Flame dynamics using the level set equation

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Introduction

Modern low emission combustion devices based on fuel lean premixed flames can display acoustic instabilities (combustion noise). The prediction of acoustic instabilities in **fully premixed Bunsen type burners** requires a transfer function that correlates the perturbations in the heat release rate and in the gas velocity. The flame is assumed to be an infinitely thin layer that separates the burnt from the unburnt gas. Assuming that the heat release rate is proportional to the **area** of the flame, the instantaneous area of the flame surface is needed. To find the area of the flame it is crucial to model the **flame dynamics**.

Flame dynamics

The surface of the flame front can be described as the G_0 **-level set** of a scalar function G (e.g., the temperature), (see Fig 1). Choosing as initial position a stationary position of the flame front, G is initialised on the grid by applying the signed distance function.

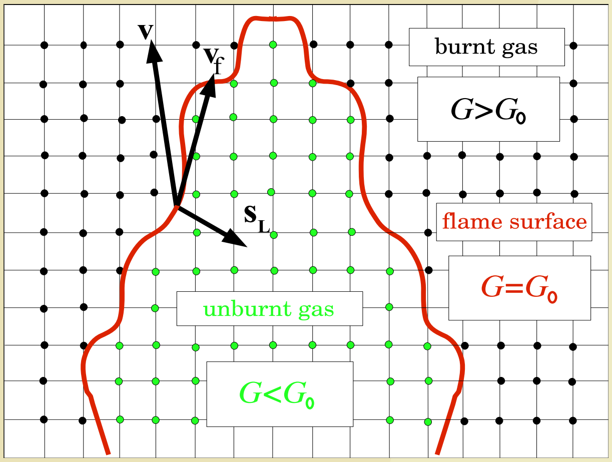

Fig 1: *Flame front dynamics.*

The flame front moves at the front velocity $\mathbf{v_f}$,

$$\mathbf{v_f} = \mathbf{v} - \mathbf{S_L}, \tag{1}$$

where $\mathbf{v}$ and $\mathbf{S_L}$ are the flow velocity and the laminar burning velocity, respectively.

$$\mathbf{S_L} = S_L \mathbf{n}, \tag{2}$$

where S_L is the laminar burning speed and $\mathbf{n}$ is the unit normal to the flame front directed towards the unburnt mixture. The motion of the flame front is given by the **level set equation (G-equation)**

$$\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_L |\nabla G|, \tag{3}$$

S_L depends on the flame curvature and flow strain, i.e.,

$$S_L = S_L^0 - S_L^0 \mathcal{L}\kappa - \mathcal{L}S. \tag{4}$$

Here S_L^0, $\mathcal{L}$, κ and S are the burning speed of the unstretched planar flame, the Markstein length, the curvature of the flame front and the strain of the flow, respectively. The curvature is given by

$$\kappa := -\frac{\nabla^2 G - \mathbf{n} \cdot \nabla(\mathbf{n} \cdot \nabla G)}{|\nabla G|}, \tag{5}$$

The strain rate is defined as

$$S := -(\mathbf{n} \cdot \nabla)(\mathbf{v} \cdot \mathbf{n}) + \nabla \cdot \mathbf{v} = \nabla v^t, \tag{6}$$

where v^t is the tangential component of the flow velocity. The **G-equation** is written in the form

$$\frac{\partial G}{\partial t} = L(G), \tag{7}$$

where

$$L(G) := H(\nabla G) + P(G), \tag{8}$$

$$H(\nabla G) := \mathbf{v} \cdot \nabla G - S_L^0 |\nabla G| + \mathcal{L} S |\nabla G| \tag{9}$$

$$P(G) := S_L^0 \mathcal{L} \kappa |\nabla G| \tag{10}$$

- H is a hyperbolic Hamilton-Jacobi term and it is approximated using a high order SLLF (Stencil Local Lax-Friedrichs) method combined with 5th order WENO scheme
- P is a parabolic term approximated with central differences,
- a 3rd order low storage TVD Runge Kutta time integration is applied to (7),
- reinitialisation is applied to keep G a signed distance function,
- C++ oriented object implementation.

Tests

The numerical scheme was tested to capture the movement of implicit defined curves. Tests indicate that the numerical scheme is robust and can be used to describe nonlinearities like flame front cusping and breaking.

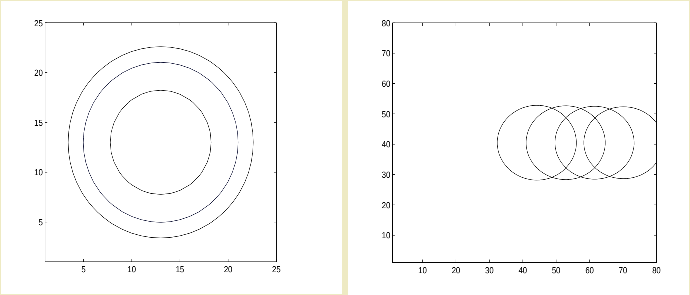

Fig 2: *Motion of a circle captured with the level set method. Left: circle collapsing in a curvature-driven flow. Right: motion of a circle due an extern velocity field*

Future developments

- apply the above numerical method to simulate the movement of a Bunsen flame due to flow perturbations
- describe the interaction between the flame and the flow by solving numerically the flow equations coupled with the G-equation.

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