Correction to Theorem 12.1.1

In [BCN], Theorem 12.1.1 the existence of a certain association scheme is claimed, and details are given for \( n = 3 \). As Frédéric Vanhove (pers.comm., Sept. 2013) observed, things are slightly different for odd \( n \geq 5 \).

Let \( q \) be a power of 2, and \( n \geq 3 \). Let \( V \) be an \( n \)-dimensional vector space over \( \mathbb{F}_q \) provided with a nondegenerate quadratic form \( Q \). If \( n \) is odd, there will be a nucleus \( N = V^\perp \).

We construct an association scheme with point set \( X \), where \( X \) is the set of projective points not on the quadric \( Q \) and (for odd \( n \)) distinct from \( N \). For \( n = 3 \) and for even \( n \), the relations will be \( R_0, R_1, R_2, R_3 \) where

\[
R_0 = \{(x, x) \mid x \in X\}, \text{ the identity relation;}
R_1 = \{(x, y) \mid x + y \text{ is a hyperbolic line (secant)}\};
R_2 = \{(x, y) \mid x + y \text{ is an elliptic line (exterior line)}\};
R_3 = \{(x, y) \mid x + y \text{ is a tangent}\}.
\]

For odd \( n \), \( n \geq 5 \), it is necessary to distinguish \( R_{3a} \) and \( R_{3n} \), defined by

\[
R_{3a} = \{(x, y) \mid x + y \text{ is a tangent not on } N\};
R_{3n} = \{(x, y) \mid x + y \text{ is a tangent on } N\}.
\]

For \( q = 2 \) a hyperbolic line contains only one nonisotropic point, so that \( R_1 \) is empty.

**Theorem 12.1.1** (corrected)

(i) \((X, \{R_0, R_1, R_2, R_3\})\) is an association scheme for even \( n = 2m \geq 4 \). It has eigenmatrix

\[
P = \begin{pmatrix}
1 & \frac{1}{2}q^{m-1}(q^{m-1} + \varepsilon)(q-2) & \frac{1}{2}q^m(q^{m-1} - \varepsilon) & q^{2m-2} - 1 \\
1 & \frac{1}{2}q^{m-2}(q + 1)(q-2) & -\frac{1}{2}q^{m-1}(q-1) & \varepsilon q^{m-2} - 1 \\
1 & \varepsilon q^{m-1} & 0 & \varepsilon q^{m-1} - 1 \\
1 & -\varepsilon q^{m-1} & 0 & \varepsilon q^{m-1} - 1
\end{pmatrix}
\]

and multiplicities \( 1, q^2(q^{n-2} - 1)/(q^2 - 1), \frac{1}{2}q(q^{m-1} - \varepsilon)(q^m - \varepsilon)/(q + 1), \frac{1}{2}(q - 2)(q^{m-1} + \varepsilon)(q^m - \varepsilon)/(q - 1) \).

(ii) \((X, \{R_0, R_1, R_2, R_{3a}, R_{3n}\})\) is an association scheme for odd \( n = 2m+1 \geq 3 \). It has eigenmatrix

\[
P = \begin{pmatrix}
1 & \frac{1}{2}q^{2m-1}(q - 2) & \frac{1}{2}q^{2m} & q(q^{2m-2} - 1) & q - 2 \\
1 & \frac{1}{2}q^{m-1}(q - 2) & \frac{1}{2}q^m & -(q^{m-1} + 1)(q - 1) & q - 2 \\
1 & -\frac{1}{2}q^{m-1}(q - 2) & -\frac{1}{2}q^m & (q^{m-1} - 1)(q - 1) & q - 2 \\
1 & \frac{1}{2}q^m & -\frac{1}{2}q^m & 0 & -1 \\
1 & -\frac{1}{2}q^m & \frac{1}{2}q^m & 0 & -1
\end{pmatrix}
\]

and multiplicities \( 1, \frac{1}{2}q(q^{m+1})(q^{m-1} - 1)/(q - 1), \frac{1}{2}q(q^m - 1)(q^{m-1} + 1)/(q - 1), \frac{1}{2}(q - 2)(q^{2m} - 1)/(q - 1) \) (twice).

When \( n = 3 \), the relation \( R_{3a} \) is empty, and the second eigenspace is absent.