ANTHROPOMORPHIC IMAGE RECONSTRUCTION VIA HYPOELLIPTIC DIFFUSION

EINDHOVEN MATHEMATICS COLLOQUIUMS

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We study a model of geometry of vision due to Petitot, Citti and Sarti. This model was studied independently by R. Duits. One of the main features of this model is that the primary visual cortex $V1$ lifts an image from $\mathbb{R}^2$ to the bundle of directions of the plane. Neurons are grouped into orientation columns, each of them corresponding to a point of this bundle. In this model a corrupted image is reconstructed by minimizing the energy necessary for the activation of the orientation columns corresponding to regions in which the image is corrupted. The minimization process gives rise to an hypoelliptic heat equation on the bundle of directions of the plane. In the original model directions are considered both with and without orientation giving rise respectively to a problem on the group of rototranslations of the plane SE(2) or on the projective tangent bundle of the plane $\mathbb{P}T\mathbb{R}^2$. We provide a mathematical proof of several important facts for this model. We first prove that the model is mathematically consistent only if direction are considered without orientation. We then prove that the convolution of a $L^2(\mathbb{R}^2, \mathbb{R})$ function (e.g. an image) with a 2-D Gaussian is generically a Morse function. This fact is important since the lift of Morse functions to $\mathbb{P}T\mathbb{R}^2$ is defined on a smooth manifold. Finally we present the main ideas of an algorithm which allows to perform image reconstruction on real non-academic images. A very interesting point is that this algorithm is massively parallelizable and needs no information on where the image is corrupted.