Circular choosability

Tobias Müller (EURANDOM)
(joint work with F. Havet, R.J. Kang and J.-S. Sereni)

31 January 2007

The circular chromatic number of a graph is defined as follows. A t-circular colouring of a graph $G$ is a map $c : V(G) \to S(t)$, where $S(t)$ denotes a circle of circumference $t$, such that the length of the arc between $c(v)$ and $c(w)$ is at least one whenever $vw \in E(G)$. The circular chromatic number $\chi_c(G)$ of $G$ is the infimum of all $t$ for which there is a $t$-circular colouring. It satisfies $\chi(G) = \lceil \chi_c(G) \rceil$, where $\chi$ denotes the chromatic number, and can thus be said to be a refinement of the ordinary chromatic number. The circular chromatic number has received considerable attention since its introduction by Vince in 1988.

More recently Mohar introduced a “list version” of the circular chromatic number, the circular choosability, denoted by $cch(G)$. I will give a survey of known results and open problems on circular choosability (after stating the definition properly of course). One question that I will address in particular is the following. Mohar asked for the value of

$$\tau := \sup\{cch(G) : G \text{ is planar}\},$$

stating that he thought it would be between 4 and 5. I will show that, on the contrary, it is between 6 and 8.