Approximating the maximum feasible subsystem problem

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Given a matrix $A \in \mathbb{R}^{m \times n}$ and a vector $b \in \mathbb{R}^m$, the *maximum feasible subsystem* (MFS) problem is to find a maximum-size collection of (in)equalities of the system

$$Ax \diamond b, \tag{1}$$

where $\diamond$ is an operator in $\{=, \leq, <\}$. The problem is NP-hard for any choice of $\diamond$ and is even NP-hard to approximate within a factor m^ϵ (for some $\epsilon > 0$) if all constraints are equalities. For inequality constraints, it can be approximated within a factor of 2 using a simple greedy algorithm.

We consider a variant in which A is a zero-one matrix and constraints are of the form

$$l_i \leq a_i^T x \leq u_i.$$

Note that if A would be a real matrix, then this variant includes the MFS problem with equality and inequality constraints. We study the complexity parameterized by $\max_i\{u_i\}$. In addition, we give a quasi-polynomial time approximation scheme for the case that A is an interval matrix.

Next, we apply our theory to the problem of pricing a collection of items so as to maximize the seller's profit.

This is joint work with Khaled Elbassioni