \) is maximal if the quantity \( \sin(\vartheta)^3 \alpha \cos^3 \vartheta \) is maximal.

We can conclude from this result that the optimal angle between the line and the ground varies from \( 17^\circ \) over sea to \( 32^\circ \) over urban areas.

Observe that in the case where all parameters can be chosen to optimize the power output the numerical optimization of the unrestricted model for the power output showed very similar results for the optimal angle.
6.2 Location on Sea versus Location on Land

To prove the first Diehl’s conjecture ”Plants will be built rather on sea than on land” we analyzed the maximal power output with respect to the surface friction coefficient $\alpha$. For a fixed line length and all other parameters chosen optimally we obtain the following result:

We can see that the maximal power output over a urban area is about 50% less than over the optimal sea location with low surface roughness.
6.3 Optimal Line Construction

We compare the one- and two-line kite-model generators in order to verify the fifth Prof. M. Diehl’s conjecture: ”Connection to ground by only one line, not two or more”.

To have the same level of safety for both kite generators we must have the same line cross section in the two cases, ensuring the same maximal tensile strength. Therefore the diameter of the tethers in the two-line construction must be $\delta = \frac{1}{\sqrt{2}} d$. Consequently, the effective glide ratio in the two-line case $G_e'$ becomes:

$$G_e' = \frac{C_L}{C_D} \frac{1}{1 + x\sqrt{2}}.$$  

in comparison to:

$$G_e = \frac{C_L}{C_D} \frac{1}{1 + x}.$$
with the dimensionless number \( x = \frac{dr}{4ACD} \).

Thus,

\[
G'_e = G_e \ast \frac{1 + x}{1 + x\sqrt{2}}.
\]

To verify the fifth Diehl’s conjecture, we compare the mechanical power output of the two constructions:

\[
\frac{P_M'}{P_M} = \frac{k'_0}{k_0} = \frac{1 + x}{1 + x\sqrt{2}} \left[ \frac{1 + x^2}{1 + x^2 + \left( \frac{C_L}{C_D} \right)^2} + \left( \frac{1 + x}{1 + x\sqrt{2}} \right)^2 \right] \frac{1}{(1 + x^2) \left( \frac{C_D}{C_L} \right)^2}
\]

\[
\lim_{x \to 0} \frac{P_M'}{P_M} = 1, \quad \lim_{x \to \infty} \frac{P_M'}{P_M} = \frac{1}{\sqrt{2}} \approx 0.7071
\]

Figure 6: Efficiency of 2-line construction

As the graph shows, the two-line construction is less efficient than the one-line construction. For a shape coefficient available with an acceptable kite, the power ratio lies between 0.75 and 0.95.
In order to design a functional and flexible wind generator that for example can be used for agricultural purposes, we considered a wind generator construction that can be mounted to the back of a tractor or similar vehicle. Since in agriculture power might be needed in outlying areas such a mobile variant is very useful.

Because our kite construction is now attached to the back of a tractor (with a mass of $M_T = 3500\, KG$ and a possible drum length of $2\, m$) and not rooted to the ground we have to impose restrictions on the area of the kite to prevent too high forces on the machinery that could damage it or compromise the safety. Therefore we restrict the area of the kite to be less than $20\, m^2$.

The following graph shows the maximum mechanical power output of the mobile kite wind generator operating at a height of approximately 300m.

With a kite of $15\, m^2$ that is designed for sport purposes we can obtain 96KW (located at a field area in the Netherlands). (A typical conventional wind generator used in agriculture e.g. for pumping fresh- and wastewater generates 100KW.)
8 Lake Kite Generator

At low altitudes the wind speed is highly influenced by the surface friction, therefore we like to design a construction that is located on a small island. For this construction we can drop the size restriction on our kite, that was a result of the limited weight of the tractor, for the simple reason that the drum is now connected to a ground station that is rooted to ground. Hence the kites we take into consideration now have an area of up to $150\,m^2$. We can see from the following power output profile that we can now gain up to 5.8MW electrical Power with a $150\,m^2$ kite.
In order to guarantee durability, economic viability and operating safety we still use a large drum with only one layer of Dyneema line that reduces abrasion effects.
9 Higher Altitudes

If we consider a project that is not restricted to a maximum height of 300m due to the federal aviation law, a much bigger project can be realized. That means much bigger kites and longer lines can be used to exploit wind energy in high altitudes. The following plot shows the maximum power that can be gained under these conditions.

![Figure 11: Power output in high altitudes](image)

As the graph shows we obtain a maximum electrical power output of 20MW at a height of approximately 6,5km (line length $r = 15km$) with a kite area of 1000m$^2$. Clearly the technical feasibility of such a huge project depends strongly on the capability to start and land the kite, but this is not of concern in this project.

10 Conclusion

Based on the power output formula for a kite generator obtained in [1], we have been able to propose optimal designs of a kite generator.

To achieve this, we have considered several limiting factors ranging from structural design constraints to environmental laws. We considered the effects, on the power output of a kite generator, whenever any of these constraints is relaxed. Figure 3, for example, showed that if the kite generator is mounted on the ground and located on an island or close to the sea, a considerably higher power could be produced. It was shown in Section 8 that if higher altitude kiting is allowed in certain places, a much more power could be realized from a kite generator.

We have also examined the effect of construction materials on the power output. We obtained results which are in agreement with the previous result of Prof. Diehl as stated in his conjectures.
It was shown in Figure 4, for example, that one-line construction is more efficient than a two-line construction. This is exactly the fifth Prof. Diehl’s conjecture.

It is worthwhile to comment here that we have examined how different constraints, on chosen design parameters, affect the power of the kite generator without considering transportation of the effective electrical power energy output to the final point of usage. For instance, in establishing Prof. Diehl’s 4th conjecture, we simply concluded that more power would be available at locations closer to the sea. Therefore more research still needs to be done, taking into consideration transportation of the power output from the source to the point of usage.
References


