Energy constraint flow for wireless sensor networks

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Overview

Problem statement

Easier cases

Approximations

Hardness

Conclusions
Wireless sensor networks

- Wireless sensor - "smart dust" - *motes*
- Microchip and miniature battery
- Batteries are typical limited and run out
- Limited range transmit/receive capacity
Ad hoc networks

- Motes have sizes 6 by 3 by 0.6 cm and smaller
- Motes form ad hoc networks
- Monitoring applications, e.g.:
  - monitoring bridges (embed motes in concrete)
  - monitoring condition of machines
  - monitoring water meters
  - monitoring pollution and water level in nature
  - military applications
A mote
Data gathering problem

- A base station
- Some sensors have information that must be sent to base station
- Sensors can transmit messages
- Energy of sensors is limited and not replenishable
- Receiving and transmitting costs energy
- Processing energy can be neglected
- Energy of sending a message depends on distance of target

- Network $G = (V, A)$
- Sink node $t$
- Set of source nodes $S$
- Each node has a battery capacity (energy) $E_i$
- For each arc $ij$, node $i$ must spend $e_{ij}$ energy to transmit a packet to $j$
- What is the maximum number of packets that can be sent to $t$?

Energy constraint version of flow
(I)LP formulation

Maximize $F = \sum_{j \in V} f_{jt}$ (t sink node), subject to:

- $f_{ij} \geq 0$ for all $ij \in A$
- $\sum_{j \in V} f_{ij} = \sum_{j \in V} f_{ji}$ for all $i \in V - S - \{t\}$
- $\sum_{j \in V} f_{ij} \cdot e_{ij} \leq E_i$ for all $i \in V$

Two variants: allowing fractional solutions, or demanding integer solutions:

- $f_{ij} \in \mathbb{N}$ for all $ij \in A$
Easier cases I: Equal costs

- If all $e_{ij}$ are equal, then problem is polynomial time solvable
- Maximum flow with node capacities
- Works for integral and for fractional case
Easier cases II: Fractional case

- The fractional case is – of course – solvable in polynomial time
- LP
- Interesting open problem: *Is there a combinatorial polynomial time algorithm (i.e., not depending on ellipsoid method)?*
- Recent research: comparing heuristics with LP solution
- We now look to integral case
Approximation algorithm 1

- Polynomial time approximation is possible with a multiplicative factor

\[ \rho = \max_{i \in V} \frac{\max_{j,i,j \in A} e_{ij}}{\min_{j,i,j \in A} e_{ij}} \]

- Set all costs \( e_{ij} \) to \( \max_{j,i,j \in A} e_{ij} \).
- Gives a polynomial solvable instance with quality \( \rho \)
Approximation algorithm II

- First solve the fractional LP-formulation, and obtain flow $f^*$
- Round down to integral flow $f$ as follows
- Start with $f = 0$ everywhere
- While there is a path $p$ from a source $s$ to the sink $t$ with for all arcs $ij$ on $p$ $f^*_{ij} - f^*_{ji} \geq 1$, do
  - Let $f_p = \min_{ij \in P} [f^*_{ij} - f_{ij}]$
  - For all $ij \in P$, increase $f^*_{ij}$ by $f_p$
Analysis of approximation algorithm II

- Note: $f^* \leq f$; $f^*$ is integral solution
- Each round, we gain at least one arc with $f^*_{ij} - f^*_{ji} < 1$
- $O(|A|)$ rounds, so $O(|A|^2)$ time
- Let $Q$ be all vertices, reachable from a source with a path with all arcs $f^*_{ij} - f^*_{ji} \geq 1$
- The capacity of the cut $(Q, V - Q)$ is less than $|A|$

Theorem

Approximation algorithm II finds in $O(|A|^2)$ time an integral flow whose value is at least the value of the maximum fractional flow minus $|A|$
The Maximum Flow in Wireless Sensor Networks with Energy Constraints problem is NP-hard, even when one of the following holds:

- There is only one source
- Energies are given in unary (strongly NP-hard)
- Nodes are points in the 2d-planes, and for all $i, j$, $e_{ij}$ is the square of the distance from $i$ to $j$
  - Also: points on a line
  - And: all nodes have the same energy
- The network has treewidth three

Some non-approximability results also follow
Starting problem for proofs

- Transform from 3-PARTITION: given positive integers $a_1, \ldots, a_{3m}, B$, with $B_4 < a_i < B_2$ for all $i$, can we make $m$ disjoint groups of three $a_i$’s, each group of sum $B$?
- This problem is strongly NP-complete
From 3-partition to Max Flow WSNEC

- $m$ sources with $B$ energy
- $3m$ forwarders, that can send exactly one message to the sink ($E_{f_i} = e_{f_i}t = 1$)
- sending from source to forwarder $i$ costs $a_i$ energy
Only one source

- Standard flow techniques give e.g., equivalence for the case there is only one source
Points in the plane

- The proof can be modified such that nodes are points in $\mathbb{R}^2$.
- For each pair of nodes, the energy to send from $i$ to $j$ is the square of the distance from $i$ to $j$. 
Hardness proof for points in the plane

1. Place $m$ source nodes on the same position here, with $B$ energy.
2. The distance from a source node to a node $f_i$ is $\sqrt{a_i}$.
3. Each $f_i$ has precisely the energy to send one message to $g_i$.
4. Only $g_i$'s can reach $t$.
2D: Proof details

- If we can send $3m$ messages to $t$, then each $g_i$ sends one message to $t$, so each $f_i$ sends one message to $g_i$, so each $f_i$ receives one message from a source. Hence: a solution to 3-partition problem.

- However: not a correct transformation as we use coordinate values like $\sqrt{a_i}$.

- What works: round all values down to multiple of $\epsilon$, for $\epsilon = 1/\Theta(m^2B^2)$.

- Or, equivalently, take coordinate-values $\lfloor mB\sqrt{a_i} \rfloor$ and adjust energies accordingly.

- Note: in these instances, each node has same energy.
1D

- Still NP-complete if all nodes on a line
- Put $t$ right of all $g_i$'s and give each $g_i$ just enough energy to send one message to $t$
1D with the same energy

- By repeating pattern, we can give all nodes the same energy.
- Induction shows that in $i$th copy, $i$ nodes can forward only one message.
Nonapproximability

- Strong NP-hardness implies that there is no FPTAS, unless $P=NP$
- A stronger result can be obtained
- Two power settings: for each node $i$, there are only two possible values for energies $e_{ij}$ (if $ij \in A$)

Theorem

*Even if for each node there are two power settings, there is no PTAS for the Integer Max Flow WSNEC problem, unless $P = NP$.***
On the no-PTAS-proof

**2-size 3-capacity generalized assignment problem (2GAP-3)**

**Instance:** A set $B$ of $m$ bins and a set $S$ of $n$ items. Each bin $j$ has capacity $c(j) = 3$ and for each item $i \in S$ and bin $j \in B$, we are given a size $s(i, j) = 1$ or $s(i, j) = 1 + \epsilon$ (for some $\epsilon > 0$) and a profit $p(i, j) = 1$.

**Objective:** Find a subset $U \subseteq S$ of maximum profit such that $U$ has a feasible packing in $B$.

**Theorem (Chekuri and Khanna, 2005)**

2GAP-3 is APX-hard and hence has no PTAS, unless $P = NP$.

- Construct similar as before
Treewidth

- Problem is solvable on trees
- Problem is solvable on graphs of treewidth two, if there is one source, and treewidth remains two if we add an edge from source to sink
- Problem is pseudopolynomial time solvable if treewidth bounded by constant
- Problem is weak NP-hard for treewidth three
Conclusions

- Wireless sensor networks: data gathering and not computing
- Problem is “Beyond Turing”, but algorithmic analysis uses classic “Turing-type” techniques