Abstract: Glimpses of $p$-adic Hodge Theory

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In arithmetic geometry, one of the principal aims is to study the absolute Galois group of a number field $F$ or, at least, the action of this group on representations coming from geometry. A good example is the $p$-adic Tate module of an elliptic curve $E$ defined over $F$. This action gives a lot of different informations: for example the reduction of the curve at various primes of $F$. The local behavior of $E$ at those primes changes considerably if a prime $p$ of $F$ divides - or not - $p$: the $p$-adic world turns out in the first case. A more general class of $p$-adic representations arising from algebraic geometry is given by the $p$-adic tale cohomology groups of a smooth and projective variety defined over a $p$-adic field $K$.

The goal of $p$-adic Hodge theory is to study and classify different classes of representation of the absolute Galois group $G_K$ of a $p$-adic field $K$. In this seminar I’ll present some motivational examples, giving particular emphasis to a theorem of Tate for Abelian Varieties over $p$-adic fields (as a particular case of the Hodge-Tate conjecture, proved by Faltings). Moreover, I’ll try to present some ingredients of the theory of Tate and Sen for the study of the category of $C$-representations of $G_K$, following Fontaine.