Problem #25 (Solved !)

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Summary: Is the $\Sigma_2$-fragment of the first-order theory of ground terms modulo AC decidable?

Consider a finite set of function symbols containing at least one AC (associative-commutative) function symbol. Let $T$ be the corresponding set of terms (modulo the AC properties). It is known from [Tre92] that the first-order theory ($\Sigma_3$ fragment) of $T$ is undecidable when $F$ contains at least a non-constant symbol (besides the AC symbol). When $F$ only contains an AC symbol and constants, the theory reduces to Presburger’s arithmetic and is hence decidable. On the other hand the $\Sigma_1$ fragment of $T$ is always decidable [Com93]. The decidability of the $\Sigma_2$ fragment of the theory of $T$ remains open.

Remark

Even more, the solvability of the following important particular case is open: given $t,t_1,\ldots,t_n \in T(F,X)$, is there an instance of $t$ which is not an instance of $t_1,\ldots,t_n$ modulo the AC axioms? This is known as complement problems modulo AC.

Several special cases have been solved [Fer93][LM93], and in unpublished work in progress.

The undecidability of the $\Sigma_2$-fragment of the first-order theory of ground terms modulo AC has been shown by [Mar99].
Bibliography


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