technische universiteit eindhoven

Development of a Maxwell's Solver[a]

E. Chtcherbakov, W.H.A. Schilders

Introduction

Maxwell's equations of electromagnetics, formulated in 1870, represent a fundamental unification of electric and magnetic fields predicting electromagnetic wave phenomena such as:

- EM behavior of complex IC's
- Wave guiding
- Radiation
- Scattering

Using *MKS* units, the time dependent Maxwell's equations are:

Faraday's Law:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} - \mathbf{J}_M \text{ or } \frac{\partial}{\partial t}\iint_A \mathbf{B} dA = -\oint_l \mathbf{E} d\mathbf{l} - \iint_A \mathbf{J}_M d\mathbf{A} \quad (1)$$

Ampere's Law:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}_E \text{ or } \frac{\partial}{\partial t}\iint_A \mathbf{D} dA = \oint_l \mathbf{H} d\mathbf{l} - \iint_A \mathbf{J}_E d\mathbf{A} \quad (2)$$

Gauss' Law for the electric field:

$$\nabla \cdot \mathbf{D} = 0 \text{ or } \oint\!\!\!\oint_A \mathbf{D} d\mathbf{A} = 0 \quad (3)$$

Gauss' Law for the magnetic field:

$$\nabla \cdot \mathbf{B} = 0 \text{ or } \oint\!\!\!\oint_A \mathbf{B} d\mathbf{A} = 0 \quad (4)$$

Applying linear relations between **D** and **E**, and between **B** and **H**, we obtain the basis of numerical algorithms for electromagnetic wave interactions in three-dimensional space:

$$\begin{aligned} \frac{\partial \mathbf{H}}{\partial t} &= -\frac{1}{\mu}\nabla \times \mathbf{E} - \frac{1}{\mu}\mathbf{J}_M \\ \frac{\partial \mathbf{E}}{\partial t} &= \frac{1}{\varepsilon}\nabla \times \mathbf{H} - \frac{1}{\varepsilon}\mathbf{J}_E \end{aligned} \quad (5)$$

Classes of algorithms

Current numerical methods differ primarily in how the space lattice is set up [1].

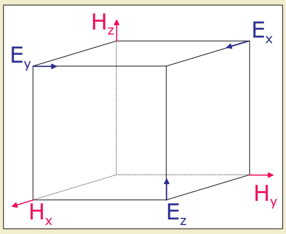
- *Almost completely structured.* Staircasing is used when a surface is not parallel to the grid coordinate axes.
- *Surface-fitted.* The space lattice is globally distorted to fit the shape of the structure of interest.
- *Completely unstructured.* The space containing the structure of interest is completely filled with collection of lattice cells of varying sizes and shapes, but conforming to the structure surface.

Time discretization is usually performed by employing a leapfrog time-stepping scheme. Since this is an explicit method, extremely small time steps have to be taken.

At present, the best choice of computational algorithm and mesh remains unclear.

Data structure

Tensor like grids provide a possibility to have both *FIT* and *FDTD* implemented within one program code. Space discretization is done as $x_i \in \mathbf{X}$, $y_i \in \mathbf{Y}$ and $z_i \in \mathbf{Z}$. All **E** and **H** components are spread over space providing interlinked *Faraday's Law* and *Ampere's Law* contours according to the following table:



	i	j	k
E_x	$2n$	$1+2n$	$1+2n$
E_y	$1+2n$	$2n$	$1+2n$
E_z	$1+2n$	$1+2n$	$2n$
H_x	$1+2n$	$2n$	$2n$
H_y	$2n$	$1+2n$	$2n$
H_z	$2n$	$2n$	$1+2n$

Research themes

In general, the Maxwell's equations result in wave equations. According to [2] many practical situations can be approximated as a combination of plane waves, which can be used for testing purposes. The following goals are feasible within this framework:

- Program and compare *FIT* and *FDTD* on orthogonal grids (uniform and non-uniform)
- Program and compare explicit and implicit time stepping
- Program local grid refinement and observe the benefits
- Investigate the usability of *Dual FIT* approach

References

[1] Taflove, A. and S.C. Hagness, *Computational Electrodynamics*. Norwood: Artech House, 2000.

[2] Ida, N., *Engineering Electromagnetics*. New York: Springer-Verlag, 2000.

Footnotes

[a] This work is done due to and as a part of the European project *CODESTAR* (Compact modelling of on-chip passive structures at high frequencies) *IST-2001-34058*. Check `http://www.imec.be/codestar/` for a detailed project description.