Geometrical problems linked to linear codes arising from incidence matrices

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The incidence matrices of finite geometrical structures can be used to construct linear codes.

In particular, the p-ary linear codes defined by the incidence matrices of the Desarguesian projective planes $\mathrm{PG}(2, q)$, $q = p^h$, p prime, $h \geq 1$, have received great attention. It is known that the minimum weight codewords have weight $q+1$ and are equal to the scalar multiples of the incidence vectors of the lines of $\mathrm{PG}(2, q)$ [1]. Regarding the second weight of these linear codes, the difference of the incidence vectors of two lines is a codeword of weight $2q$. We prove that $2q$ effectively is the second weight of these linear codes by eliminating the possibility of codewords with weight in the interval $[q + 2, 2q - 1]$ [2].

The incidence matrix of the generalized quadrangle $Q(4, q)$, q even, can be used as a parity check matrix for a binary linear code. It is known that the largest weight of this linear code is equal to $q^3 + q$, and the codewords of largest weight correspond to the ovoids of $Q(4, q)$. So here, the question arises what is the second largest weight. We prove that there are no codewords in this linear code with weight in the interval $]q^3 + \frac{5q-4}{6}, q^3 + q - 1]$ [3].

References

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