Problem #5

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*Date:* April 1991

**Summary:** Does surjective pairing conservatively extend \(\lambda\beta\eta\)-conversion?

Do the surjective pairing axioms

\[
D_1(Dxy) = x \\
D_2(Dxy) = y \\
D(D_1x)(D_2x) = x
\]

conservatively extend \(\lambda\beta\eta\)-conversion on pure untyped lambda terms? More generally, is surjective pairing always conservative, or do there exist lambda theories, or extensions of Combinatory Logic for that matter, for which conservative extension by surjective pairing fails? (Surjective pairing is conservative over the pure \(\lambda\beta\)-calculus; see [dV89]). Of course, there are lots of other \(\lambda\beta\), indeed \(\lambda\beta\eta\), theories where conservative extension holds, simply because the theory consists of the valid equations in some \(\lambda\) model in which surjective pairing functions exist, e.g., \(D_\infty\).

**Comment sent by Kristian Støvring**

*Date:* Tue, 22 Nov 2005 00:18:13 +0100

The problem has been solved with a positive answer [Stø05, Stø06]. The generalization to arbitrary lambda theories remains open.

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Bibliography

