Problem #13

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Summary: Give decidable criteria for left-linear rewriting systems to be Church-Rosser.

By a lemma of Gérard Huet [Hue80], left-linear term-rewriting systems are confluent if, for every critical pair \( t \approx s \) (where \( t = u[r\sigma] \leftarrow u[l\sigma] = g\tau \rightarrow d\tau = s \), for some rules \( l \rightarrow r \) and \( g \rightarrow d \)), we have \( t \rightarrow^\parallel s \) (\( t \) reduces in one parallel step to \( s \)). (The condition \( t \rightarrow^\parallel s \) can be relaxed to \( t \rightarrow^\parallel r \leftarrow^\parallel s \) for some \( r \) when the critical pair is generated from two rules overlapping at the roots; see [Toy88].) What if \( s \rightarrow^\parallel t \) for every critical pair \( t \approx s \)?

What if for every \( t \approx s \) we have \( s \rightarrow^= t \)? (Here \( \rightarrow^= \) is the reflexive closure of \( \rightarrow \).) What if for every critical pair \( t \approx s \), either \( s \rightarrow^= t \) or \( t \rightarrow^= s \)? In the last case, especially, a confluence proof would be interesting: one would then have confluence after critical-pair completion without regard for termination. If these conditions are insufficient, the counterexamples will have to be (besides left-linear) non-right-linear, non-terminating, and non-orthogonal (have critical pairs). See [Klo92].

Remark

Significant progress is reported in [OO97].

A new criterion based on so-called simultaneous critical pairs has been presented in [Oku98].

The history of the problem and the attempts to solve it are told in [Der05].
Bibliography


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