SUMS OF NEGATIVE POWERS MODULO A PRIME

ROB DE JEU

For an odd integer \( n \geq 2 \) and an odd prime \( p \) consider the expression

\[
S_{n,p} := \sum_{a=1}^{p-1} (-1)^a \bar{a}^{-n} \in \mathbb{Z}/p\mathbb{Z},
\]

where \( \bar{a} \) denotes the image of \( a \) in \( \mathbb{Z}/p\mathbb{Z} \). Is it true that

\[
\forall n \geq 2 \text{ odd } \exists \text{ infinitely many primes } p : S_{n,p} \neq 0?
\]

See Rob de Jeu's slides for the motivation and further details.