

# MasterMath: Representation Theory

Week 1 - September 7<sup>th</sup> 2010

Choose your exercises to hand in from the following list. You can give a hard copy of your work to Shona Yu, or email it in digital form to [s.h.yu@tue.nl](mailto:s.h.yu@tue.nl).  
Deadline for submitting your homework is Tuesday, September 21.

1. Give examples of Sylow  $p$ -subgroups of the symmetric groups  $\mathfrak{S}_5$  and  $\mathfrak{S}_6$ , for all primes  $p$ .
2. Show that  $\mathfrak{S}_9$  has a Sylow 3-subgroup isomorphic to the semidirect product  $C_3 \rtimes (C_3 \times C_3 \times C_3)$ .
3. Prove that the alternating group  $A_5$  is simple.
4. Show that there are *no* simple groups of order 28, 39 and 132, without using Burnside's Theorem.
5. Prove that any group of order  $1295 = 5 \times 7 \times 37$  is abelian.
6. If  $p, q, r$  are primes, show that all groups of order  $p^m$ ,  $p^m q$ ,  $p^2 q^2$  and  $pqr$  are solvable (and hence not simple), without using Burnside's Theorem.