technische universiteit eindhoven

# Current distribution in a gradient coil


dr. ir. J.M.B. Kroot MTD, prof. dr. W.H.A. Schilders, dr. ir. A.A.F. van de Ven and dr. ir. S.J.L. van Eijndhoven


## Introduction

Magnetic Resonance Imaging (MRI) is a revolutionary imaging technique that plays an important role in the medical community. It provides images of cross-sections of a body without ionizing radiation, and a lot of information is obtained because MR signals are sensitive for several tissue parameters. The gradient coils are part of the scanner and realize a gradient in the magnetic field in order to locate and to select a slice of the body.

## Objectives

In this project, we determine the current distribution in the strips of a gradient coil and the magnetic field inside the scanner. The industrial design is an accurate, user-friendly simulation tool that enables an analysis of (self-)eddy currents[a]. We investigate time and frequency dependence and how eddy currents affect resistance and self-inductance of a coil.

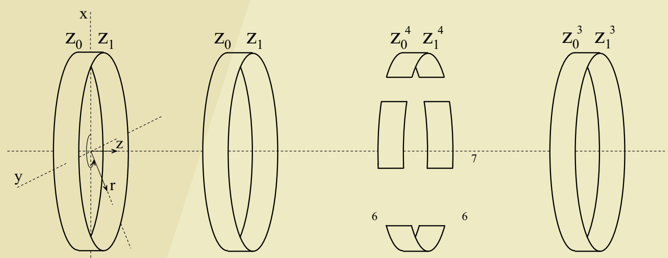
## Assumptions

Simplifications of the Maxwell equations are based on the following assumptions:

- The coil is excited by a low frequency source current (kHz).
- The media are homogeneous, isotropic, non-polarizable.
- Quasi-static approach:

$$\frac{\partial \mathbf{D}}{\partial t} \approx \mathbf{0}, \qquad \frac{\partial \rho}{\partial t} \approx 0.$$

- Strips are negligibly thin: $\mathbf{j} = j_\varphi(\varphi, z)\mathbf{e}_\varphi + j_z(\varphi, z)\mathbf{e}_z$.
- Conductors are fixed, no magneto-mechanical influence.
- Geometrically, the system can be considered as a set of rings and islands. All rings and islands are positioned on the same radius ($R$) and have the same thickness ($h$).




## Analysis

The Maxwell equations, together with the boundary (jump) conditions on the conductors, can be reduced to the integral equation

$$\mathbf{j}(\boldsymbol{x}) - \frac{i\kappa}{4\pi}\int_{\mathcal{S}_\cup} \frac{\mathbf{j}(\boldsymbol{\xi})}{|\mathbf{x} - \boldsymbol{\xi}|}\,\mathrm{d}a(\boldsymbol{\xi}) = \mathbf{j}^s(\boldsymbol{x}),$$

with $\mathbf{j}^s$ the source current, $\kappa$ a system parameter, and $\mathcal{S}_\cup$ the united surface of all strips. It is sufficient to consider the $\varphi$ component only. The component in the $z$-direction follows automatically from the law of conservation of charge $\nabla \cdot \mathbf{J} = 0$. We write

$$j_\varphi(\varphi, z) - i\kappa \int_{\mathcal{S}_\cup} \mathcal{K}_\varphi(\varphi - \theta, z - \zeta) j_\varphi(\theta, \zeta)\,\mathrm{d}a(\theta, \zeta) = j^s_\varphi(\varphi, z).$$

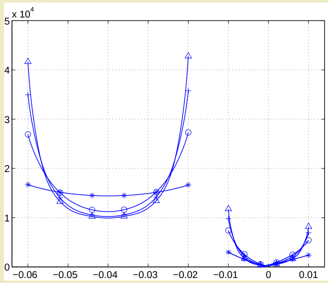
From a numerical point of view, we use the Fourier cosine series of the kernel function

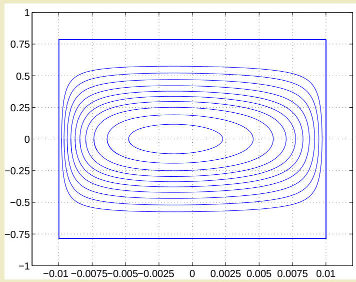
$$\mathcal{K}_\varphi(\varphi, z) = \frac{1}{4\pi^2}\Big(Q_{\frac{1}{2}}(\chi) + \sum_{k=1}^{\infty} \cos(k\varphi)[Q_{k-\frac{3}{2}}(\chi) + Q_{k+\frac{1}{2}}(\chi)]\Big),$$

where $Q_{k-\frac{1}{2}}$ are the Legendre functions of the second kind of odd-half-integer order, which have a logarithmic singularity in $\chi = (z^2 + 2)/2 = 1$, i.e. $z = 0$.

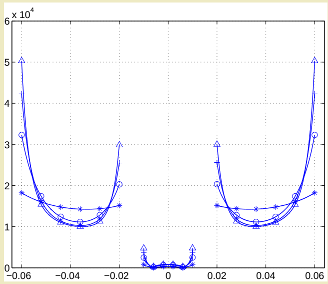
## Results

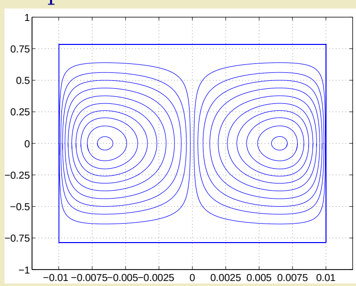
Consider one ring and one island with a length of a quarter of the ring. The first figure shows the amplitude of the current density in $\varphi$-direction, $|j_\varphi|$ (A/m), on the line $\varphi = 0$, plotted as a function of $z$ (m). Edge-effects occur in both ring and island, which become stronger as the frequency is increased. In the second figure, the streamlines of the current in the island are shown.



Consider two rings with one island in between. The currents in the rings are in phase. The two figures below show that two eddies occur in the island, with low amplitudes[b].



### Footnotes

[a]This work is done in cooperation with Philips Medical Systems

[b]For questions and remarks: mail to `j.m.b.kroot@gmail.com`