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Title: Model Order Reduction for Coupled Systems using Low-rank Approximations

The subject of the research are model order reduction (MOR) methods, that can be applied to coupled systems. The goal is to develop reduction techniques, that preserve special properties of coupled or interconnected system, e.g. block-structure of the underlying matrices. On the other hand, new techniques should also be able to benefit from the knowledge, that the system they are applied to, consists of two (or more) sub-systems or describes some phenomena in different physical domains. As a result of this study, two main approaches are proposed. Their general description is given in the following paragraphs.

First, the Separate Bases Reduction (SBR) algorithm is developed, which is a projection based MOR technique that uses Krylov subspaces as reduction bases. The novelty of this method is that SBR algorithm, unlike standard reduction methods designed for coupled problems, uses an uncoupled formulation of the system. In other words, an appropriate Krylov subspace is built for each of the sub-system constituting the interconnected system. As a result, the computational costs of application of the SBR algorithm, with respect to time and memory storage needed for calculations, is lower than in case of MOR methods that use the coupled formulation of the system. Moreover, the block-diagonal form of the reduction matrices allows for preservation of the block-structure of the system matrices and keeps the sub-systems (or different physical domains) still recognizable in the reduced-order model. The SBR algorithm was successfully applied to a few test cases, resulting in the reduced systems that approximate the original ones with accuracy comparable to the accuracy of systems reduced by means of other block-structure preserving MOR methods.

The second topic of the research focuses on the couplings between the sub-systems. Here, the off-diagonal blocks of the system matrices that correspond to the couplings, are approximated by matrices of lower rank. As a main tool, generalized singular value decomposition (GSVD) is used, which allows to find the most important components of a coupling block with respect to one of the sub-systems. Although this method does not reduce the dimension of the considered problem, it gives benefits if used before application of a MOR technique. First of all, the use of low-rank approximations of the coupling blocks can decrease the computational costs of the the Krylov subspaces construction needed for reduction. If the couplings can be approximated by sufficiently low-rank blocks, the necessary matrix inverse calculation can be performed cheaper, by application of the Sherman-Morrison formula. Moreover, the undesired growth of the reduction bases, in case of use of the SBR algorithm to sub-systems with many inputs (outputs), can be lowered by use of only dominant components of the input (output) space. The conducted experiments showed, that for some cases, the number of the components used to define the couplings can be significantly reduced.