Optimal blocking multisets

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(Ghent University; joint work with I. Landjev)

Definition 1. *An (f, m)-minihyper in* $\mathrm{PG}(t, q)$ *is an m-fold blocking multiset of size f, i.e. a multiset $\mathfrak{F}$ of f points in* $\mathrm{PG}(t, q)$ *s. t. every hyperplane contains at least m of these points.*

A natural question here would be: for given m, what is the least number f such that there exists an (f, m)-minihyper? For proper multisets, one has $f \geq \frac{v_t}{v_{t-1}} m$, with $v_i = \frac{q^i - 1}{q - 1}$.

Definition 2. *In* $\mathrm{PG}(t, q)$*, an optimal blocking multiset is an (xv_t, xv_{t-1})-minihyper, for some $x \in \mathbb{N}$.*

Hence, these parameters are a very particular choice of minihypers, and many examples of them are known. The study on minihypers was originally started by Hamada in the context of linear codes meeting the Griesmer bound [2], and coincidently, it turns out that such codes are most highly divisible when it minihyper's parameters are of the form (xv_t, xv_{t-1}).

When two different problems point to the same structure, it is no surprise that they have been studied before [1, 4, 5]. This study was however always performed from a purely combinatorial point of view.

Starting from a theorem of Landjev and Storme [5], I will present a simple natural problem in linear algebra, which turns out to have exactly these optimal blocking multisets as its set of solutions. So we now have 3 different natural problems pointing to this very same structure.

Using this new characterization, we could greatly extend and improve upon most known results on this structure [3, 4, 5]. In this talk, I will attempt to give some insight in this characterization and its strengths, to convince the audience that these optimal blocking multisets are very special combinatorial structures and that they are also an example of structures where purely combinatorial methods are really not the proper tool for the job.

References

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