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Local Defect Correction in Time-Dependent Problems

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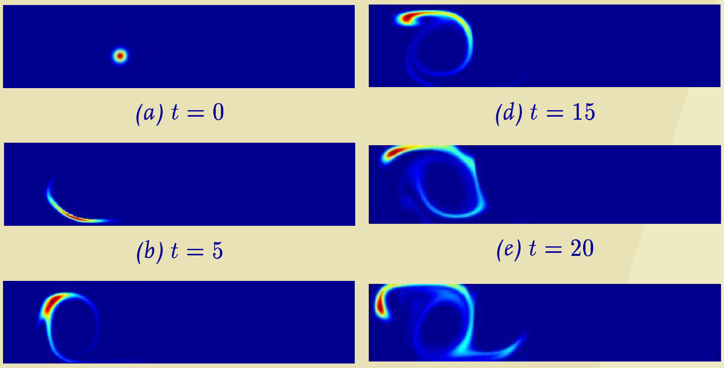
Motivation: transport in turbulent flow fields

Solutions of Partial Differential Equations (PDEs) describing transport of passive tracers in turbulent flow fields are often characterized by regions where spatial gradients are quite large compared to those in the rest of the domain, where the solution presents a relatively smooth behavior (Figure 1).



(a) $t = 0$ *(d) $t = 15$* *(b) $t = 5$* *(e) $t = 20$* *(c) $t = 10$* *(f) $t = 25$*

Figure 1 - Particles introduced in the flow at time $t = 0$ are transported by the velocity field remaining mostly confined in a certain part of the domain

An efficient numerical method is needed to solve transport problems in turbulent flow fields.

The Local Defect Correction (LDC) method

Local Defect Correction (LDC) is a method suitable for problems whose solutions exhibit a high activity in a small part of the domain. Through the defect, which is an approximation of the local discretization error, LDC iteratively combines the solution over a global uniform coarse grid and the solution over one or more local uniform fine grids (Figure 2); the fine grids are located at each time step where the high activity occurs.

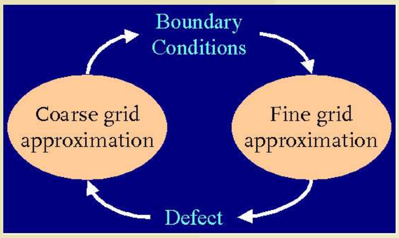


Figure 2 - LDC scheme

The main advantages of LDC are:

- adaptivity;
- simple data structures;

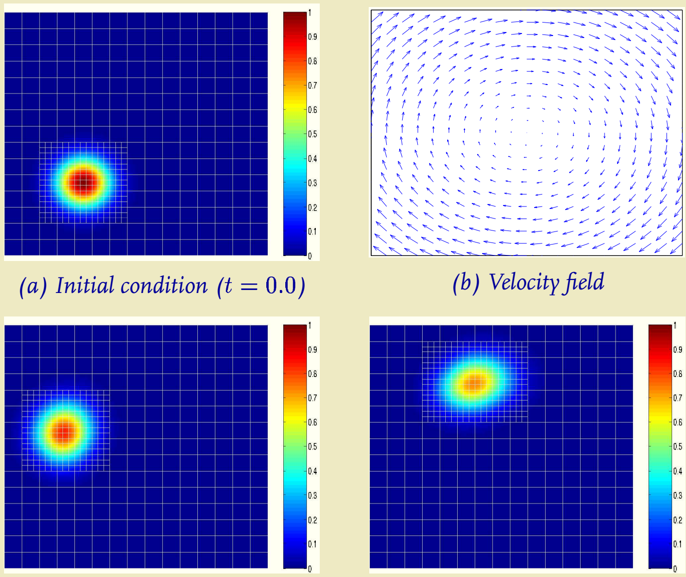
Solution of time-dependent PDEs using LDC

At each time step the following operations are performed:

1. Integrate on the coarse grid.
2. Determine a local fine grid at forward time.
3. Provide initial values on the local grid at backward time.
4. Use space and time interpolation to provide boundary conditions to the local problem.
5. Integrate on the local fine grid.
6. Until convergence
   - Compute a defect at forward time.
   - Solve a modified coarse grid problem.
   - Provide new boundary conditions locally.
   - Integrate on the fine grid with updated boundary conditions.

Results

We solve a 2D convection-diffusion equation using LDC (see Figure 3) and an equivalent uniform grid having the same size as LDC's fine grid.



(a) Initial condition ($t = 0.0$) *(b) Velocity field* *(c) Solution at $t = 1.0$* *(d) Solution at $t = 2.0$*

Figure 3 - Test problem and representation of LDC coarse and fine grid.

Table 1 shows that in our example LDC achieves the same accuracy as the equivalent uniform grid solver while having to compute a minor complexity.

	$\Delta x = \Delta y = \Delta t$		ϵ_∞	unkn
LDC	coarse	1/40	$1.2 \cdot 10^{-1}$	$2.1 \cdot 10^{6}$
	fine	1/200		
Uniform grid		1/200	$1.2 \cdot 10^{-1}$	$7.9 \cdot 10^{6}$
LDC	coarse	1/80	$3.2 \cdot 10^{-2}$	$3.4 \cdot 10^{7}$
	fine	1/400		
Uniform grid		1/400	$3.2 \cdot 10^{-2}$	$1.3 \cdot 10^{8}$

Table 1 - Results of numerical experiment; ϵ_∞ is the maximum error at $t = 2$ and unkn the total number of unknowns to be computed.

Acknowledgement

This project is funded by the Netherlands Organisation for Scientific Research (NWO) under Computational Science research grant 635.000.002.