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Mathematical Modelling of Press-section of Paper Machine

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Introduction

In the **press-section** of a paper machine, water is squeezed out of the **paper** web into the **felt** by applying a pressure pulse as the paper together with the felt passes through a press-nip, see Figure A. Even a small improvement in the efficiency may lead to a significant saving of the production time and energy consumption.
Limitation of experimental approach (high processing speed, small thickness of the paper etc) motivates mathematical modelling of the pressing process. In particular, we are interested in the qualitative properties of the solutions.

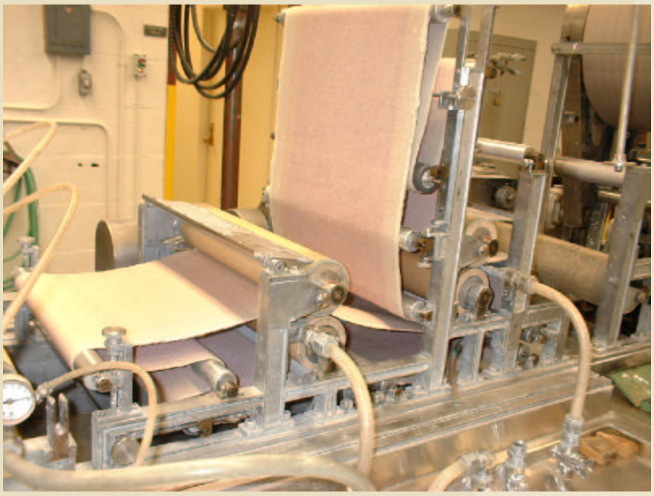

Fig. A Press section of paper machine.

Assumptions

- Both the paper web and the felt are considered as deformable, **perfectly elastic** porous media.
- Motivated by scaling arguments, only **transversal flow** is considered.
- The water and solid phase is assumed incompressible while air is assumed to be an ideal gas.

The model

The resulting mathematical model is the following system of partial differential equations:

$$\begin{cases} (us)_t = (c(u)k_{rw}(s)u_x)_x, \\ (\rho(u,t)u(1-s))_t = (\rho(u,t)c(u)k_{ra}(s)u_x)_x. \end{cases} \quad (1)$$

These equations describe the flow of water and air, respectively. Here u is void ration, s is saturation, ρ is pressure of air. Since the total pressure is assumed to be a known function of time t, ρ can be expressed as a function of u and t. k_{rw} and k_{ra} are relative water and air permeabilities, given as functions of s. Further

$$c(u) = k(u)p_s'(u),$$

where k and p_s are permeability and structural pressure given as functions of u. **Material co-ordinates** are used to fix the computational domain. Thus the scaled equations are considered in domain $Q^p \cup Q^f$, where subdomains

$$Q^p := (0, x_c) \times (0,1) \text{ and } Q^f = (x_c, 1) \times (0,1)$$

correspond to the paper and felt, successively and $x_c \in (0,1)$ corresponds to cross section. Different functions k and p_s result in different coefficients c in Q^p and Q^f.

Boundary and initial conditions

- $x = 0$, $x = 1$: Impermeable press rolls imply no flow condition on the external boundaries.
- $x = x_c$: The continuity of mass discharges and pressures is assumed on the internal boundary.
- $t = 0$: Layers are assumed to be initially undeformed.

Transformation

Introducing a new unknown $r = R(u,s)$ by a suitable regular transformation R we rewrite (1) as a **parabolic-hyperbolic system** in terms of u and r:

$$\begin{cases} u_t = a(r,u,u_x,t)u_{xx} + b(r,r_x,u,u_x,t), \\ r_t = c(r,u,u_x,t)r_x + d(r,u,u_x,t). \end{cases} \quad (2)$$

Assuming reasonably smooth coefficients (corresponding to a single-layer case) this system is known to have a unique local classical solution.

Computational results

An **explicit scheme** is used for time integration. For the space discretization, we use **finite volumes** for the parabolic equation and the **upwind scheme** for the hyperbolic equation. The results show good qualitative resemblance with observed effects, see Figure B.

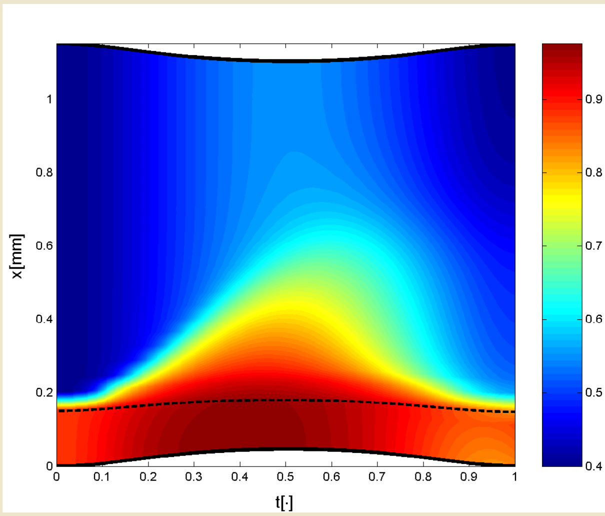

Fig. B Saturation in spatial coordinate.

References

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