DISTANCE-REGULAR GRAPHS WITH A SMALL NUMBER OF VERTICES COMPARED TO THE VALENCE

In this talk we study distance-regular graphs with a small number of vertices compared to the valency. We show that for a given $\alpha > 2$, there are finitely many distance-regular graphs $\Gamma$ with valency $k$, diameter $D \geq 3$ and $v$ vertices satisfying $v \leq \alpha k$ unless ($D = 3$ and $\Gamma$ is imprimitive) or ($D = 4$ and $\Gamma$ is antipodal and bipartite). As a consequence of this result, we also show that there are finitely many distance-regular graphs with valency $k \geq 3$, diameter $D \geq 3$ and $c_2 \geq \varepsilon k$ for a given $0 < \varepsilon < 1$ unless ($D = 3$ and $\Gamma$ is imprimitive) or ($D = 4$ and $\Gamma$ is antipodal and bipartite).