Problem #101

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Summary: Are universality and inclusion of AC-recognizable languages decidable?

An AC-tree automaton as defined by [Ohs01] is given by a signature $\Sigma$, a set of AC-axioms (that is, associativity and commutativity) for some function symbols of $\Sigma$, and a set of rewrite rules $R$ of the form

\begin{align*}
f(q_1, \ldots, q_n) & \rightarrow q \\
f(q_1, \ldots, q_n) & \rightarrow f(p_1, \ldots, p_n) \\
q & \rightarrow p
\end{align*}

where the $q$'s and $p$'s are state symbols. Such an automaton accepts a term $t$ iff it rewrites $t$ modulo the given AC-axioms to some final state. $L(A)$ denotes the language recognized by an AC-tree automaton $A$; a language $L$ is called AC-recognizable iff $L = L(A)$ for some AC-tree automaton $A$.

Are the following questions decidable?

- **Universality:** Given an AC-tree automaton $A$, is $L(A)$ equal to the set of all ground terms over the given signature $\Sigma$?

- **Inclusion:** Given AC-tree automata $A$ and $B$, is $L(A)$ a subset of $L(B)$?

It has been shown [OT02] that emptiness of AC-recognizable languages is decidable. Furthermore, as a consequence of the results of [ZL03], universality and inclusion are decidable if transition rules of the form $f(q_1, \ldots, q_n) \rightarrow f(p_1, \ldots, p_n)$ are not allowed (this is the sub-class of so-called *regular AC tree-automata*). However, both questions are still open in the general case.

**Remark**

The inclusion problem of AC-tree automata is undecidable [OTTR05]. Decidability of universality is still an open question.

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Bibliography


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