On local search and LP and SDP relaxations for k-set packing

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In this presentation we consider the k-set packing problem, where given a universe $\mathcal{U}$ and a collection $\mathcal{C}$ of subsets over $\mathcal{U}$, each of cardinality k, the goal is to find the largest collection of mutually disjoint subsets. This is a fundamental problem equivalent to hypergraph matching and strongly related to the maximum independent set problem (on $k+1$-claw free graphs).

We present a new bound on the integrality gap of a strengthened linear programming relaxation for the problem called the intersecting family LP. The previous bound was $\frac{k+1}{2}$ [2] and we improved it to $\frac{k}{3}+1+\varepsilon$. By the results of [2], this immediately implies that there exist a polynomially sized LP and SDP formulation for k-set packing with this bounds on its integrality gap.

We also give a rough sketch of a simplified proof of the main lemma of the currently best weighted $\frac{k+1}{2}$-approximation algorithm from [1]. The current proof is very clever but also very algebraic, and we found a simple observation that allows us to reveal more intuition on what is really happening behind the math.

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References

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[2] Yuk Hei Chan and Lap Chi Lau. On linear and semidefinite programming relaxations for hypergraph matching. In *Proceedings of the Twenty-First Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA '10, pages 1500–1511, Philadelphia, PA, USA, 2010. Society for Industrial and Applied Mathematics.