Elastic stationary analysis of a cracked plate

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Introduction

The existence of small crack flaws, even microscopic ones, may lead to the failure of the structures that contain them.

London Bridge, Great Ocean Road, before 1990.

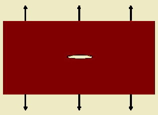
London Bridge after collapse.

It is important to obtain a proper understanding of how the size, geometry and location of a crack can be related to the load a structure may withstand, as well as being able to predict the rate at which a crack can approach a critical size, the direction of growth of a rapidly propagating crack and under which conditions its growth may be prevented.

Governing equations

Consider an ellipse shaped crack on a thin plate $\boldsymbol{\Omega}$, as shown in the picture. The plate is loaded in the plane on its upper and lower edges, so that plane stress conditions can be assumed. The surface of the crack is considered to be stress free.

If the medium is linearly elastic, homogeneous, isotropic, the relevant continuous mechanics equations for this problem are given in terms of the stress tensor $\boldsymbol{\sigma}$, the strain tensor $\boldsymbol{\epsilon}$ and the displacements $\mathbf{u}$:



Equation of motion in the quasistatic case

$\text{div}\,\boldsymbol{\sigma} + \mathbf{b} = 0;$

Strain-displacement relation

$\boldsymbol{\epsilon}(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^T);$

Elastic Constitutive Equation

$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\epsilon}.$

Here, $\mathbf{b}$ represents the body force applied on the body and $\mathbf{C}$ is the elasticity tensor.

We may take the displacement vector field as the primary unknown and eliminate the stress and strain from the governing equations by substitution, hence obtaining

$\text{div}\,(\mathbf{C}\boldsymbol{\epsilon}(u)) + \mathbf{b} = 0,$

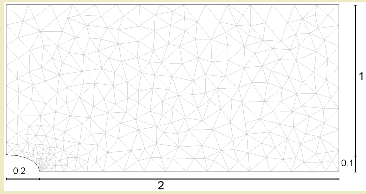
to which boundary conditions must be added:

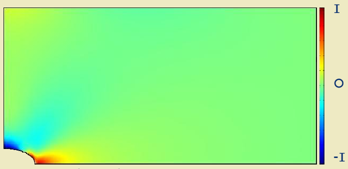
Boundary conditions

$\boldsymbol{\sigma}.\mathbf{n} = \boldsymbol{\sigma}_0, \text{ on } \partial\Omega$

Computational results

Due to the double symmetry, only one quarter of the plate needs to be analyzed. The displacement vector is numerically obtained using *FEM* on a triangular mesh with linear elements. The mesh has been refined close to the crack tip. The normal stress in the x direction and the normal stress in the y direction, respectively represented by $\boldsymbol{\sigma}_x$ and $\boldsymbol{\sigma}_y$, can be determined by numerical differenciation.





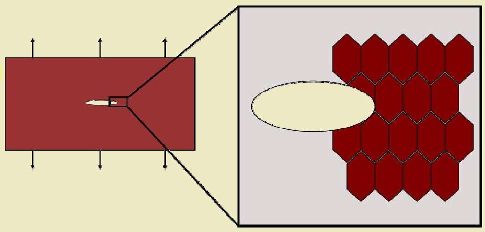
Stress in the x direction

Stress in the y direction

Results were obtained for an aluminum plate by taking $\boldsymbol{\sigma}_0 = 1$. There is a stress accumulation near the crack tip.

Microstructural aspect

In the regions near the tips of the crack, several aspects which may be neglected for the remainder of the plate might here play an important role, such as the geometry and orientation of the grains, the manner in which they are connected and the crack geometry. This gives rise to a new material model, which must be linked with the previously described analysis.



Ongoing Research

Our goal is to combine the Continuous Mechanics approach with a microstructural analysis in the region close to the crack tip.

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