Cameron-Liebler line classes

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Cameron-Liebler line classes are sets of lines in $\mathrm{PG}(3, q)$ having many nice combinatorial properties; they were originally studied as generalizations of line orbits under the action of a collineation group with equally many orbits on points and lines. They can also be thought of as generalizations of line classes in symmetric tactical decompositions of $\mathrm{PG}(3, q)$. A Cameron-Liebler line class $\mathcal{L}$ shares precisely x lines with every spread of the space for some non-negative integer x, called the parameter of the set. There has been much recent work showing the nonexistence of line classes with small parameter.

We will discuss some of the known results on Cameron-Liebler line classes, and describe the known examples. We will also detail ongoing work in finding new examples of these line classes. A Cameron-Liebler line class with parameter x contains $x(q^2 + q + 1)$ elements. Using this fact, we aim to combine orbits of a cyclic group of order $q^2 + q + 1$ in various ways to construct new line sets. Examples have been found using computational matrix methods, and their description leads to an interesting algebraic question.