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EXPERIMENTAL AND NUMERICAL ANALYSIS OF A SINGLE ELBOW PIPE FILLED WITH WATER

HT-62/01/019/A

Summary :
This study takes part of the University of Dundee Project (Scotland) dealing with FSI in piping systems and of the EDF R&D project called VICI which includes an item on guidelines to model them. A vibration analysis is realized on a clamped-free L-shaped piping system filled with water, called '*Casto1*', which plane waves acoustical modes and beam modes 'superimpose'. This mock-up has been independently designed by TNO (Holland). The objectives are to validate numerical models with the Circus code and Code_Aster, and to compare fully coupled models with added mass ones. The uses of fully coupled models with the Circus code and Code_Aster are experimentally validated on Casto1. The errors on the estimation of 8 modal frequencies on the frequency range 0-500 Hz is around 3%. An added mass model is shown to be sufficient to represent the pipe as the effects of fluid-structure coupling in the elbow are shown to be weak. It is necessary to pay attention to the model of the pipe with no fluid in order to get a trustful model of Casto1 filled with water.

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Synthesis

Context :

EDF R&D and Dundee University (Scotland) have started a collaborative study within the 'Dundee Project' on fluid-structure interaction in liquid-carrying pipes. The collaborative study includes experimental parts to be carried out by EDF R&D. The first experimental set up, presented in this document, concerns a clamped-free L-shaped piping system, called '*Casto1*' which has been independently designed by TNO (Holland). This experimental validation is connected to the EDF R&D project called VICI which includes an item on guidelines to model piping systems.

Objectives :

1. to realize an experimental validation of fully coupled numerical models on the L-shape piping system which plane waves acoustical modes and beam modes 'superimpose' using the Circus code and Code_Aster.
2. to analyze the numerical models by comparing fully coupled with added mass models.

Conclusions :

1. The uses of fully coupled models with the Circus code and Code_Aster are experimentally validated on Casto1 with errors on the estimation of frequencies around 3%.
2. An added mass model may be sufficient to represent the one-elbow pipe as the effects of fluid-structure coupling in the elbow are shown to be weak.
3. The error induced by the model of the pipe with no fluid may be much more important than the error induced by the representation of fluid-structure effects. Then, it is necessary to pay attention to the model of the pipe with no fluid in order to get a trustful model in water.

Synthèse

Contexte :

EDF R&D et l'Université de Dundee (Ecosse) ont commencé une collaboration au sein du « Dundee Project » sur l'interaction fluide-structure dans les tuyauteries en eau. Cette collaboration inclut des tâches expérimentales réalisées par EDF/AMV. La première maquette est un tuyau encastré-libre coudé en L appelé « Casto1 » qui a été conçu indépendamment par TNO (Pays-Bas). Cette validation expérimentale est connectée au projet EDF R&D VICI qui inclut une tâche sur la définition de règles de modélisation des réseaux de tuyauteries.

Objectifs :

1. réaliser une validation expérimentale de modèles numériques entièrement couplés sur le tuyau coudé en L dont les modes acoustiques d'ondes planes et mécaniques de type poutre se « superposent » avec le code Circus et Code_Aster.
2. analyser les modèles numériques en comparant un modèle entièrement couplé avec un modèle en masse ajoutée.

Conclusions :

1. L'utilisation de modèles couplés avec Circus et Code_Aster est validée sur Casto1 avec des erreurs sur l'estimation des fréquences voisines de 3%.
2. un modèle en masse ajoutée semble suffisant pour représenter le tuyau coudé comme les effets de couplage fluide-structure dans le coude sont relativement faibles..
3. L'erreur induite par le modèle du tuyau en air peut être aussi importante que celle induite par la modélisation des effets de couplage fluide-structure. Il est donc nécessaire de porter une attention particulière à la modélisation du tuyau sans fluide pour obtenir un modèle réaliste en eau.

Summary

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1. Introduction

1.1. Context

EDF R&D and Dundee University (Scotland) have started a collaborative study within the 'Dundee Project' on fluid-structure interaction (FSI) in liquid-carrying pipes (contract number: T64/D27018/RNE875). The general objectives of the Dundee Project are to improve the understanding of the coupling between fluid pressure pulsations and structural vibrations in piping systems and to provide guidelines that will clearly identify where numerical FSI analysis is required. The collaborative study includes experimental parts to be carried out by EDF R&D. The first experimental set up, presented in this document, concerns a clamped-free L-shaped piping system, called '*Casto1*'[1], which is a typical subset of French nuclear power plants. The first objective of the present study is to realize an experimental validation of fully coupled numerical models on a L-shape piping system. The second objective consists in connecting the results to the EDF R&D project called VICI which includes an item on guidelines to model piping systems.

1.2. Independent design of the single elbow pipe by TNO

The complete definition of this simple piping system, has been realized with the help of C. de Jong (TNO - Holland) (contract number: P54/C26818/EP938). The TNO code Presto [5] is dedicated to the calculation of the pulsations and vibrations of fluid-filled pipe systems in the transfer matrix models that have also been implemented in the code Circus. Presto is then applied to design the L-shaped piping system.

- **Schedule of conditions**

The design should respect this initial schedule of conditions:

- the system is a L-shaped stainless steel pipe including one elbow. The coupling effects should only occur in the elbow.
- the system should allow the superposition of at least two acoustical modes on mechanical modes, by changing the acoustical and/or mechanical boundary conditions or pipe lengths, in the frequency range corresponding to plane waves in the fluid.
- the system should be mounted in the 'banc Evadyn' at EDF/AMV, and hence fit within the geometry of this test rig. It should be smaller than the inside of Evadyn (L×W×H=2.70×1.20×1.90 m^3).
- to keep the system manageable, the pipe diameter should be in the order of magnitude of 100 mm. The manufacturer came with the following two suggestions:
    1) outer diameter D_e = 101.6 mm, wall thickness t = 3.6 mm, elbow radius of curvature R_c = 133.5 mm
    2) outer diameter D_e = 88.9 mm, wall thickness t = 3.2 mm, elbow radius of curvature R_c = 114.5 mm

- **Data for the design calculations**

For the design calculations, the following approximate material properties have been used:

Young's modulus:	$E_s \approx 200$ GPa,	loss factor:	$\eta_s \approx 0.002$,
density:	$\rho_s \approx 8000$ kg/m^3	Poisson's ratio:	$\nu \approx 0.3$

The pipes is partially filled with clean water:

density: $\rho_f \approx 1000$ kg/m^3 sound velocity: $c_f \approx 1480$ m/s,

[1] '*To validate coupling effects, just go to CASTO (French store of do-it-yourself) and buy an elbow with 2 pieces of pipe and hit it*', J.L. Trollé, personal communication.

The cut-on frequency [1 ; 5] of propagating n=2 lobar waves in the pipe wall ($f_{n=2}$) for the two given water-filled steel pipes are:

1) $f_{n=2} \approx 713$ Hz
2) $f_{n=2} \approx 831$ Hz

Hence, Presto [5] and Circus [1] models may be used confidently at frequencies below 700 Hz.

For planar pipe systems, three wave types play a role: compressional waves in the fluid and extensional and flexural waves in the pipe wall. The inertia of the fluid will influence the flexural wave velocity and the elasticity of the wall the fluid wave velocity. The approximate wave velocities in the two pipes are:

1) $c_F \approx 1299$ m/s, $c_E \approx 5000$ m/s, $c_B \approx 11.4\sqrt{\omega}$ m/s
2) $c_F \approx 1301$ m/s, $c_E \approx 5000$ m/s, $c_B \approx 10.7\sqrt{\omega}$ m/s

The acoustical differences between the two pipes appear to be small.

Given the maximum size of the system, the demand for at least two fluid modes in the frequency range up to 700 Hz and the need of a possibility to adjust the natural frequencies so that acoustical and mechanical modes are superimposed, it is advantageous to have open-closed boundary conditions for the fluid column. These boundary conditions lead to the lowest natural frequency for a given length of the fluid column (‘quarter wavelength resonator’). The length of the fluid column can be easily adjusted by filling up with fluid at the open end, or by draining fluid via a small valve at the closed end. To keep the design simple, the mechanical boundary conditions could be ‘clamped’ at the closed end and ‘free’ at the open end. A practical realization of a 'free' end is relatively simple. The design of a ‘clamped’ end requires some more attention, since nothing is perfectly rigid. A sketch of this system is given in Figure 1.

Presto calculations have been carried out for a number of possible design options. In an iterative process, the design has been modified until a system was found that allowed superimposing mechanical and acoustical modes by adapting the water level.

- **Results**

A series of Presto calculations has been performed in which the water level in the vertical pipe was decreased in 50 subsequent steps of 3 cm. A change of the water level will change both the structural response and the fluid response, even in the case without fluid-structure interaction in the elbow, because it changes the length of the fluid column and the added fluid mass that determines the flexural motion of the pipe. The effect of fluid-structure interaction is clearly visible when we compare the response that is calculated with and without coupling (Figure 2). It shows the level of the response of fluid and wall as a function of the frequency and the water level (expressed as the height of the air column between the water level and the top of the vertical pipe). The actual levels are not significant in this figure. The light colored lines indicate the structural and acoustic resonance peaks for the coupled and uncoupled calculations. It is shown that coupling has a strong impact on the resonant behaviour. The coupled modes (bottom figures) cannot be recognized as a superposition of the uncoupled fluid modes (left upper figure) and structural modes (right upper figure), but have their own behaviour.

Figure 2 shows that structural-added-mass modes and acoustics-Allievi modes superpose both around 100 and 350 Hz when the level of air is between 30 and 70 cm. The tests will be realized varying the level of air in this range.

The final sketches of Casto1 derived from the specifications of TNO have been realized by EDF/AMV and are given in Annex A.

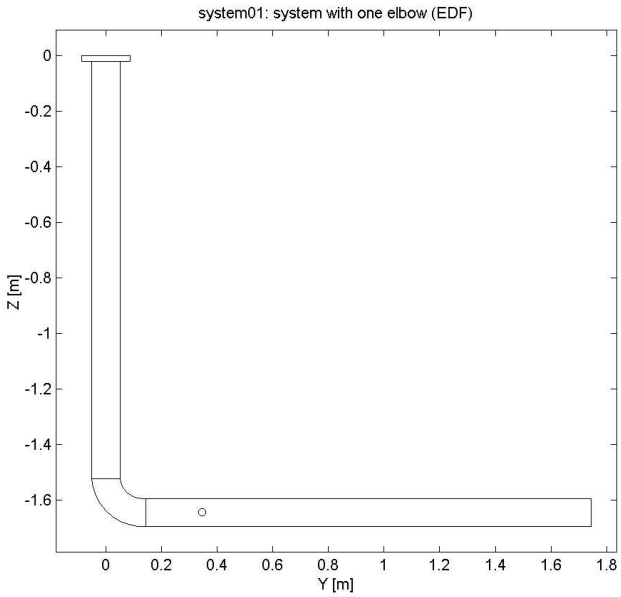

Figure 1: sketch of the proposed experimental single elbow pipe system by TNO

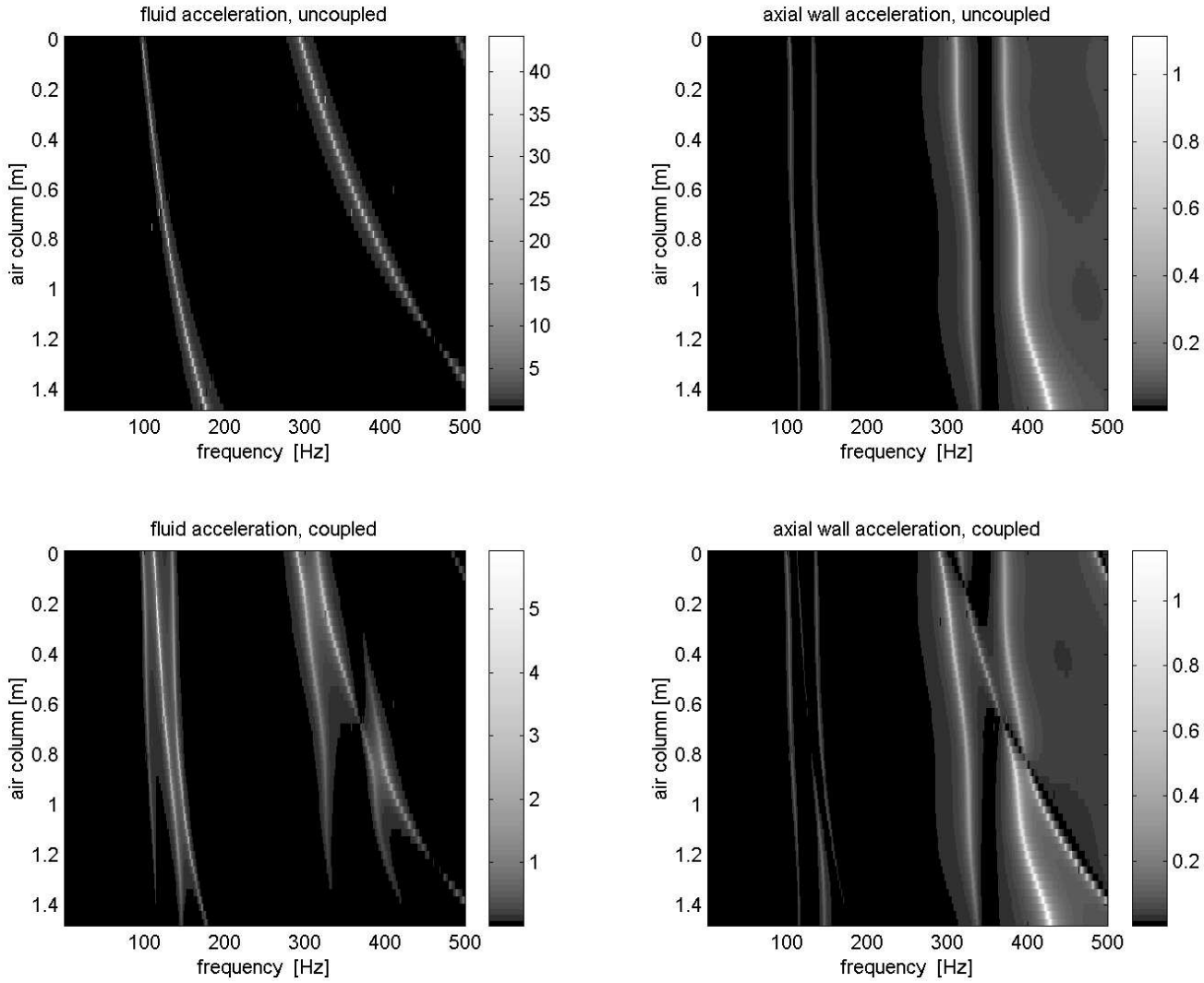

Figure 2: resonance frequencies of the coupled and uncoupled systems

2. Structural analysis

The structural analysis in air of Casto1 is detailed in[15]. The objective of the present analysis in air is to provide numerical models of the pipe alone with Circus and Code_Aster which fit well with experimental data.

2.1. Experimental analysis

The excitation is provided by hammer hit near location A (Figure 10 - Annex A) and the frequency range studied is lying from 1 Hz to 500Hz as defined by TNO. The response devices consist in 22 accelerometers (B&K 4370 connected with B&K 2635 charge amplifiers) positioned every 150 mm along the pipe to observed every in-plane mode shapes. Three sets of experimental studies have been realized. The first 2 sets correspond to an in plane excitation due to an hammer hit in the X and Z directions and the last one corresponds to an out of plane excitation in the Y direction. Results are summarized in Table 3 of Annex C and in-plane modal shapes are given in Figure 3 which are only interesting for the incoming coupled analysis. The specifications of TNO have shown that the last 4 modes will superpose . At the exception of a slight difference for the first mode, the experimental in-plane modes fit well for the excitations in the X and Z directions.

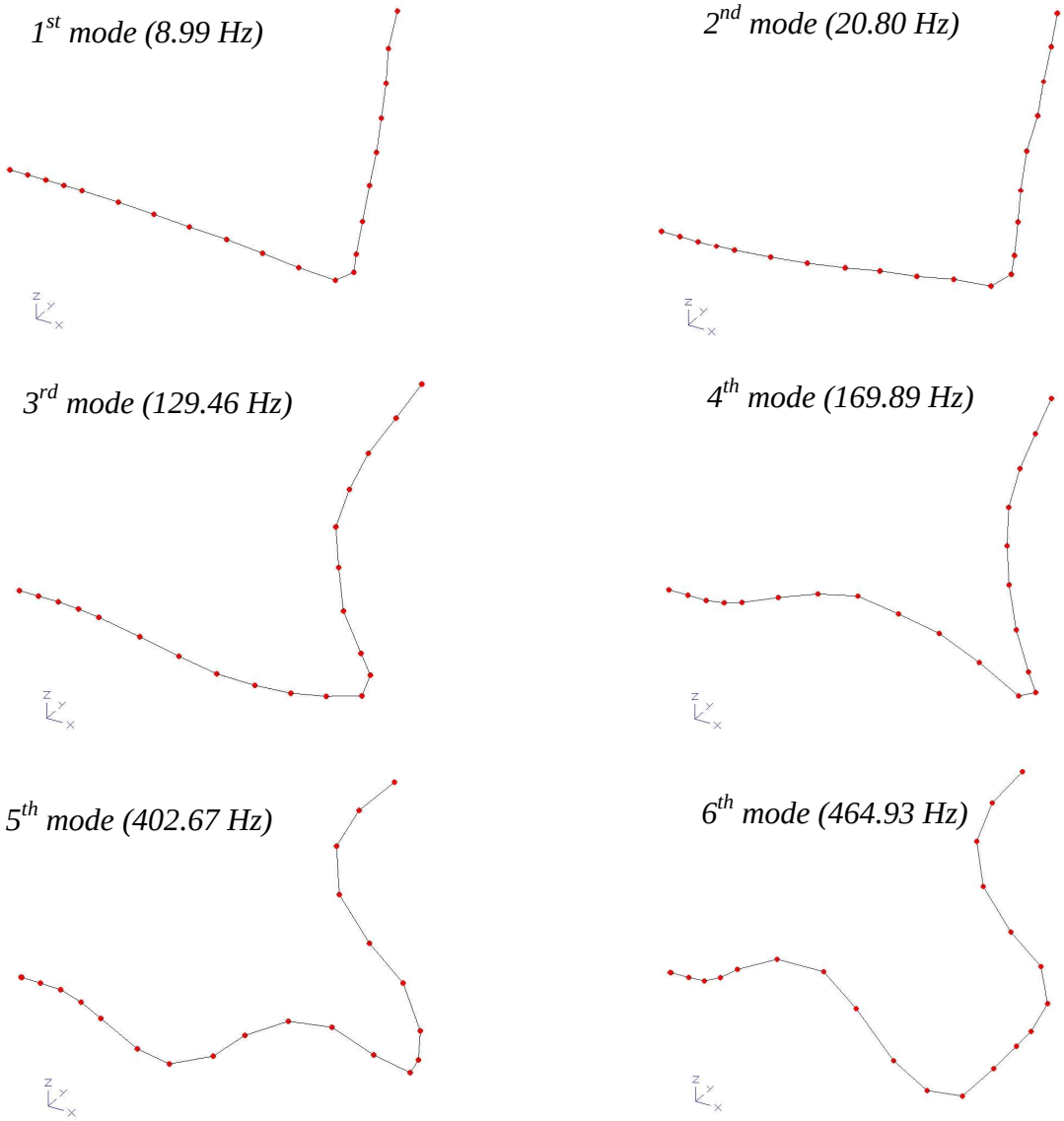

Figure 3: experimental in-plane mode shapes in air

2.2. Numerical analysis

The numerical analysis are performed using:

- the Circus code [1] which is dedicated to the vibrational analysis of piping systems using beam theory coupled with plane waves theory.
- Code_Aster which is a FEM mechanical code. A fully coupled model of the pipe may be done using a (u, p, ϕ) vibroacoustical formulation [9]. Shell theory and 3D-acoustics are respectively used for the pipe and the fluid as in [2 ; 3 ; 7].

- **Simulations with Circus (3.4.00)**

This analysis with the Circus code is divided in 2 steps:

1. first simulations on a simplified model
2. user-dependent updating.

Circus code performs frequency-domain analysis and does not have any specific research algorithm for natural frequencies. The calculated values are in fact the local maximums of the response spectra which implies that obtained natural frequencies could be slightly affected by the frequency step and by the specific spectrum chosen.

Circus describes the experimental set up by the use of 17 elements as pictured in Figure 4:

- the element 1 describes the clamped anchor. This element assumes rigidity values for displacements and rotations. As example values of 10^{12} N/m and 10^{12} N.m/rad drive to a totally clamped structure.
- the elements 2, 4, 6 ,8, 10, 12, 14 and 16 are straight beams.
- the element 11 is a curved beam element representing the elbow. Circus allows to take into account elbow flexibility by dividing inertia value by an appropriate factor. Using RCC-M rules, Circus calculated an elbow flexibility factor of 7.164.
- the element 17 is a straight beam with a larger thickness in order to represent the final flange.
- the elements 3, 5, 7, 9, 13 and 15 are concentrated mass (centered on the axis) which represent the total weight of two symmetrical pressure taps. Each concentrated mass was estimated at 0.16 kg by reading the sketches of Annex A.

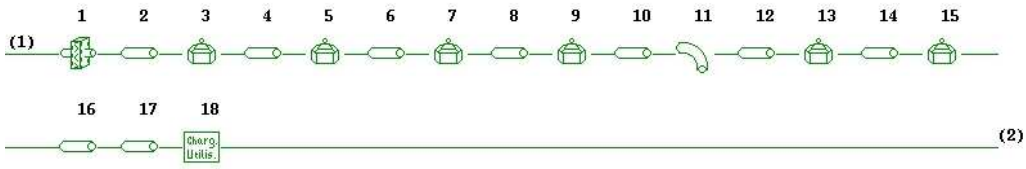

Figure 4: Circus model of Casto1

With the previous modeling and considering a perfectly clamped structure the values obtained for the modes are given in the column entitled 'initial model' of Table 1 (the error columns indicate the difference with experimental results).

The parameter chosen for the updating procedure are boundary stiffness and the mass of pressure taps. Results obtained are reported in the columns entitled 'updated model' of Table 1. All stiffness are of 1.10^{12} N/m at the exception of the rotational in Z which is of 5.10^{7} N.m/rd. The top flange density is equal to 7500 kg/m^3, pressure sensor mass is of 0.175 Kg (for two sensors), the thickness is of 4.17 mm and the elbow flexibility factor of 5.5. The results obtained may appear quite good but it does not mean we get a trustful model. For instance, we observed that the thickness may vary along the pipe.

- **Simulations with Code_Aster (V6.02.00)**

Code_Aster has specific algorithms based on modal analysis to determine natural frequencies and directly gives actual modes in- and out-of-plane. Code_Aster modeling with shell elements takes automatically into account elbow flexibility. Nevertheless, at the opposite of the results issued from the Circus code, the results of Code_Aster are meshing dependent.

Different transversal meshes for pipe sections have already been tested using DKQ finite-elements at EDF/AMV [2 ; 3 ; 7 ; 15] and drove to a good enough approximation with 16 elements by section. Longitudinal meshing has been determined for the analysis in air of Casto1 in [15]. Nevertheless, a new mesh (Figure 5) with more elements has been tested here as CPU time was not so long (24 elements per section, 48 longitudinal elements in the elbow, 25 longitudinal elements for each straight pipes, 3 radial elements for the fluid used for the coupled analysis). This finite-elements model is derived from the updated Circus model built before using the Astus 1.0 software [8] , which provides a mesh file and a command file for Code_Aster. The density of the top flange is corrected in order to keep its weight constant as its geometry is different from reality. The data (*donnees_circus.dat*) and discretisation (*discretisation.dat*) input files for Astus and an example of the command file for Code_Aster (*astus.comm*) are given in Annex B.

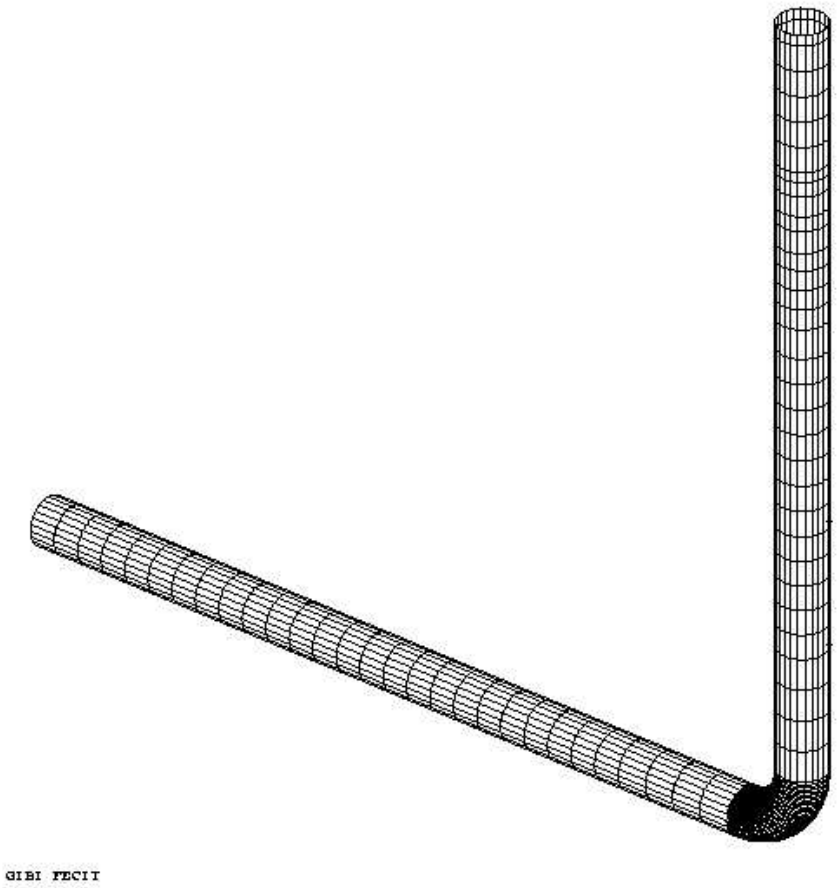

Figure 5: mesh of Casto1 for Code_Aster

2.3. Conclusion

Numerical and experimental mode shapes are found to be similar as in [15] (and mode shapes given by fully coupled simulations with Code_Aster are given in Annex D).

The comparison between experimental and numerical results with no-updating is good as the error on the frequencies is less than 3%. Nevertheless, the user-dependent updating based on a initial updating using FEMTools [15] improves the results. Finally, the design and the manufacturing of Casto1 are found to be good and particularly, the anchor end.

We dispose of validated numerical models of the in-plane structure for the fully coupled analysis.

Mode	*experimental frequency (Hz)*	*initial model*		*updated model*	
		frequency (Hz)	*error (%)*	*frequency (Hz)*	*error (%)*
In-plane					
1	8.99	8.74	2.8	9.01	0.2
2	20.80	19.86	4.5	20.87	0.3
3	129.46	127.55	1.5	128.2	1.0
4	169.89	166.09	2.2	168.6	0.8
5	402.67	393.60	2.3	396.2	1.6
6	464.93	466.47	0.3	467.8	0.6
Out-of-plane					
1	9.18	8.53	7.1	8.71	-5.1
2	25.32	25.90	2.3	26.05	2.9
3	127.25	128.40	0.9	128.61	1.1
4	188.45	193.42	2.6	193.71	2.8
5	286.30	284.41	0.7	294.96	3.0
6	408.89	411.96	0.8	414.17	1.3

Table 1: in-plane and out-of-plane results in air with Circus

Mode	*experimental frequency (Hz)*	*Aster*	
		frequency (Hz)	*error (%)*
In-plane			
1	8.99	9.05	0.7
2	20.80	20.68	-0.6
3	129.46	130.16	0.5
4	169.89	169.57	-0.2
5	402.67	407.00	1.1
6	464.93	471.58	1.4
Out-of-plane			
1	9.18	8.72	-5.0
2	25.32	26.24	3.6
3	127.25	130.63	2.7
4	188.45	195.34	3.7
5	286.30	321.79	12.4
6	408.89	423.30	3.5

Table 2: in-plane and out-of-plane results in air with Code_Aster

3. Fully coupled analysis

3.1. Experimental analysis

The excitation signal is provided using an electromagnetic shaker to measure the structural response (accelerometers B&K 4370) and using a gun inside the top flange generating a sufficient pressure field to measure the acoustic pressure (pressure sensors Kistler 701A). This method allows a good sensibility on the responses of the structure and the fluid. Moreover, the sound celerity was derived from the response of 3 pressure sensors using an intensimetry technique with a precision of 57 m/s (Table 4 - Annex C). The experimental sound velocity is close to the sound velocity of 1301 m/s calculated using the correction of Allievi [1 ; 2 ; 5 ; 6].

The frequency range studied is lying from 1 Hz to 500 Hz and 11 levels of water has been analyzed L-28, L-33, L-37, L-40, L-44.5, L-48, L-52, L-56, L-60, L-66 where L-X represents the height X of the air column. This procedure allows to observed the shifts of frequencies (Figure 6 and Figure 7) has predicted by TNO (Figure 2).

The frequencies are derived from the power spectral densities (PSD) of the sum of all the responses by searching the peaks. In Table 4 and Table 5 of Annex C, the bold values correspond to frequencies which are hardly identified. The modes n°6 and n°7, which are supposed to have a major acoustic contribution, are not observed on the structural responses.

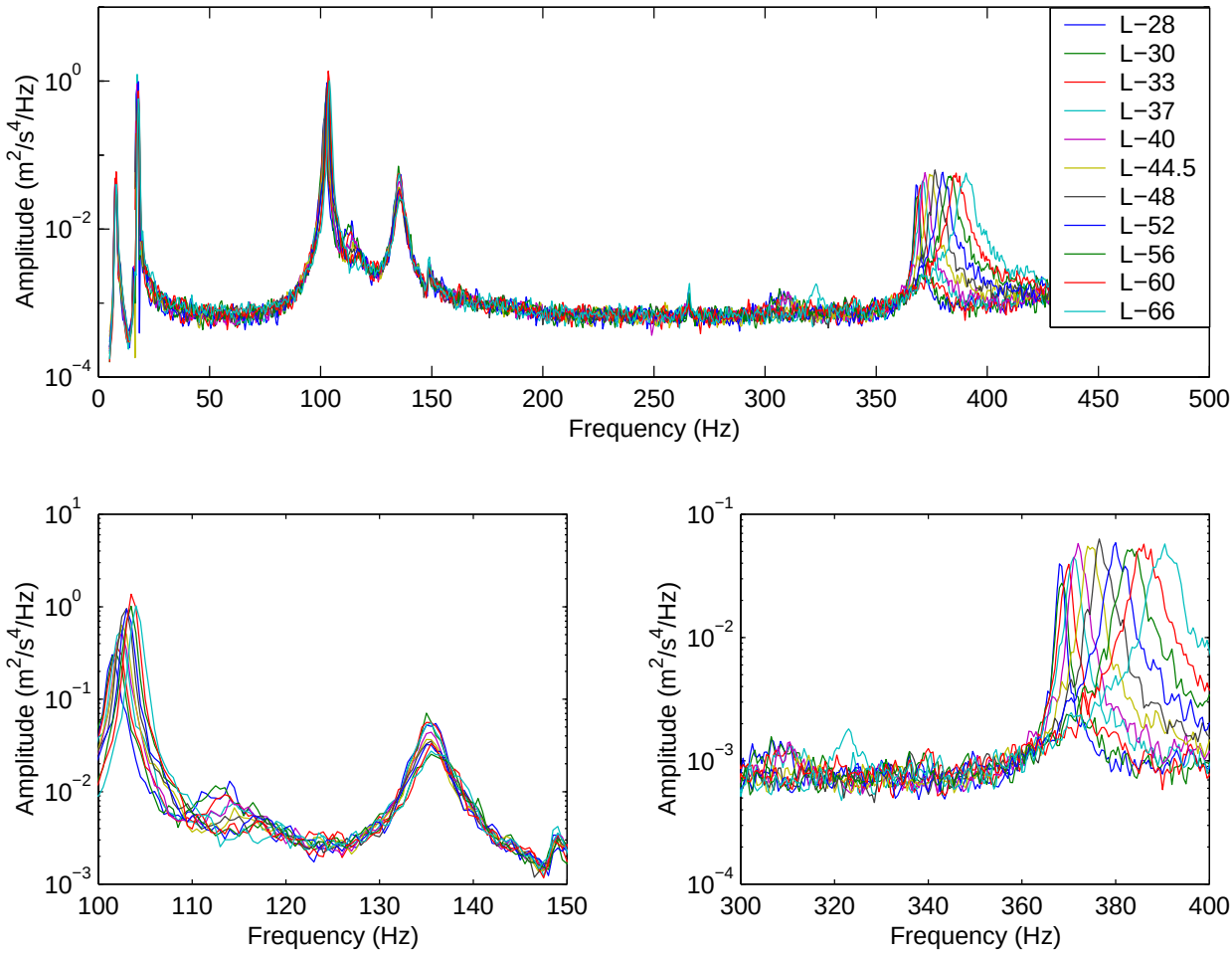

Figure 6: experimental PSD on accelerometers - Casto1 filled with water

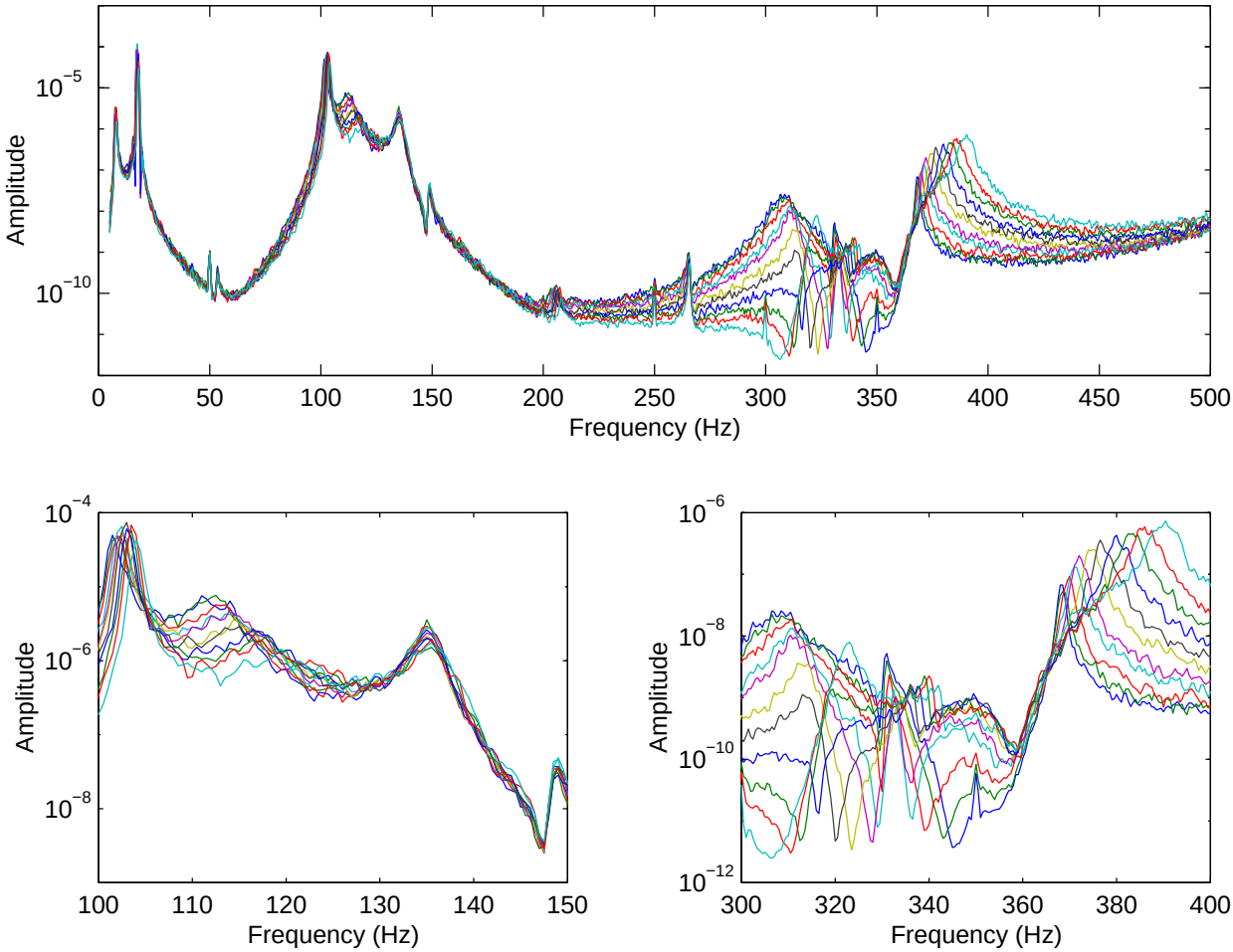

Figure 7: experimental PSD on pressure sensors in water

Remarks:

1. during the first series of tests, the sound velocity was observed to be quite low near sound velocity in air. This was attributed to bubbles of air in the pipe. To avoid this problem, when the pipe was filled with water, it was inclined and we awaited 1 month before proceeding with the tests.
2. between the tests in air and water, the pipe has been dismounted. The clamped end boundary conditions may have changed.
3. the test procedure is detailed in [4].

3.2. Numerical analysis

- **Simulations with Circus (3.4.00)**

The numerical analysis with the Circus code is realized using fully coupled, added mass and acoustical models of Casto1. The objective is to compare the three models with experimental tests and to evaluate the accuracy of each one. The Circus models are derived from the analysis in air (Figure 4). The sound velocity is that of Allievi (1301 m/s) for the coupled and acoustical analysis and is not modified.

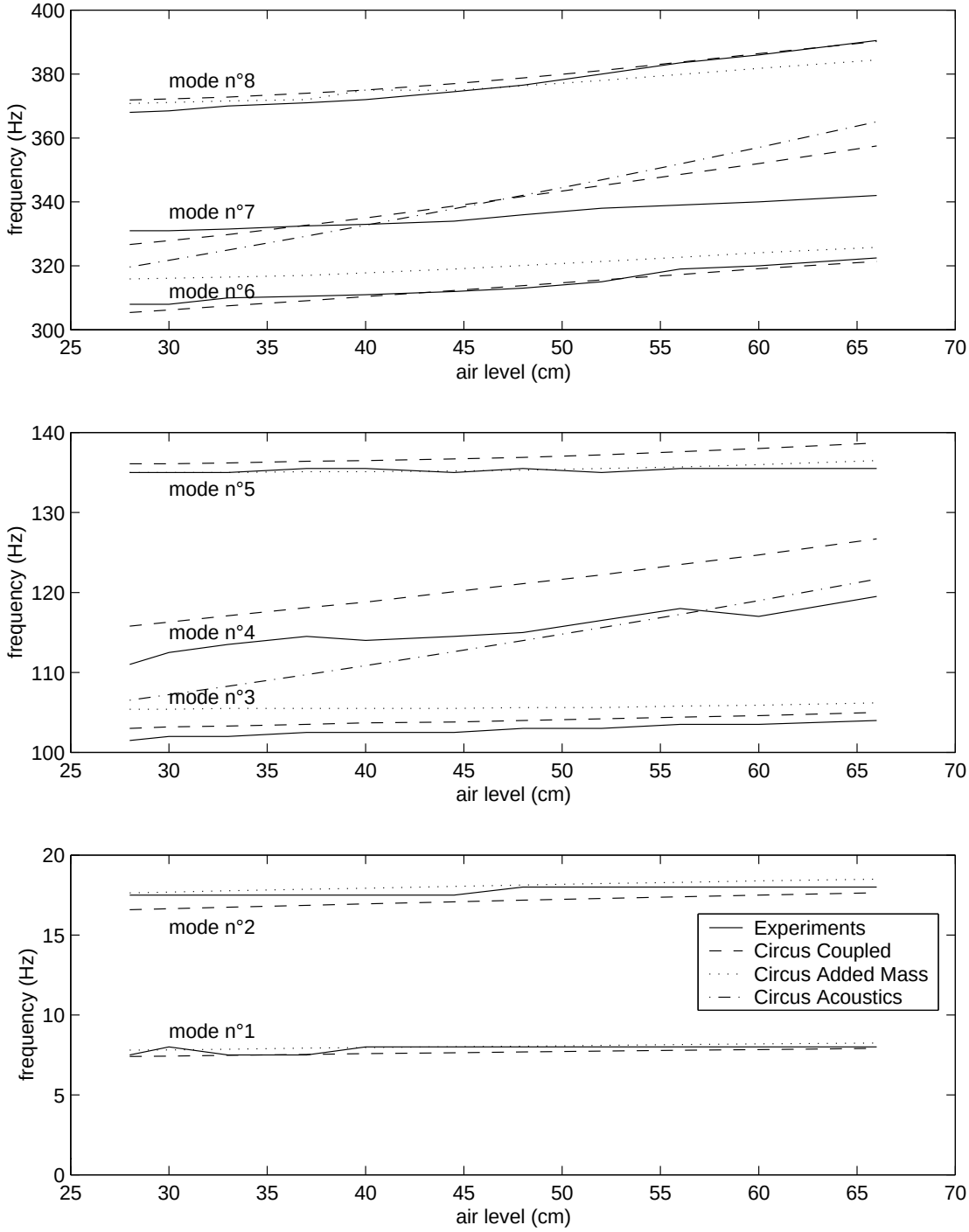

Figure 8: frequencies in water with experiments and Circus analysis

- **Simulations with Code_Aster (V6.02.00)**

The numerical model and the calculation procedure used here are identical to that of the analysis in air. Some examples of files used for Astus and Code_Aster are given in Annex B. Mode shapes for L-28 are in Annex D.

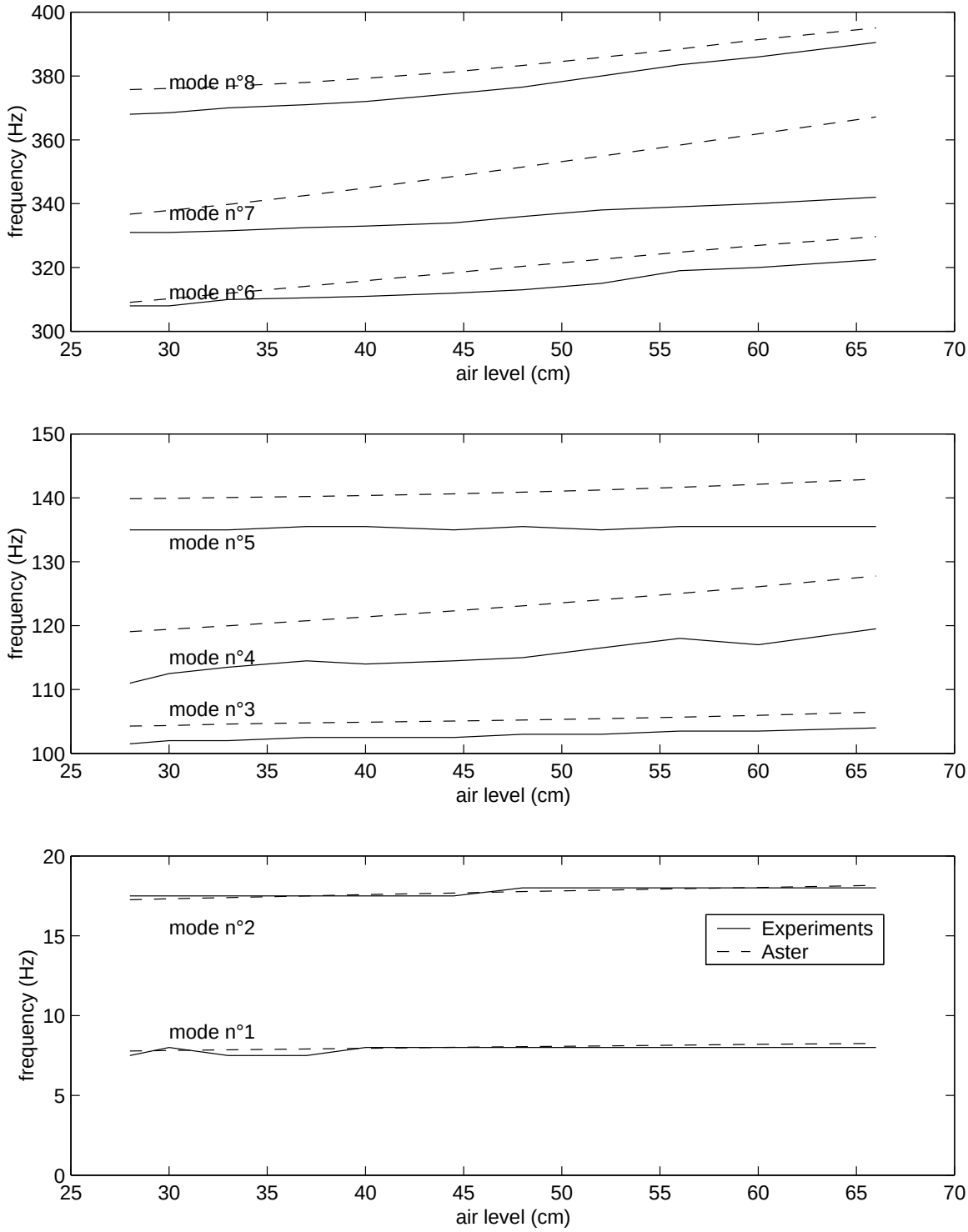

Figure 9: frequencies in water with experiments and Code_Aster analysis

4. Discussion

- **experimental and numerical results**

With the Circus Code using the fully coupled model, the error on the modal frequencies is around 3% for every modes and every air levels except for the mode n°4 where the error is of 5% (Table 7 - Annex C). This may be attributed to the estimation of the Allievi sound velocity. Nevertheless, these results validate the Circus model of Casto1 which estimates correctly the 8 modal frequencies on the range 1-500 Hz.

With Code_Aster, the same results are obtained (Table 8 - Annex C). Nevertheless, the error is a little bit more important on modes n°4, 5 and 7. This may be due to the representation of the acoustics inside the pipe as the model in air of the pipe is quite accurate (Table 2). The experimental conditions may be perfectible.

- **added mass, acoustical and fully coupled analysis**

Using the Circus code, it is possible to compare the fully coupled model with the added mass and acoustical models of Casto1 (Table 7). The added mass model can not represent the contribution of the 2 acoustic modes. The errors using the added mass and acoustical models are of the same order as the fully coupled model. This may lead to two conclusions:

1. the effects of fluid-structure coupling in the elbow are quite weak and the effects of the flexural added mass are dominant. The two acoustic modes do not modify the structural mode shapes. In Annex D, the mode shapes n°4 and 5, and n°6 and 7 are identical.
2. the coupling effects may be finally neglected which was already predicted by the design calculations (Figure 2).

This may explain the weak observation of the acoustic contributions on the accelerometers.

Finally, the coincidence of frequencies is not the only condition to get strong fluid-structure coupling. This condition corresponds to a time coincidence which must be associated to a spatial coincidence in order to get strong coupling. This spatial coincidence may be illustrated by fluid and structure wave lengths which are of the same order and by mode shapes which are compatible.

- **efforts to model the pipe**

The initial model of the pipe using the Circus code leads to an error which is of approximately 3% (Table 1). This error on the representation of the pipe in air is equivalent to that induced by the representation in water. This leads to 2 conclusions:

1. Casto1 is an analytical mock-up which boundary conditions are supposed to be well designed. Then, on an industrial piping system which boundary conditions are not well known , the error induced by the model of the pipe with no fluid may be much more important than the error induced by the representation of the fluid-structure effects.
2. It is not necessary to have an accurate representation of fluid-structure coupling as soon as the model of the pipe with no fluid is the major source of error.

5. Conclusions

1. The uses of fully coupled models with the Circus code and Code_Aster are experimentally validated on Casto1.
2. An added mass model may be sufficient to represent the one-elbow pipe as the effects of fluid-structure coupling in the elbow are shown to be weak.
3. It is necessary to pay attention to the model of the pipe with no fluid in order to get a trustful model of Casto1 filled with water.

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Annex A : Experimental set up of Casto1

The main characteristics of the cantilever pipe are the following ones:

- Outer diameter: D_e = 0.1016 m
- Thickness: t = 0.00417 m
- Young Modulus: E =1.94 10^{11} Pa
- Poisson coefficient: ν = 0.265
- Pipe density: ρ_s = 7970 kg/m^3
- Elbow curvature radius: R_c= 0.1335 m

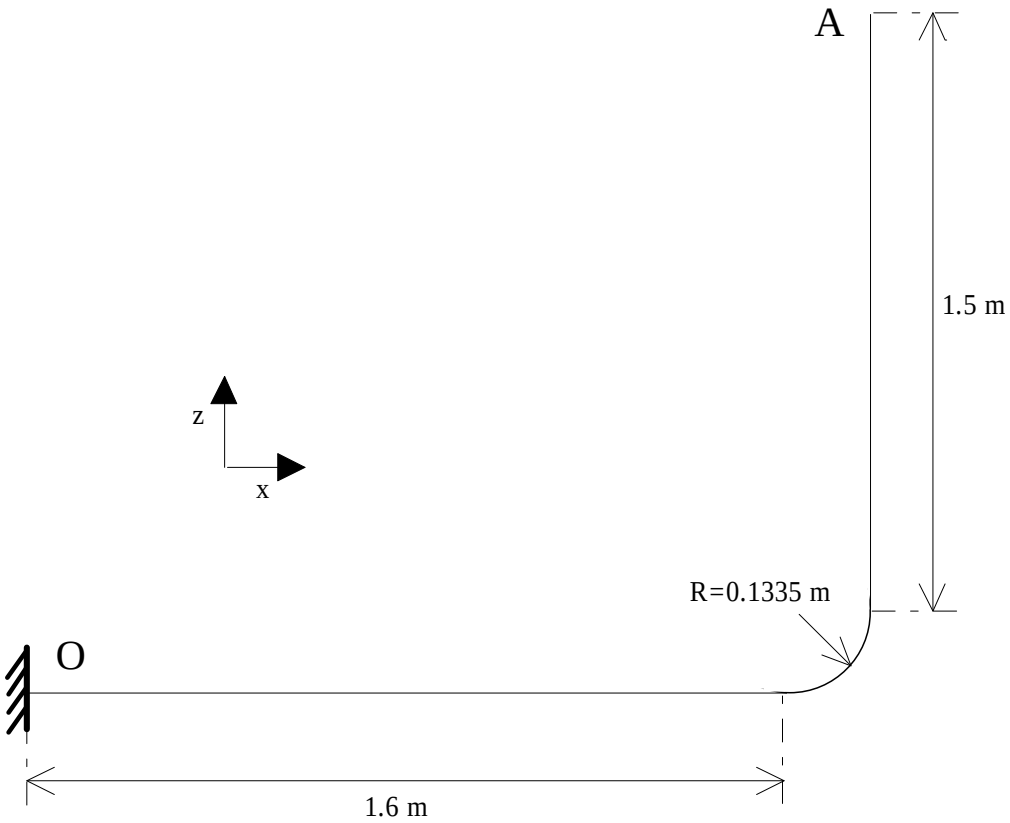

Figure 10: Final description of the experimental set-up

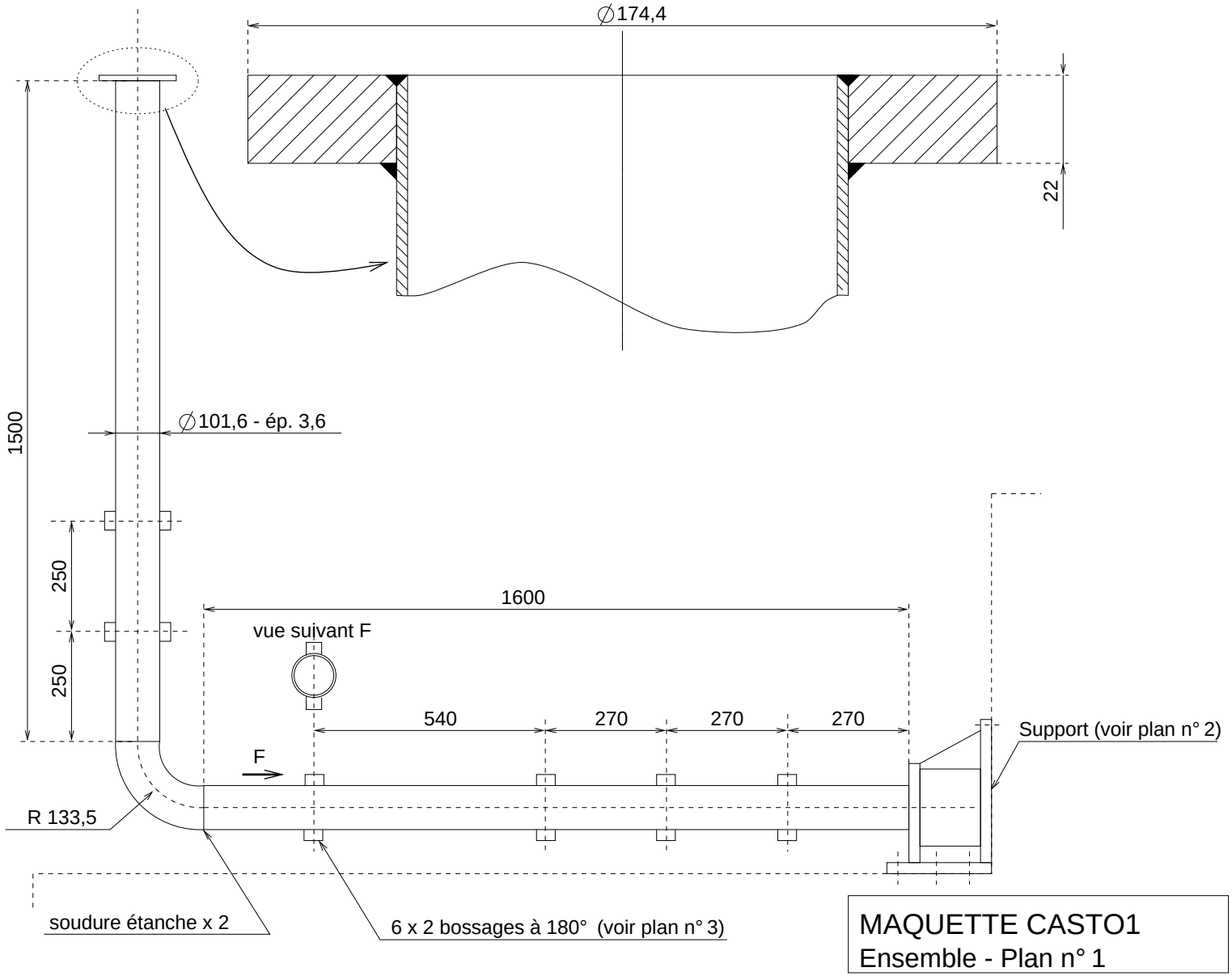

Figure 11 : final sketch of the experimental set-up

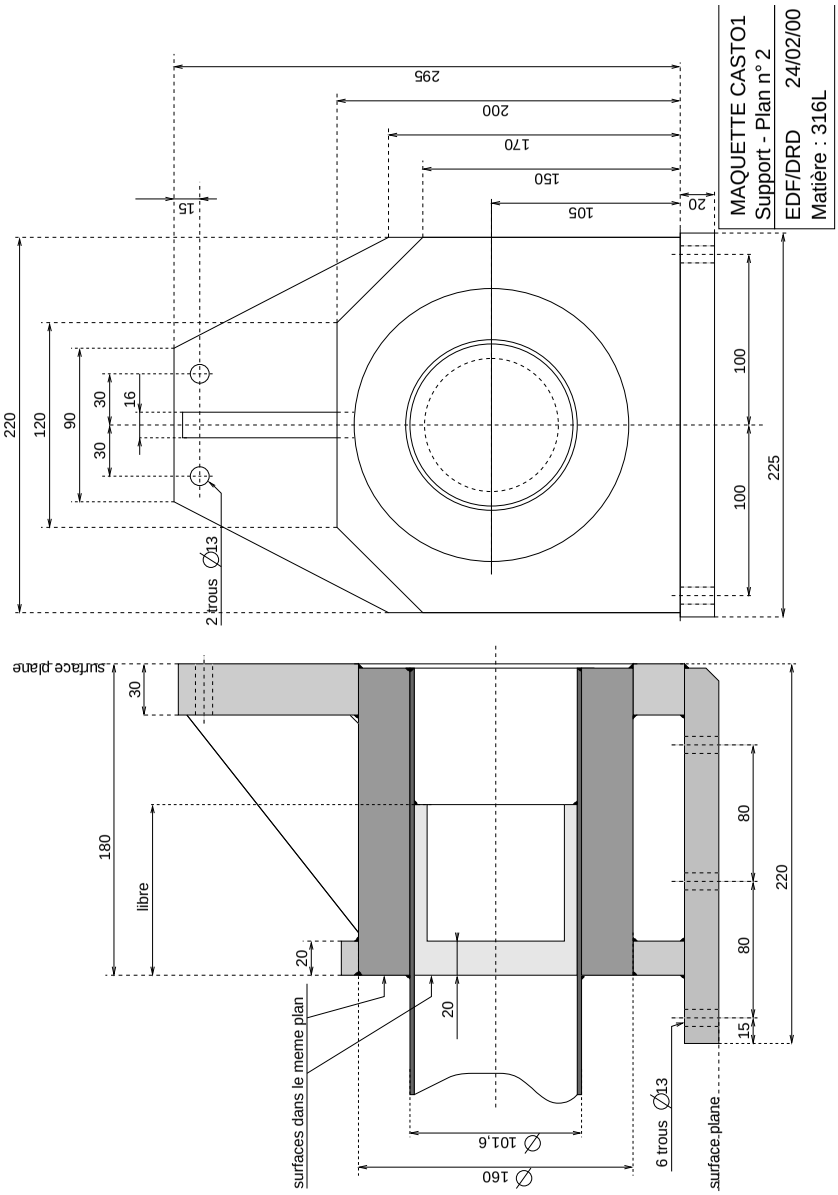

Figure 12: design of the clamped end

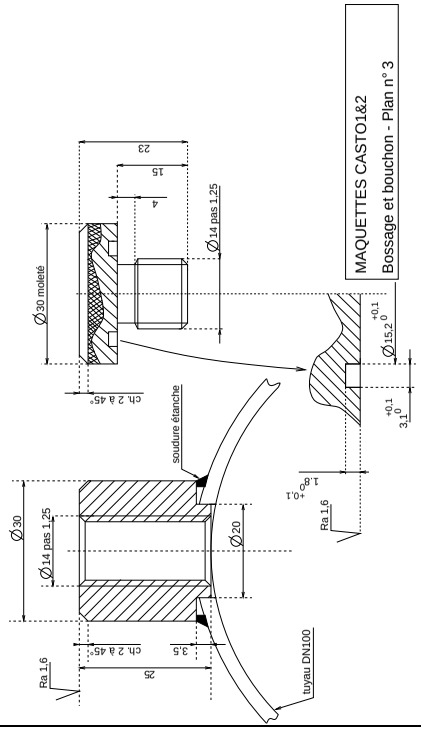

Figure 13 : design of the pressure taps

Annex B : Files

- ***donnees_circus.dat* file for Astus**

```
# ---------------------------------------------------------------------------
# fichier de donnees genere manuellement
# maquette CASTO1 recale - valeurs identique a CIRCUS recale
# ---------------------------------------------------------------------------
#
#
#lecture des noeuds        : No du noeud ; coordonnees ; rayon ext ; epaisseur ; type_fond
#lecture des elements      : No ; type CIRCUS; type GEOM (1=droit 2=convergent 3=coude 4=Te)
#                          : type COUPLAGE (1=couple 2=decouple 3=fluide seul 4=tuyau seul)
#                          : No noeud1; No noeud2; No Te (0 si pas de Te)
#                          : coordonnees centre (element coude: 5 et 6 CIRCUS; 0. pour les autres)
#                          : angle au centre (element coude: 5 et 6 CIRCUS; 0. pour les autres)
#                          : rho fluide; celerite; viscosite ; rho solide nu E amortissement
#lecture des frequences   : freq_ini ; freq_pas
#lecture des efforts       : No ; No noeud; ddl local(1->8); type (1->val 2->fichier); valeur
#                          : coefficient multiplicatif; nom du fichier
#lecture des condli        : No ; No noeud; ddl local(1->8); type (1->val 2->fichier); valeur
#                          : coefficient multiplicatif; nom du fichier
#
22        ! nombre de noeuds Circus
 1    0.0000    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 2    2.700000E-01    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 3    5.400000E-01    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 4    8.100000E-01    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 5    1.3500    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 6    1.6000    0.0000    0.0000    0.52885E-01    4.1700E-03    0
 7    1.7335    0.0000    1.335000E-01    0.52885E-01    4.1700E-03    0
 8    1.7335    0.0000    3.835000E-01    0.52885E-01    4.1700E-03    0
 9    1.7335    0.0000    6.335000E-01    0.52885E-01    4.1700E-03    0
10    1.7335    0.0000    9.955000E-01    0.52885E-01    4.1700E-03    0
11    1.7335    0.0000    1.0555    0.52885E-01    4.1700E-03    0
12    1.7335    0.0000    1.0955    0.52885E-01    4.1700E-03    0
13    1.7335    0.0000    1.1355    0.52885E-01    4.1700E-03    0
14    1.7335    0.0000    1.1755    0.52885E-01    4.1700E-03    0
15    1.7335    0.0000    1.2105    0.52885E-01    4.1700E-03    0
16    1.7335    0.0000    1.2555    0.52885E-01    4.1700E-03    0
17    1.7335    0.0000    1.2855    0.52885E-01    4.1700E-03    0
18    1.7335    0.0000    1.3255    0.52885E-01    4.1700E-03    0
19    1.7335    0.0000    1.3555    0.52885E-01    4.1700E-03    0
20    1.7335    0.0000    1.3755    0.52885E-01    4.1700E-03    0
21    1.7335    0.0000    1.6335    0.52885E-01    4.1700E-03    0
22    1.7335    0.0000    1.6555    0.52885E-01    4.1700E-03    0
21        ! nombre d"elements Circus
 1  4  1  1  1  2  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 2  4  1  1  2  3  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 3  4  1  1  3  4  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 4  4  1  1  4  5  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 5  4  1  1  5  6  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 6  6  3  1  6  7  0  1.600  0.000  1.335E-01  90.00  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 7  4  1  1  7  8  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 8  4  1  1  8  9  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
 9  4  1  1  9  10  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
10  4  1  1  10  11  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
11  4  1  1  11  12  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
```

```
12  4  1  1  12  13  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
13  4  1  1  13  14  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
14  4  1  1  14  15  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
15  4  1  1  15  16  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
16  4  1  1  16  17  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
17  4  1  1  17  18  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
18  4  1  1  18  19  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
19  4  1  1  19  20  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
20  4  1  4  20  21  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  7970.  0.265  1.940E+11  0.010
21  4  1  4  21  22  0  0.000  0.000  0.000  0.000  1000.  1450.  0.0  1.7562E+04  0.265  1.940E+11  0.010
7        ! nombre d"elements masse-ressort
1  1  2  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
2  1  3  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
3  1  4  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
4  1  5  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
5  1  8  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
6  1  9  1.750000E-01  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
7  2  1  1.00E+12  1.00E+12  1.00E+12  1.00E+12  1.00E+12  1.00E+12  0.0000  0.0000  0.0000  0.0000
2        ! nombre de frequences
1. 500.
0        ! nombre de chargements
1        ! nombre de conditions limites
1  20  7  1  0.000  0.000  0.000  NOFIC
```

- ***discretisation.dat* file for Astus**

```
'ETOILE'   ! forme de la section
1          ! elements lineaires
1          ! Orientation des elements coques rentrante
1          ! maillage des elements tes s'il y en a
6          ! Discretisation circulaire sur 90 degres
3          ! Discretisation radiale
1.         ! Pas
2                    ! iflag=1: modelisation uniforme par elem / 2:modelisation specifique par elem
5   2     ! Discretisation longitudinale et dimension geometrique de l'element  1
5   2     ! Discretisation longitudinale et dimension geometrique de l'element  2
5   2     ! Discretisation longitudinale et dimension geometrique de l'element  3
10   2     ! Discretisation longitudinale et dimension geometrique de l'element  4
4   2     ! Discretisation longitudinale et dimension geometrique de l'element  5
48   2     ! Discretisation longitudinale et dimension geometrique de l'element  6
4   2     ! Discretisation longitudinale et dimension geometrique de l'element  7
5   2     ! Discretisation longitudinale et dimension geometrique de l'element  8
7   2     ! Discretisation longitudinale et dimension geometrique de l'element  9
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  10
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  11
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  12
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  13
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  14
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  15
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  16
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  17
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  18
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  19
5   2     ! Discretisation longitudinale et dimension geometrique de l'element  20
1   2     ! Discretisation longitudinale et dimension geometrique de l'element  21
```

- ***astus.comm* command file for Code_Aster used for L-66**

```
# ************************* ASTUS V1.0 ****************************************
# GENERATION AUTOMATIQUE DU FICHIER ASTUS.COMM POUR LE CODE_ASTER
# POUR DES CALCULS DE TUYAUTERIES AVEC COUPLAGE FLUIDE-STRUCTURE
# REPERTOIRE DE TRAVAIL : .
#
# VERSION ASTER COMPATIBLE : 6
#
# DEMARRAGE DU FICHIER DE COMMANDES ET LECTURE DU MAILLAGE
# ======================================================================
DEBUT();
#
MAIL0=LIRE_MAILLAGE();
MAIL=CREA_MAILLAGE(
   MAILLAGE=MAIL0,
   CREA_POI1=(_F(NOM_GROUP_MA='PMRA1',
         GROUP_NO='NMRA1',),
       _F(NOM_GROUP_MA='PMRA2',
         GROUP_NO='NMRA2',),
       _F(NOM_GROUP_MA='PMRA3',
         GROUP_NO='NMRA3',),
       _F(NOM_GROUP_MA='PMRA4',
         GROUP_NO='NMRA4',),
       _F(NOM_GROUP_MA='PMRA5',
         GROUP_NO='NMRA5',),
       _F(NOM_GROUP_MA='PMRA6',
         GROUP_NO='NMRA6',),
       _F(NOM_GROUP_MA='PMRA7',
         GROUP_NO='NMRA7',),
   ),
);
#
# AFFECTATION DE LA MODELISATION
# ==============================
MODELE=AFFE_MODELE(
   INFO=2,
   MAILLAGE=MAIL,
   AFFE=(_F(GROUP_MA='EF3D1',
         PHENOMENE='MECANIQUE',
         MODELISATION='3D_FLUIDE',),
       _F(GROUP_MA='EI2D1',
         PHENOMENE='MECANIQUE',
         MODELISATION='FLUI_STRU',),
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    MODELISATION='DIS_T',),
  _F(GROUP_MA='PMRA6',
    PHENOMENE='MECANIQUE',
    MODELISATION='DIS_T',),
```

```
        _F(GROUP_MA='PMRA7',
          PHENOMENE='MECANIQUE',
          MODELISATION='DIS_TR',),
   ),
);
#
# PROPRIETES MECANIQUES
# ======================
# DEFINITION DES MATERIAUX
# ------------------------
ACIER=DEFI_MATERIAU(
   ELAS=_F(RHO= 0.78000E+04,
          NU= 0.30000E+00,
          AMOR_HYST= 0.00000E+00,
          E= 0.21000E+12,),
);
M_FL1   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO1   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL2   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO2   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL3   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO3   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL4   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO4   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL5   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO5   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL6   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO6   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL7   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO7   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL8   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO8   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL9   =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO9   =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL10  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO10  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL11  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO11  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
```

```
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL12  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO12  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL13  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO13  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL14  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO14  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL15  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO15  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL16  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO16  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL17  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO17  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL18  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO18  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_FL19  =DEFI_MATERIAU(
   FLUIDE=_F(RHO= 0.10000E+04,
          CELE_R= 0.14500E+04,),
);
M_SO19  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_SO20  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.79700E+04,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
M_SO21  =DEFI_MATERIAU(
   ELAS=_F(RHO= 0.10449E+05,
          NU= 0.26500E+00,
          AMOR_HYST= 0.10000E-01,
          E= 0.19400E+12,),
);
#
# AFFECTATION DES MATERIAUX
# -------------------------
CH_MAT=AFFE_MATERIAU(
   MAILLAGE=MAIL,
   AFFE=(_F(GROUP_MA='EF3D1',
          MATER=M_FL1,),
        _F(GROUP_MA='EI2D1',
          MATER=M_FL1,),
        _F(GROUP_MA='ES2D1',
          MATER=M_SO1,),
        _F(GROUP_MA='EF3D2',
          MATER=M_FL2,),
        _F(GROUP_MA='EI2D2',
          MATER=M_FL2,),
        _F(GROUP_MA='ES2D2',
          MATER=M_SO2,),
        _F(GROUP_MA='EF3D3',
          MATER=M_FL3,),
        _F(GROUP_MA='EI2D3',
          MATER=M_FL3,),
        _F(GROUP_MA='ES2D3',
          MATER=M_SO3,),
        _F(GROUP_MA='EF3D4',
          MATER=M_FL4,),
        _F(GROUP_MA='EI2D4',
```

```
      MATER=M_FL4,),
    _F(GROUP_MA='ES2D4',
      MATER=M_SO4,),
    _F(GROUP_MA='EF3D5',
      MATER=M_FL5,),
    _F(GROUP_MA='EI2D5',
      MATER=M_FL5,),
    _F(GROUP_MA='ES2D5',
      MATER=M_SO5,),
    _F(GROUP_MA='EF3D6',
      MATER=M_FL6,),
    _F(GROUP_MA='EI2D6',
      MATER=M_FL6,),
    _F(GROUP_MA='ES2D6',
      MATER=M_SO6,),
    _F(GROUP_MA='EF3D7',
      MATER=M_FL7,),
    _F(GROUP_MA='EI2D7',
      MATER=M_FL7,),
    _F(GROUP_MA='ES2D7',
      MATER=M_SO7,),
    _F(GROUP_MA='EF3D8',
      MATER=M_FL8,),
    _F(GROUP_MA='EI2D8',
      MATER=M_FL8,),
    _F(GROUP_MA='ES2D8',
      MATER=M_SO8,),
    _F(GROUP_MA='EF3D9',
      MATER=M_FL9,),
    _F(GROUP_MA='EI2D9',
      MATER=M_FL9,),
    _F(GROUP_MA='ES2D9',
      MATER=M_SO9,),
    _F(GROUP_MA='EF3D10',
      MATER=M_FL10,),
    _F(GROUP_MA='EI2D10',
      MATER=M_FL10,),
    _F(GROUP_MA='ES2D10',
      MATER=M_SO10,),
    _F(GROUP_MA='EF3D11',
      MATER=M_FL11,),
    _F(GROUP_MA='EI2D11',
      MATER=M_FL11,),
    _F(GROUP_MA='ES2D11',
      MATER=M_SO11,),
    _F(GROUP_MA='EF3D12',
      MATER=M_FL12,),
    _F(GROUP_MA='EI2D12',
      MATER=M_FL12,),
    _F(GROUP_MA='ES2D12',
      MATER=M_SO12,),
    _F(GROUP_MA='EF3D13',
      MATER=M_FL13,),
    _F(GROUP_MA='EI2D13',
      MATER=M_FL13,),
    _F(GROUP_MA='ES2D13',
      MATER=M_SO13,),
    _F(GROUP_MA='EF3D14',
      MATER=M_FL14,),
    _F(GROUP_MA='EI2D14',
      MATER=M_FL14,),
    _F(GROUP_MA='ES2D14',
      MATER=M_SO14,),
    _F(GROUP_MA='EF3D15',
      MATER=M_FL15,),
    _F(GROUP_MA='EI2D15',
      MATER=M_FL15,),
    _F(GROUP_MA='ES2D15',
      MATER=M_SO15,),
    _F(GROUP_MA='EF3D16',
      MATER=M_FL16,),
    _F(GROUP_MA='EI2D16',
      MATER=M_FL16,),
    _F(GROUP_MA='ES2D16',
      MATER=M_SO16,),
    _F(GROUP_MA='EF3D17',
      MATER=M_FL17,),
    _F(GROUP_MA='EI2D17',
      MATER=M_FL17,),
    _F(GROUP_MA='ES2D17',
      MATER=M_SO17,),
    _F(GROUP_MA='EF3D18',
      MATER=M_FL18,),
    _F(GROUP_MA='EI2D18',
      MATER=M_FL18,),
    _F(GROUP_MA='ES2D18',
      MATER=M_SO18,),
    _F(GROUP_MA='EF3D19',
      MATER=M_FL19,),
    _F(GROUP_MA='EI2D19',
      MATER=M_FL19,),
    _F(GROUP_MA='ES2D19',
      MATER=M_SO19,),
    _F(GROUP_MA='ES2D20',
      MATER=M_SO20,),
    _F(GROUP_MA='ES2D21',
      MATER=M_SO21,),
  ),
);
#
# AFFECTATION DES CARACTERISTIQUES
ELEMENTAIRES (ELEM POU,COQ,DISCRET)
# ------------------------------------------------------------
----
CARAELEM=AFFE_CARA_ELEM(
  MODELE=MODELE,
  COQUE=(_F(GROUP_MA='ES2D1',
      ANGL_REP=(0.,0.),
      EPAIS= 0.41700E-02,),
    _F(GROUP_MA='ES2D2',
      ANGL_REP=(0.,0.),
      EPAIS= 0.41700E-02,),
    _F(GROUP_MA='ES2D3',
      ANGL_REP=(0.,0.),
      EPAIS= 0.41700E-02,),
    _F(GROUP_MA='ES2D4',
      ANGL_REP=(0.,0.),
      EPAIS= 0.41700E-02,),
    _F(GROUP_MA='ES2D5',
      ANGL_REP=(0.,0.),
      EPAIS= 0.41700E-02,),
    _F(GROUP_MA='ES2D6',
```

```
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D7',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D8',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D9',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D10',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D11',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D12',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D13',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D14',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D15',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D16',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D17',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D18',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D19',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D20',
        ANGL_REP=(0.,0.),
        EPAIS= 0.41700E-02,),
      _F(GROUP_MA='ES2D21',
        ANGL_REP=(0.,0.),
        EPAIS= 0.40000E-01,),
  ),
  DISCRET=(_F(GROUP_MA='PMRA1',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA2',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA3',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA4',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA5',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA6',
        VALE= 0.17500E+00,
        CARA='M_T_D_N',),
      _F(GROUP_MA='PMRA7',
        VALE=(    0.10000E+13,    0.10000E+13,
0.10000E+13,
            0.10000E+13,            0.50000E+08,
0.50000E+08),
        CARA='K_TR_D_N',),
    ),
);
#
# CONDITIONS LIMITES ET CHARGEMENTS
# ================================
# CONDITIONS LIMITES
# -------------------
CONDLI=AFFE_CHAR_MECA(
   MODELE=MODELE,
   DDL_IMPO=_F(GROUP_NO='NFPT20',
        PRES    = 0.00000E+00,
        PHI     =0.,),
   LIAISON_SOLIDE=_F(GROUP_NO=('NMRA7
','NSPT1   '),),
);
#
# CALCUL MODAL
# =============
# MATRICES ET VECTEURS ELEMENTAIRES
EN MODAL
# ----------------------------------------
RIGELMOD=CALC_MATR_ELEM(
   MODELE=MODELE,
   CHAM_MATER=CH_MAT,
   CARA_ELEM=CARAELEM,
   CHARGE=CONDLI,
   OPTION='RIGI_MECA',
);
#
MASELMOD=CALC_MATR_ELEM(
   MODELE=MODELE,
   CHAM_MATER=CH_MAT,
   CARA_ELEM=CARAELEM,
   CHARGE=CONDLI,
   OPTION='MASS_MECA',
);
#
# ASSEMBLAGE DES MATRICES ET
VECTEURS ELEMENTAIRES
# ---------------------------------------------
NUDDLMOD=NUME_DDL(
   MATR_RIGI=RIGELMOD,
   METHODE='MULT_FRONT',
   RENUM='MD',
);
#
RIGASMOD=ASSE_MATRICE(
   MATR_ELEM=RIGELMOD,
   NUME_DDL=NUDDLMOD,
);
```

```
#
MASASMOD=ASSE_MATRICE(
   MATR_ELEM=MASELMOD,
   NUME_DDL=NUDDLMOD,
);
#
# CALCUL MODAL DES MODES ENTRE F(1) ET
F(NB_FREQ)
# ----------------------------------------------
MODES=MODE_ITER_SIMULT(
   MATR_A=RIGASMOD,
   MATR_B=MASASMOD,
   CALC_FREQ=_F(OPTION='BANDE',
         FREQ=( 0.10000E+01, 0.50000E+03),),
   VERI_MODE=_F(STOP_ERREUR='NON',),
);
#
IMPR_RESU(
   RESU=_F(RESULTAT=MODES,
         TOUT_CHAM='NON',
         TOUT_PARA='OUI',),
);
#
FIN();
```

Annex C : Tables

Mode	*Frequency (Hz)*	*Damping (%)*
In-plane X excitation		
1	8.99	3.94
2	20.80	0.6
3	129.46	0.12
4	169.89	0.14
5	402.67	0.06
6	464.93	0.08
In-plane Z excitation		
1	8.89	2.43
2	20.79	0.59
3	129.50	0.11
4	169.87	0.15
5	402.71	0.07
6	464.85	0.09
Out-of-plane Y excitation		
1	9.18	2.28
2	25.32	0.5
3	127.25	0.11
4	188.45	0.08
5	286.30	0.07
6	408.89	0.07

Table 3: in-plane and out-of-plane EXPERIMENTAL results in AIR

Air level (cm)	*Sound velocity (m/s)*
L-28	1263
L-33	1310
L-37	1310
L-40	1263
L-44.5	1263
L-48	1263
L-52	1263
L-56	1310
L-60	1310
L-66	1263

Table 4: EXPERIMENTAL sound velocities inside the pipe

Air level (cm)	*Frequencies using accelerometers (Hz)*					
	mode n°1	*mode n°2*	*mode n°3*	*mode n°4*	*mode n°5*	*mode n°8*
L-28	7.5	17.5	101.5	**111**	135	368
L-30	8	17.5	102	**112.5**	135	368.5
L-33	7.5	17.5	102	**113.5**	135	370
L-37	7.5	17.5	102.5	**114.5**	135.5	371
L-40	8	17.5	102.5	**114**	135.5	372
L-44,5	8	17.5	102.5	**114.5**	135	**374.5**
L-48	8	18	103	**115**	135.5	376.5
L-52	8	18	103	**116.5**	135	380
L-56	8	18	103.5	**118**	135.5	**383.5**
L-60	8	18	103.5	**117**	135.5	**386**
L-66	8	18	104	**119.5**	135.5	**390.5**

Table 5: experimental frequencies in water using accelerometers

Air level (cm)	*Frequencies using pressure sensors (Hz)*							
	mode n°1	*mode n°2*	*mode n°3*	*mode n°4*	*mode n°5*	*mode n°6*	*mode n°7*	*mode n°8*
L-28	7.5	17.5	101.5	**111**	135	308	331	368
L-30	8	17.5	102	**112.5**	135	308	331	368.5
L-33	7.5	17.5	102	**113.5**	135	310	331.5	370
L-37	7.5	17.5	102.5	**114.5**	135.5	310.5	332.5	371
L-40	8	17.5	102.5	**114**	135.5	311	333	372
L-44,5	8	17.5	102.5	**114.5**	135	312	334	**374.5**
L-48	8	18	103	**115**	135.5	313	336	376.5
L-52	8	18	103	**116.5**	135		338	380
L-56	8	18	103.5	**118**	135.5	**319**	339	**383.5**
L-60	8	18	103.5	**117**	135.5	**320**	340	**386**
L-66	8	18	104	**119.5**	135.5	**322.5**	342	**390.5**

Table 6: experimental frequencies in water using pressure sensors

air level (cm)	*mode n°1*					*mode n°2*				
	exp.	*coupled*	*error (%)*	*added mass*	*error (%)*	*exp.*	*coupled*	*error (%)*	*added mass*	*error (%)*
L-28	7,5	7.41	-1.2	7.80	4.0	17,5	16.59	-5.2	17.64	0.8
L-30	8	7.43	-7.1	7.82	-2.2	17,5	16.65	-4.8	17.69	1.1
L-33	7,5	7.48	-0.3	7.86	4.8	17,5	16.74	-4.3	17.77	1.5
L-37	7,5	7.53	-.4	7.93	5.7	17,5	16.86	-3.6	17.87	2.1
L-40	8	7.58	-5.2	7.96	-0.5	17,5	16.95	-3.1	17.93	2.5
L-44,5	8	7.64	-4.5	8.01	0.1	17,5	17.08	-2.4	18.05	3.1
L-48	8	7.69	-3.9	8.05	0.6	18	17.18	-4.6	18.13	0.7
L-52	8	7.74	-3.2	8.10	1.2	18	17.29	-3.9	18.22	1.2
L-56	8	7.79	-2.6	8.15	1.9	18	17.40	-3.3	18.30	1.7
L-60	8	7.84	-2.0	8.19	2.4	18	17.50	-2.8	18.39	2.2
L-66	8	7.91	-1.1	8.25	3.1	18	17.65	-1.9	18.50	2.8

Air level (cm)	*mode n°3*					*mode n°4*				
	exp.	*coupled*	*error (%)*	*added mass*	*error (%)*	*exp.*	*coupled*	*error (%)*	*acoustics*	*error (%)*
L-28	101,5	103.0	1.5	105.4	3.8	111	115.8	4.3	106.5	-4.1
L-30	102	103.2	1.2	105.4	3.3	112,5	116.3	3.4	107.2	-4.7
L-33	102	103.3	1.3	105.5	3.4	113,5	117.1	3.2	108.3	-4.6
L-37	102,5	103.5	1.0	105.5	2.9	114,5	118.1	3.1	109.7	-4.2
L-40	102,5	103.7	1.2	105.5	2.9	114	118.8	4.2	110.9	-2.7
L-44,5	102,5	103.8	1.3	105.5	2.9	114,5	120.1	4.9	112.6	-1.7
L-48	103	104.0	1.0	105.6	2.5	115	121.1	5.3	114.0	-0.9
L-52	103	104.2	1.2	105.6	2.5	116,5	122.2	4.9	115.6	-0.8
L-56	103,5	104.4	0.9	105.8	2.2	118	123.5	4.7	117.3	-0.6
L-60	103,5	104.6	1.1	105.9	2.3	117	124.7	6.6	119.0	1.7
L-66	104	105.0	1.0	106.2	2.1	119,5	126.7	6.0	121.7	1.8

Air level (cm)	*mode n°5*					*mode n°6*				
	exp.	*coupled*	*error (%)*	*added mass*	*error (%)*	*exp.*	*coupled*	*error (%)*	*added mass*	*error (%)*
L-28	135	136.1	0.8	135.0	0	308	305.4	-0.8	315.9	2.6
L-30	135	136.1	0.8	135.0	0	308	306.2	-0.6	316.1	2.6
L-33	135	136.2	0.9	135.0	0	310	307.5	-0.8	316.5	2.1
L-37	135,5	136.4	0.7	135.1	-0.3	310,5	309.1	-0.4	317.0	2.1
L-40	135,5	136.5	0.7	135.1	-0.3	311	310.4	-0.2	317.8	2.2
L-44,5	135	136.7	1.3	135.2	0.1	312	312.3	0.1	319.0	2.2
L-48	135,5	136.9	1.0	135.3	-0.1	313	313.8	0.3	320.1	2.3
L-52	135	137.2	1.6	135.5	0.4	-	315.6	-	321.4	-
L-56	135,5	137.6	1.5	135.7	0.1	319	317.3	-0.5	322.7	1.2
L-60	135,5	138.0	1.8	136.0	0.4	320	319.1	-0.3	324.1	1.3
L-66	135,5	138.7	2.4	136.5	0.7	322,5	321.4	-0.3	325.8	1.0

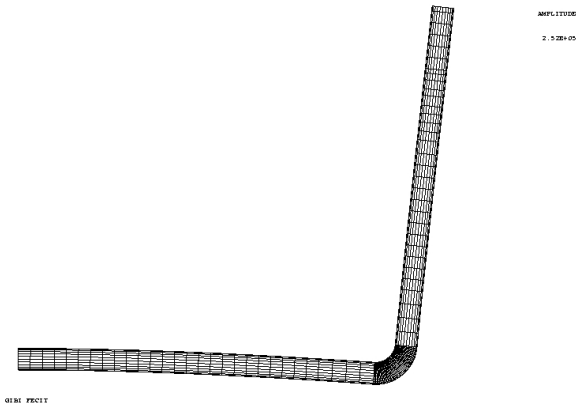
Air level (cm)	*mode n°7*					*mode n°8*				
	exp.	*coupled*	*error (%)*	*acoustics*	*error (%)*	*exp.*	*coupled*	*error (%)*	*added mass*	*error (%)*
L-28	331	326.7	-1.3	319.6	-3.4	368	371.9	1.1	370.8	0.8
L-30	331	327.9	-0.9	321.7	-2.8	368,5	372.2	1.0	371.1	0.7
L-33	331,5	329.8	-0.5	324.9	-2.0	370	372.8	0.8	371.6	0.4
L-37	332,5	332.7	0.1	329.3	-1.0	371	374.0	0.8	372.0	0.3
L-40	333	335.0	0.6	332.7	-0.1	372	375.0	0.8	375.0	0.8
L-44,5	334	338.7	1.4	337.9	1.2	374,5	377.0	0.7	374.9	0.1
L-48	336	341.7	1.7	342.0	1.8	376,5	378.8	0.6	376.3	-0.1
L-52	338	345.1	2.1	346.9	2.6	380	381.1	0.3	378.0	-0.5
L-56	339	348.6	2.8	351.9	3.8	383,5	383.7	0.1	379.9	-0.9
L-60	340	352.0	3.5	357.1	5.0	386	386.4	0.1	381.8	-1.1
L-66	342	357.5	4.5	365.1	6.8	390,5	390.2	-0.1	384.4	-1.6

Table 7: frequencies (Hz) in water with experiments and Circus analysis

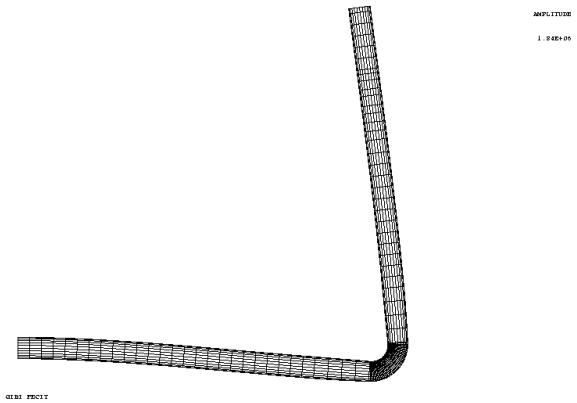
Air level (cm)	*mode n°1*			*mode n°2*		
	exp.	*coupled*	*error (%)*	*exp.*	*coupled*	*error (%)*
L-28	7.5	7.78	3.7	17.5	17.26	-1.4
L-30	8	7.81	-2.4	17.5	17.32	-1.0
L-33	7.5	7.86	4.8	17.5	17.40	-0.6
L-37	7.5	7.91	5.5	17.5	17.50	0.0
L-40	8	7.95	-0.6	17.5	17.58	0.5
L-44,5	8	8.01	0.1	17.5	17.68	1.0
L-48	8	8.06	0.8	18	17.77	-1.3
L-52	8	8.10	1.3	18	17.86	-0.8
L-56	8	8.15	1.9	18	17.95	-0.3
L-60	8	8.20	2.5	18	18.03	0.2
L-66	8	8.26	3.3	18	18.16	0.9
Air level (cm)	*mode n°3*			*mode n°4*		
	exp.	*coupled*	*error (%)*	*exp.*	*coupled*	*error (%)*
L-28	101.5	104.28	2.7	111	119.05	7.3
L-30	102	104.36	2.3	112.5	119.42	6.2
L-33	102	104.60	2.5	113.5	119.98	5.7
L-37	102.5	104.77	2.2	114.5	120.76	5.5
L-40	102.5	104.89	2.3	114	121.36	6.5
L-44,5	102.5	105.07	2.5	114.5	122.31	6.8
L-48	103	105.23	2.2	115	123.10	7.0
L-52	103	105.43	2.4	116.5	124.03	6.5
L-56	103.5	105.66	2.1	118	125.03	6.0
L-60	103.5	105.95	2.4	117	126.08	7.8
L-66	104	106.45	2.4	119.5	127.76	6.9
Air level (cm)	*mode n°5*			*mode n°6*		
	exp.	*coupled*	*error (%)*	*exp.*	*coupled*	*error (%)*
L-28	135	139.86	3.6	308	309.10	0.4
L-30	135	139.92	3.6	308	310.24	0.7
L-33	135	140.04	3.7	310	311.93	0.6
L-37	135.5	140.21	3.5	310.5	314.14	1.2
L-40	135.5	140.36	3.6	311	315.85	1.6
L-44,5	135	140.64	4.2	312	318.37	2.0
L-48	135.5	140.89	4.0	313	320.36	2.4
L-52	135	141.24	4.6	-	322.61	-
L-56	135.5	141.65	4.5	319	324.79	1.8
L-60	135.5	142.13	4.9	320	326.94	2.2
L-66	135.5	142.95	5.5	322.5	329.68	2.2
Air level (cm)	*mode n°7*			*mode n°8*		
	exp.	*coupled*	*error (%)*	*exp.*	*coupled*	*error (%)*
L-28	331	336.70	1.7	368	375.71	2.1
L-30	331	337.86	2.1	368.5	376.08	2.1
L-33	331.5	339.78	2.5	370	376.80	1.8
L-37	332.5	342.61	3.0	371	378.01	1.9
L-40	333	344.93	3.6	372	379.24	1.9
L-44,5	334	348.55	4.4	374.5	381.32	1.8
L-48	336	351.49	4.6	376.5	383.31	1.8
L-52	338	354.90	5.0	380	385.80	1.5
L-56	339	358.35	5.7	383.5	388.47	1.3
L-60	340	361.85	6.4	386	391.37	1.4
L-66	342	367.20	7.4	390.5	395.08	1.2

Table 8: frequencies (Hz) in water with experiments and Code_Aster analysis

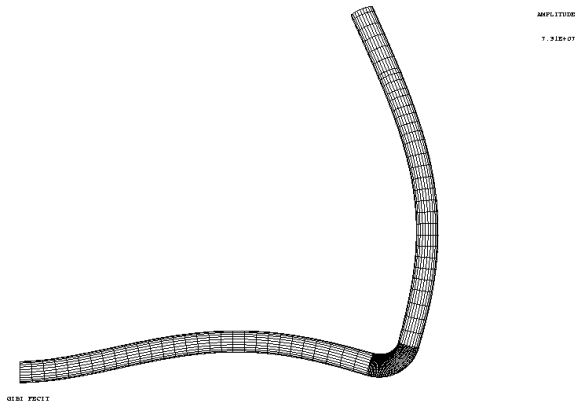
Annex D : Code_Aster mode shapes for L-28



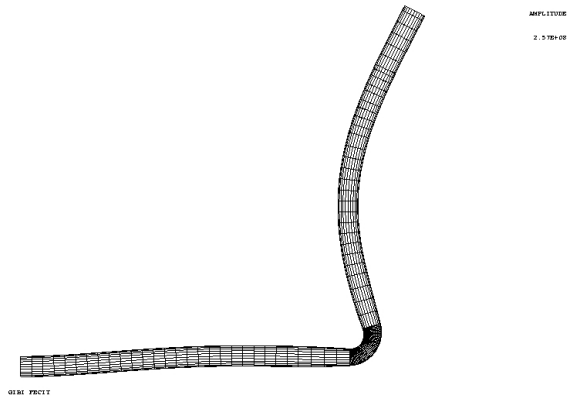
mode n°1 - 7.78 Hz



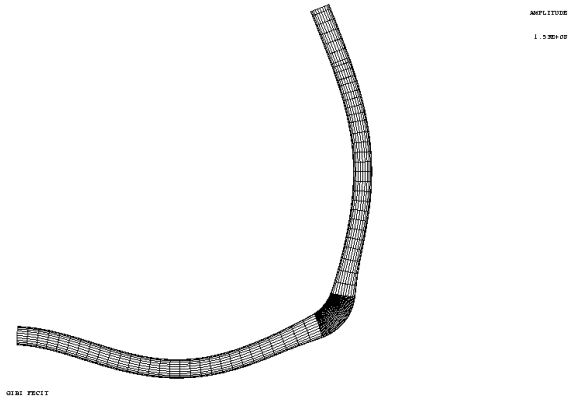
mode n°2 - 17.26 Hz



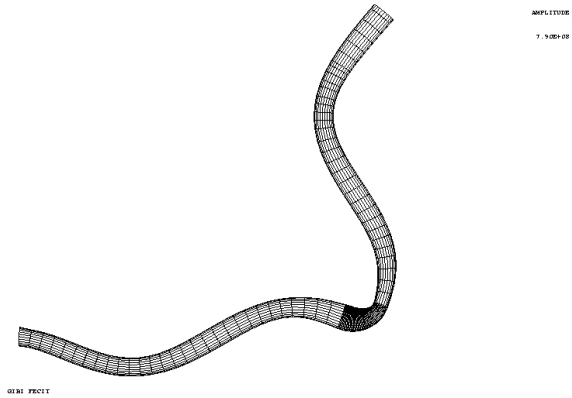
mode n°3 - 104.28 Hz



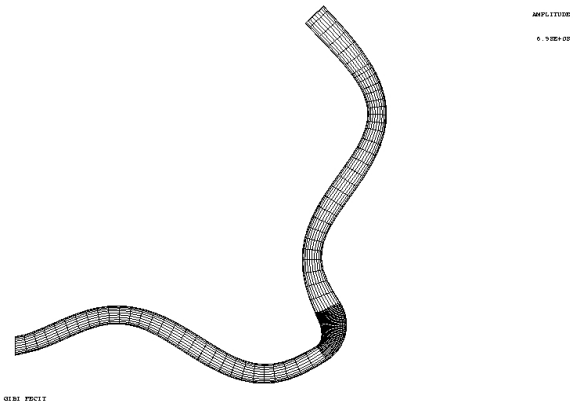
mode n°4 - 119.05 Hz



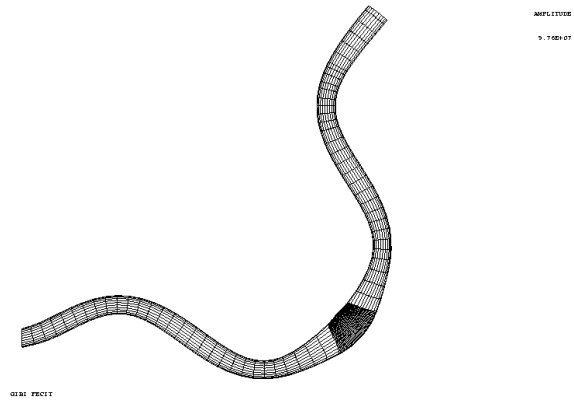
mode n°5 - 139.86 Hz



mode n°6 - 309.10 Hz



mode n°7 - 336.70 Hz



mode n°8 - 375.71 Hz