We present a simple method for power line reduction in electrocardiographic (ECG) recordings based on adaptive interference cancellation. The method can track amplitude and phase of the interference signal, while a reference power line recording is not needed. Its behavior at line frequency deviations is analyzed and validated experimentally at several adaptation time constants. Performance is assessed in terms of signal to interference ratio of the cleaned ECG signal. The method performs adequately up to power line frequency deviations of 0.2%.

INTRODUCTION

A proper recording environment does not always avoid power line interference in ECG recordings sufficiently when a high quality analysis is to be made. The worst case power line amplitude that can be expected is as large as 1/4 times the peak-to-peak QRS amplitude [1]. This is equivalent to an input signal to interference ratio (SIR_{in}) of around 0 dB, where SIR_{in} is the ratio between ECG signal power \( P_s \) and line interference power \( P_i \), i.e. \( \text{SIR}_{\text{in}} = \frac{P_s}{P_i} \). Furthermore, the power line frequency can vary fractions of a Hertz, or even a few Hertz in some countries [2]. Often the power line signal contains harmonics. To reduce the power line interference after the recording stage one has to use signal processing techniques. In most cases an interference amplitude of less than 1/200 times peak-to-peak QRS amplitude is acceptable [3], corresponding to a signal to interference ratio (SIR) of around 30 dB. An appropriate power line interference reduction technique should therefore be able to remove line interference and harmonics with a varying line amplitude and deviating line frequency, thereby achieving an SIR of at least 30 dB.

Current techniques that have been proposed include notch filtering and adaptive interference cancellation. A disadvantage of notch filtering is that it induces a transient response at QRS pulses or removes ECG signal components. Adaptive interference cancellation (ANC) [4] implies the use of an adaptive filter operating on a reference interference signal to produce an estimate of the interference in the

---

1Technische Universiteit Eindhoven, Eindhoven, the Netherlands
2Maxima Medisch Centrum, Veldhoven, the Netherlands
corrupted signal. Widrow [4] mentioned the use of a two-weight adaptive filter acting on an external reference line signal for reduction of the power line in ECG recordings. Ziarani [2] introduced a more complex ANC method. This method uses an internal reference signal and can track not only line amplitude and phase but also deviating line frequency.

In this paper we elaborate, extend and analyze the simple ANC method by Widrow. The external reference signal is replaced by an internal signal. An error filter improves performance. We first present the system model and the adaptive filter structure. What follows is an analysis of the behavior of the system in response to line frequency deviations and presence of ECG signal. We neglect the presence of power line harmonics in this paper.

**SYSTEM MODEL AND NOMENCLATURE**

Fig. 1 shows the system model for reduction of line interference in ECG signals. A corrupted ECG signal \(d(k)\) may to a first order approximation be expressed as \(d(k) = s(k) + i(k)\), where \(s(k)\) is the clean ECG signal and \(i(k)\) represents the power line interference signal. When neglecting harmonics, \(i(k)\) can be defined as \(i(k) = A_i(k) \sin(\omega_i k + \varphi_i)\), with \(A_i(k)\), \(\omega_i\) and \(\varphi_i\) the power line amplitude, normalized frequency in radians per sampling interval and phase in radians, respectively. The power line frequency \(\omega_i\) may not be fixed. To account for this we write \(\omega_i = \omega_i^n + d\omega_i\), \(\omega_i^n\) being the nominal frequency and \(d\omega_i\) a small frequency deviation. In that case we can rewrite \(i(k)\) as

\[
i(k) = w_1(k) \sin(\omega_i^n k) + w_2(k) \cos(\omega_i^n k),
\]

(1)

with

\[
\{w_1(k), w_2(k)\} = \{A_i(k) \cos(d\omega_i k + \varphi_i), A_i(k) \sin(d\omega_i k + \varphi_i)\}.
\]

(2)

Signal \(r(k)\) serves as a reference signal for interference \(i(k)\). Instead of an external reference signal, we use the synthetic signal \(r(k) = \sin(\omega_i^n k)\). This reference signal is passed through a transversal filter containing filter weights \(\tilde{w}_1\) and \(\tilde{w}_2\). The
filter inputs $y_1(k)$ and $y_2(k)$ are equal to $y_1(k) = \sin(\omega_i^n k)$ and $y_2(k) = \cos(\omega_i^n k)$. Signal $x(k)$ is in that case $x(k) = \hat{w}_1 y_1 + \hat{w}_2 y_2 = \hat{w}_1 \sin(\omega_i^n k) + \hat{w}_2 \cos(\omega_i^n k)$. The filter weights $\hat{w}_1$ and $\hat{w}_2$ serve as estimates of $w_1$ and $w_2$, respectively. The error signal $e(k)$, which is the difference between $d(k)$ and $x(k)$, is the input for the adaptive process (block 'LMS' in Fig. 1).

**ADAPTIVE INTERFERENCE CANCELLATION**

Let us denote by $z(k)$ the residual power line signal in $e(k)$, i.e. $z(k) = i(k) - x(k)$. We can re-express $z(k)$ in terms of misadjustments $\Delta_1(k) = w_1 - \hat{w}_1(k)$ and $\Delta_2(k) = w_2 - \hat{w}_2(k)$:

$$z(k) = \Delta_1(k) \sin(\omega_i^n k) + \Delta_2(k) \cos(\omega_i^n k).$$

(3)

When both $\Delta_1(k)$ and $\Delta_2(k)$ are zero, $z(k)$ vanishes and $e(k)$ only contains the signal of interest $s(k)$, i.e. $e(k) = s(k)$. In other cases $e(k)$ also contains residual power line interference, i.e. $e(k) = s(k) + z(k)$. By minimizing the mean squared error $E[e(k)^2]$ the error signal converges to the best least-squares estimate of signal $s(k)$ for the given filter structure and reference signal. An iterative way for finding the least mean squares error solution for the weights is the LMS algorithm:

$$\hat{w}_m(k + 1) = \hat{w}_m(k) + \mu \eta_m(k) \quad m \in \{1, 2\},$$

(4)

where $\mu$ is the constant that regulates the convergence rate. In this equation $\eta_m(k)$ is a noisy estimate of the gradient $-\partial E[e(k)^2]/\partial \hat{w}_m$:

$$\eta_m(k) \triangleq - \frac{\partial e(k)^2}{\partial \hat{w}_m} = -2 e(k) \frac{\partial e(k)}{\partial \hat{w}_m} = 2 e(k) y_m(k).$$

(5)

**PERFORMANCE MEASURE**

As a performance measure of the adaptive noise cancellation scheme we adopt the output SIR (SIR$_{out}$), defined as the ratio between ECG signal power $P_s$ and residual line interference power $P_z$ in the error signal $e(k)$, i.e.

$$\text{SIR}_{out} = \frac{P_s}{P_z}.$$

(6)

If the misadjustments $\Delta_m(k)$ in (3) are constant or fluctuate at a frequency that is much smaller than $\omega_i^n$, $P_z$ in (6) can be expressed in terms of misadjustment powers $\sigma_{\Delta_1}$ and $\sigma_{\Delta_2}$:

$$P_z \approx \frac{1}{2} \sigma_{\Delta_1}^2 + \frac{1}{2} \sigma_{\Delta_2}^2.$$

(7)
DYNAMIC BEHAVIOR

We will now analyze the dynamic behavior of the adaptive system of Fig. 1. As this behavior does not depend on $s(k)$, we neglect $s(k)$ for the moment. In that case $e(k) = z(k)$ and (5) can be written as

$$\eta_m(k) = 2z(k)y_m(k) \quad m \in \{1, 2\}. \quad (8)$$

For the system behavior we are interested in the average values of the gradient estimates, i.e.

$$\bar{\eta}_1 = E \{2\Delta_1(k)\sin^2(\omega k) + 2\Delta_2(k)\cos(\omega k)\sin(\omega k)\} = \Delta_1$$

and

$$\bar{\eta}_2 = E \{2\Delta_1(k)\sin(\omega k)\cos(\omega k) + 2\Delta_2(k)\cos^2(\omega k)\} = \Delta_2.$$  

These equations show that $\bar{\eta}_1$ does not respond to misadjustment $\Delta_2$ and $\bar{\eta}_2$ not to misadjustment $\Delta_1$, i.e. the loops for $\hat{w}_1$ and $\hat{w}_2$ are orthogonal. Now the loop behavior can be approximated as

$$\hat{w}_m(k + 1) \approx \hat{w}_m(k) + \mu \Delta_m(k). \quad (9)$$

Fig. 2 shows the corresponding first-order loop model for $w_m$.

![Figure 2: First-order approximation of loop model for the weights.](image)

According to this figure, misadjustment $\Delta_m(k)$, the difference between $w_m(k)$ and $\hat{w}_m(k)$, is scaled by a gain factor $\mu$ and integrated in order to produce $\hat{w}_m(k)$. The loop forces a time-average of $\Delta_m$ towards zero. The loop can be characterized by means of transfer function $H_{\Delta_m}(z) = \Delta_m(z)/W_m(z) = (z - 1)/(z - 1 + \mu)$, where capitals denote $z$ transforms. This leads to the following time constant for the weights:

$$\tau = \frac{1}{\ln\left(\frac{1}{1 - \mu}\right)} \approx \frac{1}{\mu}, \quad (10)$$

where $\tau$ is the normalized time constant in sampling intervals and $0 < \mu < 1$. The approximation applies for $\mu \ll 1$, the case of greatest practical interest. The time constant of adaptation for the weights is thus independent of the amplitudes of both the ECG signal and power line interference, which is desirable.

We will now analyze the impact of a small line frequency deviation $d\omega_i$. We assume that the line amplitude $A_i$ is constant, so that $P_i = \frac{1}{2}A_i^2$ and SIR$_{in} = P_i/\frac{1}{2}A_i^2$. Without loss of generality we take $\varphi = 0$ and (2) changes into \{$w_1(k), w_2(k)$\} = \{\cos(d\omega_i k), \sin(d\omega_i k)\}. The misadjustments $\Delta_1$ and $\Delta_2$ will vary sinusoidally with frequency $d\omega_i$ and amplitude $A_i|H_{\Delta_m}(e^{j\omega_i})|$. This yields misad-
justment powers $\sigma^2_{\Delta_m} = \frac{1}{2} A^2 |H_{\Delta_m}(e^{j\omega})|^2$. Using this in (7) the output SIR becomes $\text{SIR}_{\text{out}} \simeq \{(\mu^2 + 4(1 - \mu) \sin^2(\frac{d\omega_i}{2})/(4 \sin^2(\frac{d\omega_i}{2}))\} \text{SIR}_{\text{in}}$. By assuming that $\{d\omega_i, \mu\} \ll 1$ and using (10) this expression can be simplified to

$$\text{SIR}_{\text{out}} \simeq \left( \frac{1}{(\tau d\omega_i)^2} + 1 \right) \text{SIR}_{\text{in}}$$

(11)

Performance apparently depends on the amplitudes of the ECG signal and the power line interference. For $\text{SIR}_{\text{out}}$ to be much larger than $\text{SIR}_{\text{in}}$ we must dimension the loop such that $\tau \ll \frac{1}{d\omega_i}$. In that case (11) can be rewritten as $\text{SIR}_{\text{out}} \simeq \frac{1}{(\tau d\omega_i)^2} \text{SIR}_{\text{in}}$, i.e. decreasing $\tau$ by a factor of 10 yields a 20 dB $\text{SIR}_{\text{out}}$ improvement.

**GRADIENT NOISE**

We will now analyze the effect of the ECG signal on the performance of our method. The ECG signal appears as an extra term $\xi_m(k) \triangleq 2s(k)y_m(k)$ in (8). Fig. 3 shows the first-order loop model for $\tilde{w}_m$ in presence of $\xi_m$.

![Figure 3: First-order approximation of loop model for the weights in the presence of ECG.](image)

Note that this figure equals Fig. 2 except for the term $\xi_m(k)$. This term corrupts misadjustment $\Delta_m$ and causes fluctuations of the weights around the optimal value, called weight gradient noise. This noise manifests itself in that $\Delta_m(k)$ fluctuates slowly about zero with a power $\sigma^2_{\Delta_m}$, causing $\text{SIR}_{\text{out}}$ to decrease (see (7) and (6)). In case $\xi_m(k)$ is colored, a whitening procedure can decrease gradient noise. For this purpose we can add an error filter that reduces correlation of $s(k)$ and thus of $\xi_m(k)$. Let us denote by $F(z)$ the transfer function of the error filter. Now $y_m$ in (5) should be filtered accordingly. This error filter should not affect the power of frequency components in the frequency band of $i(k)$ (i.e. $|F(e^{j\omega})| = 1$) in order to have no impact on the dynamic behavior of the scheme.

Fig. 4 shows the results of a simulation with a synthetic ECG and power line interference signal. In this figure the measured $\text{SIR}_{\text{out}}$ is rendered as a function of the cut-off frequency $f_c$ when a first order recursive high pass filter is used as error filter. Note that by using an error filter about 10 dB of $\text{SIR}_{\text{out}}$ can be gained. The optimal cut-off frequency is about 30 Hz, although the precise cut-off frequency is not very critical.
Let us now try to estimate $SIR_{\text{out}}$. The power of the weight gradient noise $\sigma_{\beta m}^2$ can be expressed as:

$$\sigma_{\beta m}^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_\xi(e^{j\omega})|H_{\beta m}(e^{j\omega})|^2 d\omega \quad m \in \{1, 2\},$$  

(12)

where $P_\xi(e^{j\omega})$ is the power spectral density of $\xi_m(k)$. We denote by $s_F(k)$ the filtered ECG signal. By using the fact that $s_F(k)$ and $y_m(k)$ are statistically independent and that $E[y_m^2] = \frac{1}{2}$, $P_\xi(e^{j\omega})$ can be expressed as $P_\xi(e^{j\omega}) \simeq 2P_{sF}$, where $P_{sF}$ is the power of the filtered ECG signal. In case $s_F(k)$ is perfectly white (12) changes into $\sigma_{\beta m}^2 = 2\frac{\mu}{2-\mu}P_{sF}$. The same power is present in present in $\Delta_m(k)$, i.e. $\sigma_{\Delta m}^2 = \sigma_{\beta m}^2$. An estimate for the steady state signal to interference ratio is then $SIR_{\text{out}} \simeq \left(1 - \frac{1}{\mu}\right)\frac{P_s}{P_{sF}}$. By assuming that $\mu \ll 2$ and using (10) this expression can be simplified to

$$SIR_{\text{out}} \simeq \tau \frac{P_s}{P_{sF}}.$$

(13)

This means that increasing $\tau$ by a factor of 100 yields a 20 dB $SIR_{\text{out}}$ improvement. Performance also depends on the characteristics of the error filter. Furthermore, $SIR_{\text{out}}$ is independent of the amplitudes of both the ECG signal and the power line interference, hence also independent of $SIR_{\text{in}}$.

**NUMERICAL RESULTS**

In this section we will validate our analytical results by means of simulations with synthetic signals with worst case situations (i.e. $SIR_{\text{in}} = 0$ dB). Two situations are considered: (1) a power line interference with a small frequency deviation and constant amplitude in the absence of an ECG signal and (2) a power line interference with constant amplitude and no frequency deviation but in the presence of an ECG signal. In simulation (2) an error filter with cut-off frequency of 30 Hz is used. Fig. 5 shows the performance in these two situations. Fig. 5(a) shows that the theoretical and simulation results agree very well in case of line frequency deviations. The larger the frequency deviation, the smaller $\tau$ should be. Fig. 5(b) shows the performance in presence of the ECG signal. The
use of an error filter improves performance, especially at small $\tau$. The simulation results for the situation with error filter are significantly better than the theoretical analysis predicts, except for the lowest time constants. The reason is that the analysis assumes that the ECG signal is perfectly white. However, a simple high pass filter can only whiten the complex ECG spectrum very coarsely. According to Fig. 5(b) a time constant of 20 sampling intervals is a safe choice for achieving an adequate performance, i.e. an $\text{SIR}_{\text{out}}$ of more than 30 dB. From Fig. 5(a) we derive that in that case the performance of our method is adequate up to relative line frequency deviations of about 0.2 % (see also Fig. 6).

**DISCUSSION**

In this paper, the adaptive line interference cancelling technique proposed by Widrow has been extended and optimized for ECG recordings. A simple high pass error filter with a cut-off frequency around 30 Hz improves performance especially at small time constants. This time constant $\tau$ is always a consideration between accuracy and tracking speed. Based on theoretical analysis and simulation results, we adopted a $\tau$ of 20 sampling intervals. In worst case situations, i.e. an $\text{SIR}_{\text{in}}$ of about 0 dB, the proposed technique can handle line frequency deviations of about 0.2 % (i.e 0.1 Hz in case the line frequency is 50 Hz), which is sufficient in the Netherlands [5]. In case of larger line frequency deviations one could switch to an external reference signal or to a more complex ANC technique (e.g. [2]).
Figure 6: Simulation with synthetic ECG and power line signal (including line frequency deviation); (a) shows the corrupted ECG signal, (b) its power spectrum, (c) the cleaned ECG signal in case $\Delta f = 0.2\% \ (\tau = 20)$ and (d) the cleaned ECG signal in case $\Delta f = 2\% \ (\tau = 20)$.

REFERENCES


